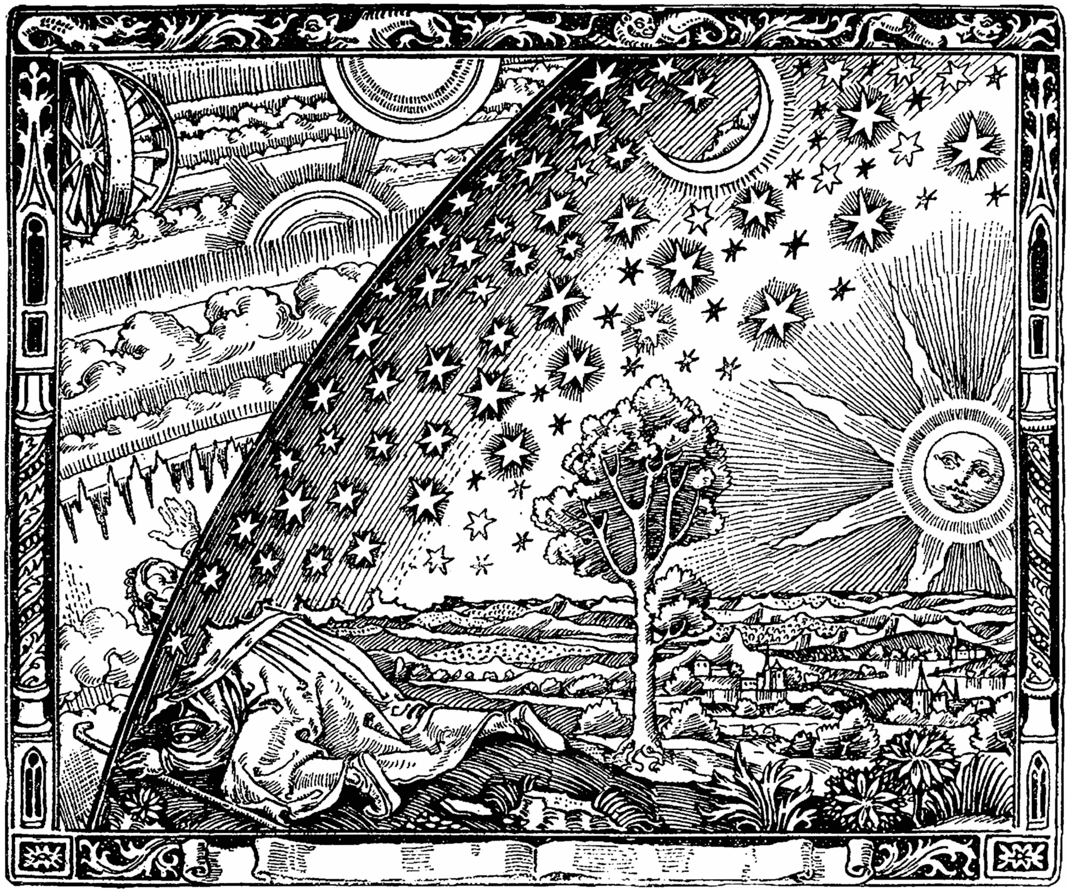
LIGER (LIght cones using GEneral Relativity) method



Flammarion engraving, Paris 1888

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CosmoBack, Marseille 28-31/05/2018

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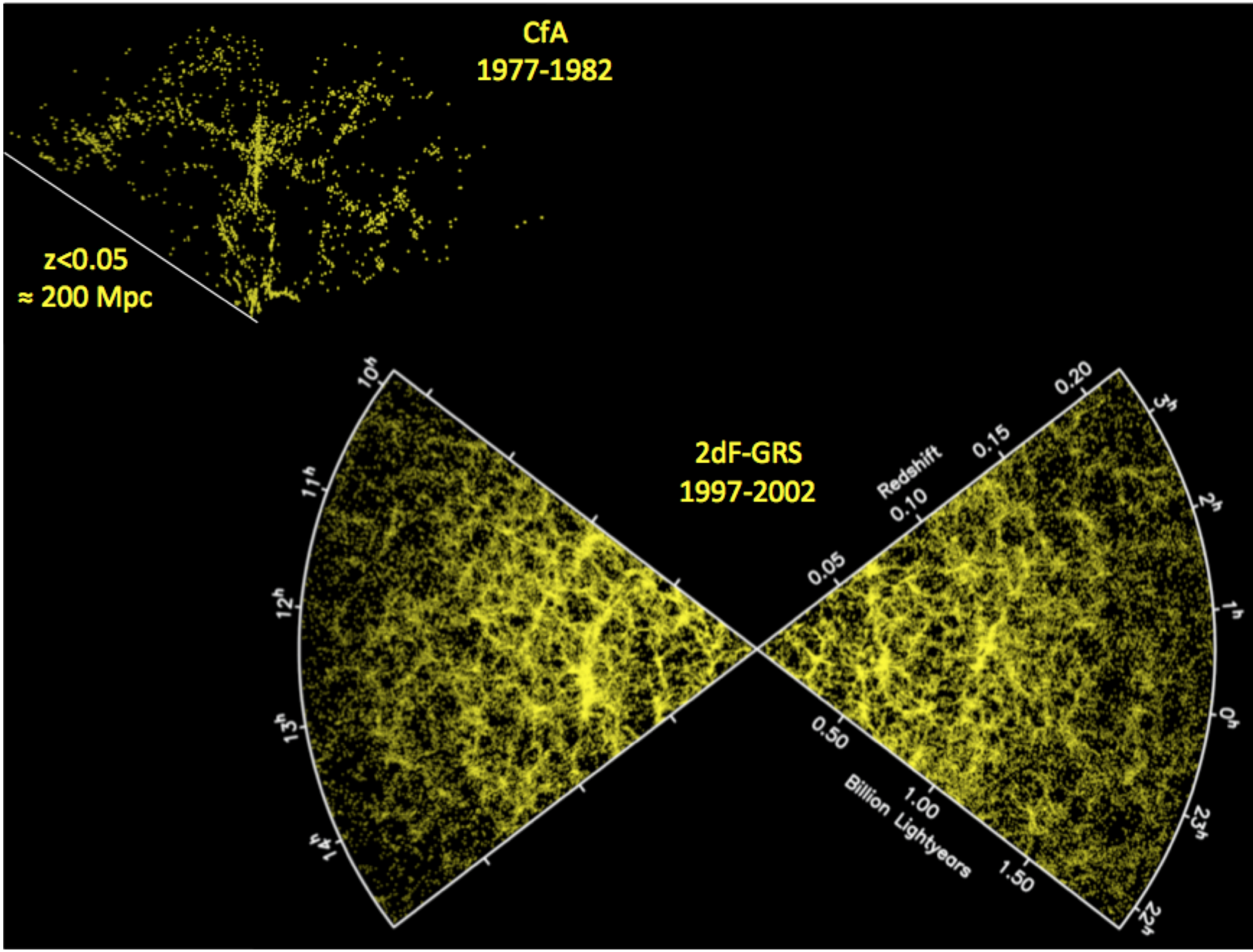
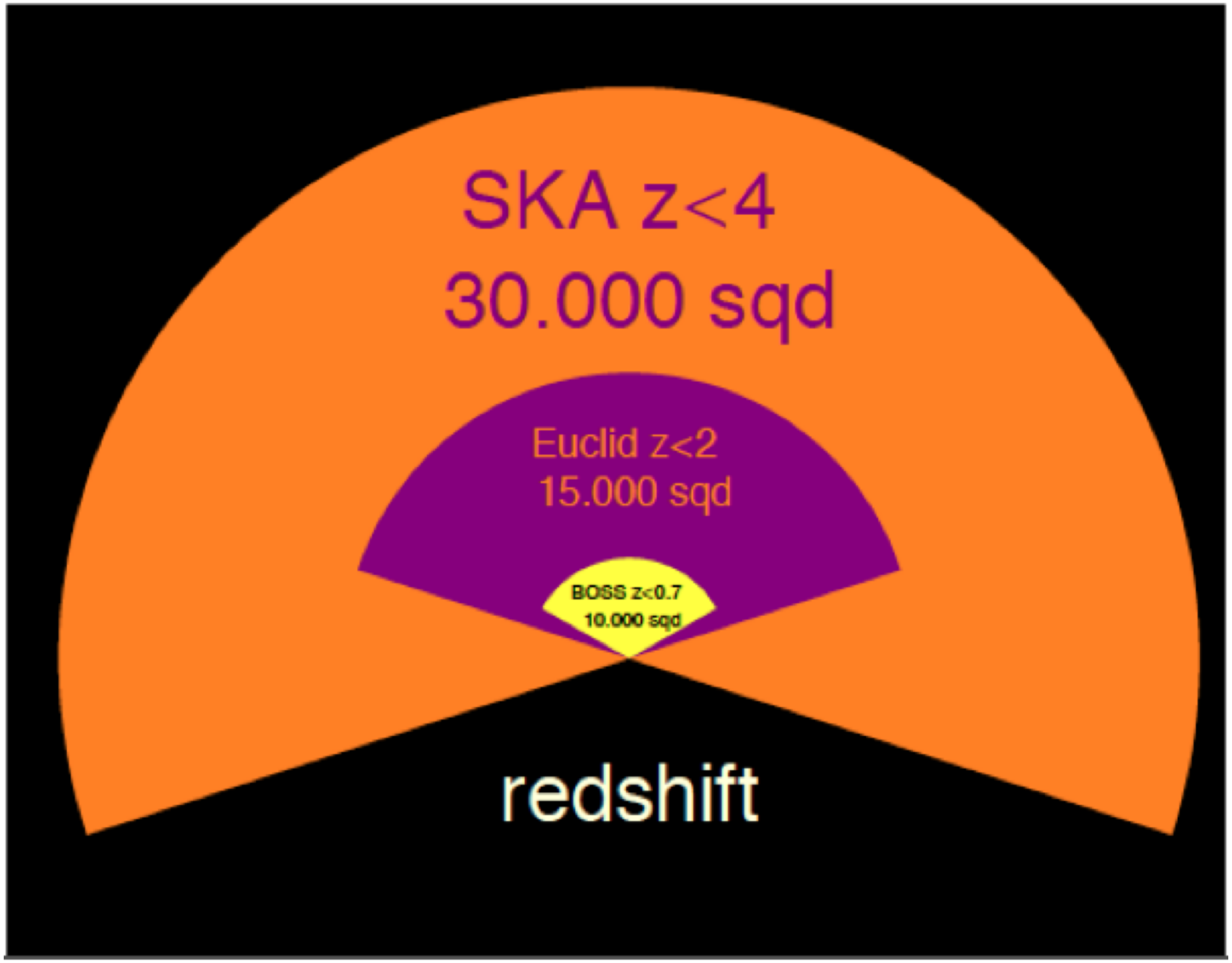
Dipartimento di Fisica e Astronomia Galileo Galilei
INFN
asi COSMOS



Since the 1970s, the size of galaxy catalogs has constantly increased in terms of solid-angle and redshift coverage as well as in sampling rate.

General relativistic effects:



Forthcoming surveys will cover large volumes of the observable Universe and will reach to high redshifts.

General relativistic effects:

- **On large scales, we need to ensure that we are using a correct general relativistic (GR) analysis.**

- **It is important to compute these effects:**

  - to avoid wrong predictions on scales ~ $1/H(z)$
  - to detect the Doppler terms
  - in order to extract the primordial non-Gaussianity
  - to compute GR corrections at 2^{nd} order
  - in order to provide the best constraints on dark energy and modified gravity models
  - to estimate the neutrino masses
  - to measure the spatial curvature parameter

General relativistic effects: Δ_g case

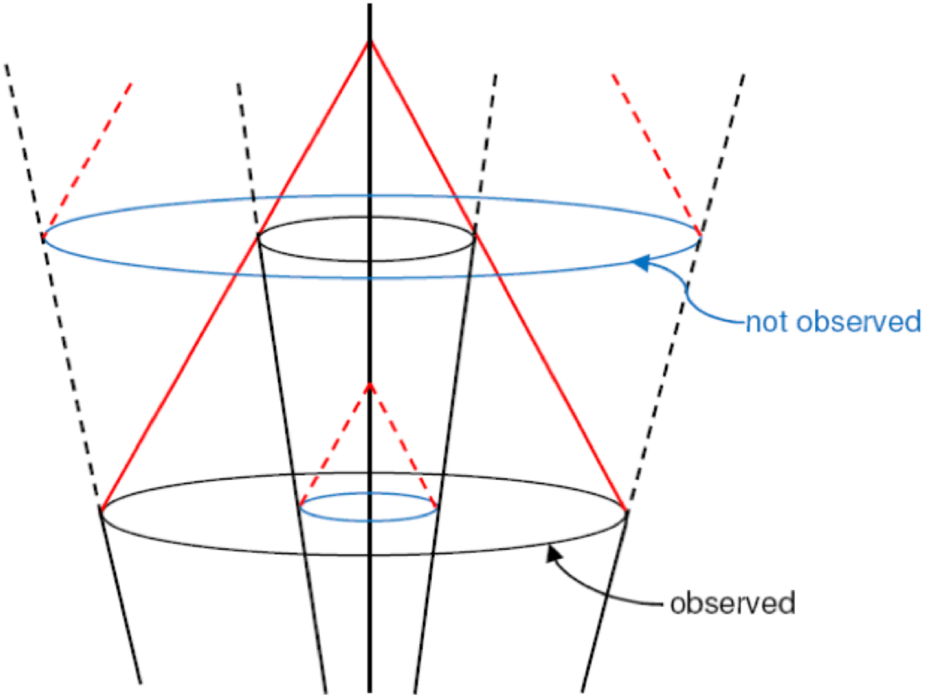
- There are two fundamental issues:

General relativistic effects: Δ_g case

- There are two fundamental issues:

1) Correctly identify the galaxy overdensity Δ_g that we observe on the past light cone.

   - it is unique
   - it is automatically gauge-invariant



General relativistic effects: Δ_g case

- There are two fundamental issues:

1) Correctly identify the galaxy overdensity Δ_g that we observe on the past light cone.

2) We need to account for all the distortions arising from observing on the past light cone:

Redshift, Magnification and Volume Distortions

see also Ruth talk

for example, see Yoo et al 2009, Bonvin et al 2011, Challinor et al 1011, Jeong et al 2011

Distortions have already been measured:

- Redshift space: the redshift is affected by galaxies velocity redshift-space distortions (Kaiser1987)
- Bias: the distribution of galaxies is a biased tracer.
- Magnification bias: gravitational lensing changes the solid angle and the threshold of observation (e.g. Broadhurst, Taylor and Peacock 1995)

Distortions have already been measured:

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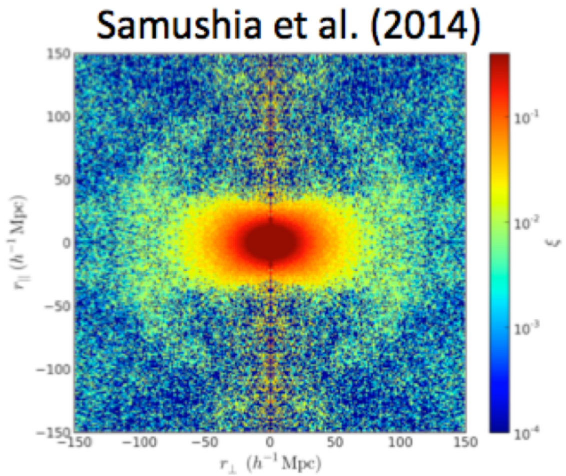
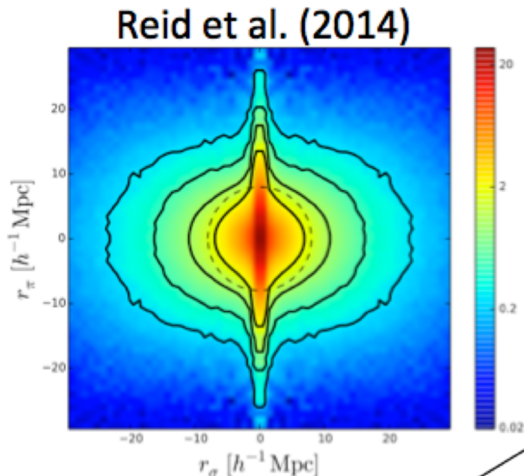
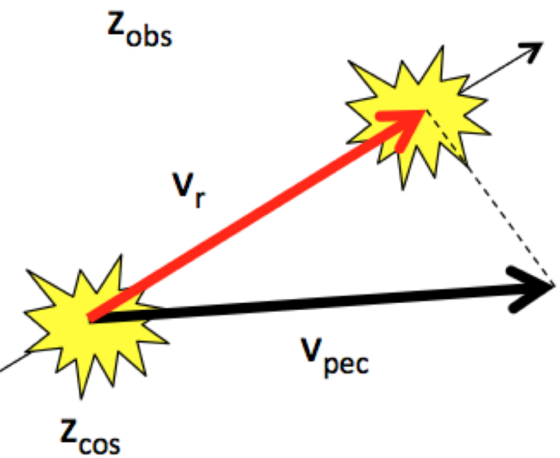
These contributions are added in ad hoc manner!

Is this everything? or are there more contributions? we need unified treatments!

Redshift-space distortions







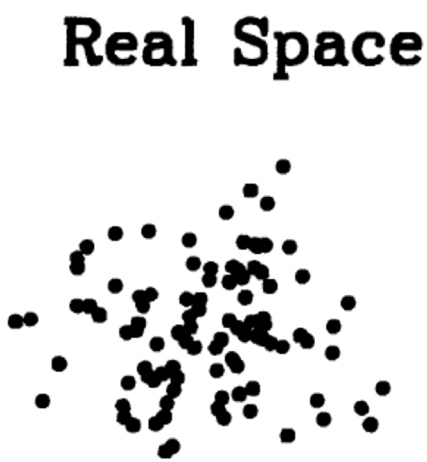
$$1 + z_{obs} = (1 + z_{\cos})(1 + \frac{v_r}{c})$$

Over-dense regions (eg galaxy clusters) and under-dense regions (eg voids) induce additional peculiar velocities relative to the Hubble flow.

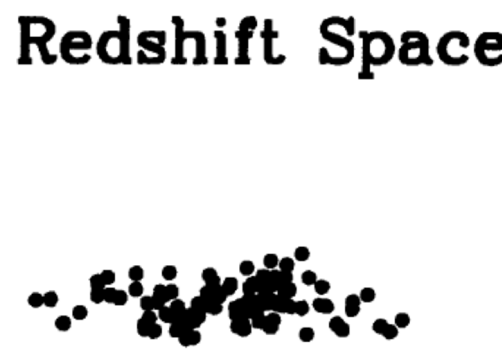
Credit: Cristiano Porciani

Redshift-space distortions

e.g. Kaiser 1987 , Hamilton 1997



$N^{\mathcal{R}}$



$N^{\mathcal{S}}$

$$N^{\mathcal{R}}(r)\, dr^3 = N^{\mathcal{S}}(s)\, ds^3$$

$\mathbf{s}(\mathbf{r}) = \mathbf{r} + \mathcal{v}_r(\boldsymbol{r})\, \hat{e}_r$ where $\mathcal{v}_r(\boldsymbol{r}) = \hat{e}_r \bullet \mathbf{v}/aH$

Redshift-space distortions

e.g. Kaiser 1987 , Hamilton 1997

Real Space

$N^{\mathcal{R}}$

Redshift Space

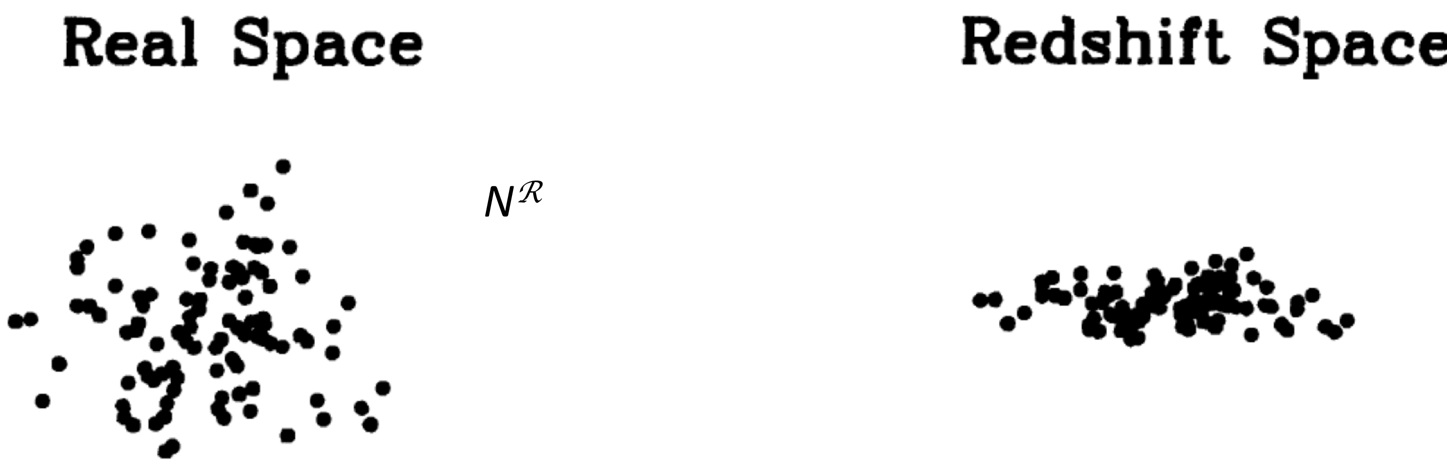
$N^{\mathcal{S}}$

$$1 + \delta^{\mathcal{S}}(\mathbf{s}) = [1 + \delta^{\mathcal{R}}(\mathbf{r})]\left(1 + \frac{\partial v_r}{\partial r}\right)^{-1}\left(1 + \frac{v_r}{r}\right)^{-2}\frac{\bar{N}(r)}{\bar{N}(r + v_r)}$$

$$\delta^{\mathcal{S}}(\mathbf{r}) = \delta^{\mathcal{R}}(\mathbf{r}) - \left(\frac{\partial v_r}{\partial r} + \frac{\alpha(\mathbf{r})v_r}{r}\right) \qquad \alpha(\mathbf{r}) = \frac{\partial \ln r^2 \bar{N}^s(\mathbf{r})}{\partial \ln r}$$

Redshift-space distortions

e.g. Kaiser 1987 , Hamilton 1997



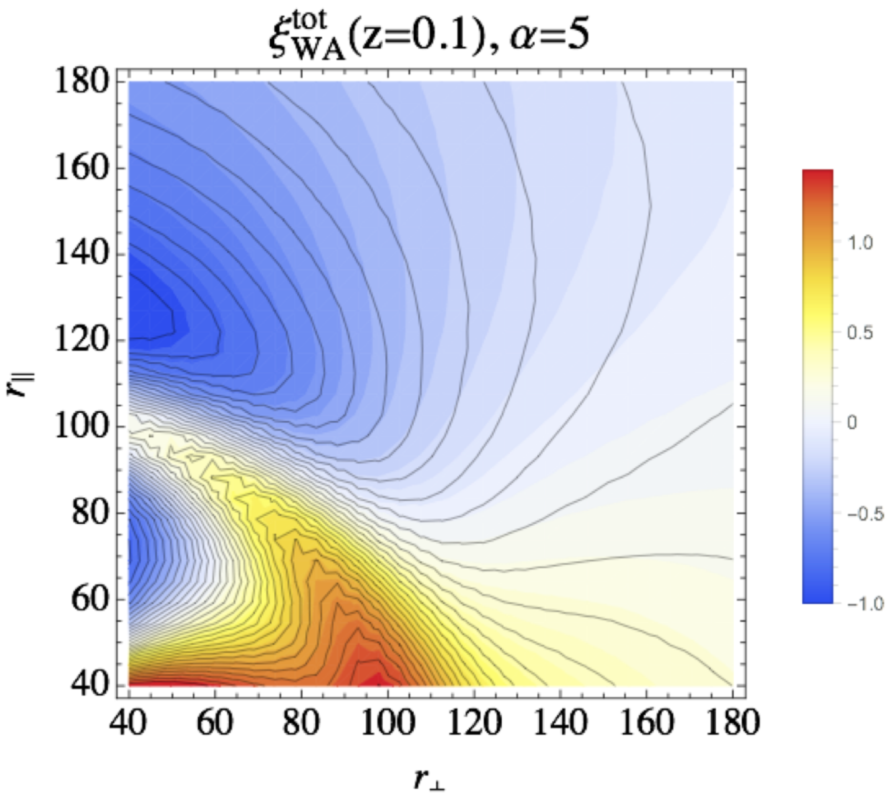
$N^{\mathcal{R}}$ $N^{\mathcal{S}}$

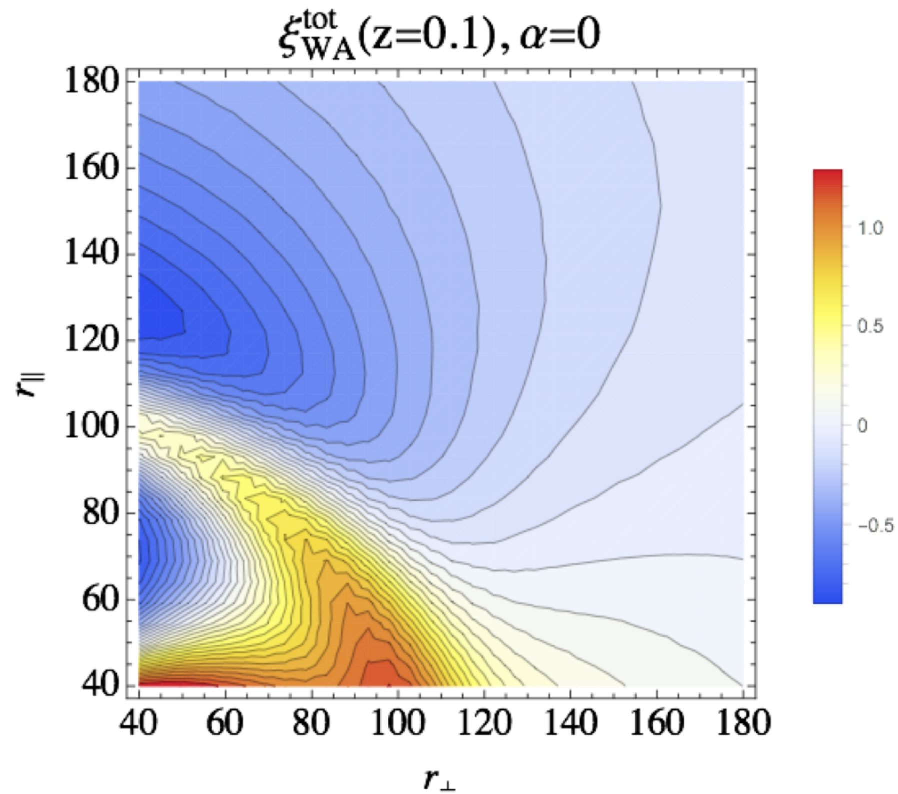
$$\delta^{\mathcal{S}}(\mathbf{r}) = \delta^{\mathcal{R}}(\mathbf{r}) - \left(\frac{\partial v_r}{\partial r} + \cancel{\frac{\alpha(\mathbf{r}) v_r}{r}} \right)$$

- **Peculiar velocities** v_r of galaxies are **small** compared to their distances r from the observer (NB: for future wide surveys probing wide angular scales, $v_r/r \approx \partial v_r/\partial r$ term, and in general cannot be neglected!)
- **Flat-sky approximation** (or plane-parallel case) $\hat{e}_r$ is the same for all galaxies considered
- **Doppler term**: $\alpha v_r/r$, does not naturally disappear, but in flat-sky approximation it is usually neglected.

Velocity and Doppler terms: α

Bertacca et al 2012, 1205.5221
Raccanelli (+ Bertacca) et al. 2016, 1602.03186

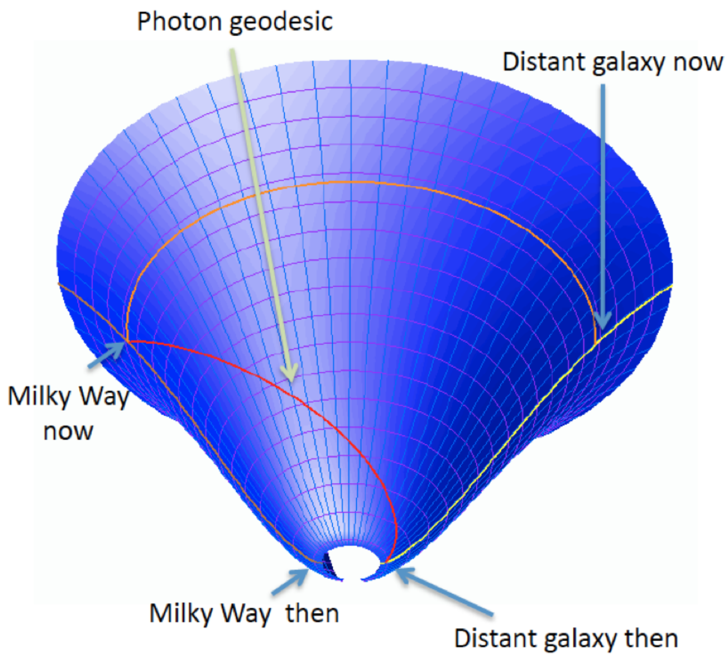




2D redshift-space galaxy correlation function including wide-angle terms.
The effect of $\alpha = 0$ and $\alpha = 5$ corresponds to the value obtained from a gaussian galaxy distribution centered at $z = 0.1$ and with $\sigma = 0.1$.
As expected, the deviation from the $\langle\delta\delta\rangle$ case increases with α.

Now we need a complete description of different effects!!

- holds in Newtonian & GR descriptions



Now we need a complete description of different effects!!

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Jeong, Hirata & Schmidt 2011
Schmidt & Jeong 2012
Bertacca, Maartens & Clarkson 2014a,b
Bertacca 2014

COSMIC LABORATORY

COSMIC LABORATORY

(cosmic rulers)

Jeong, Hirata & Schmidt 2011
Schmidt & Jeong 2012
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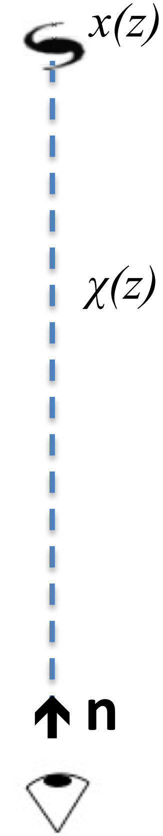
Bertacca 2014

What we observe (the galaxy) is the *apparent* position at which it appears in a given direction **n** and redshift *z* **(the redshift space)**.

In background and in observers frame (*i.e. in uniform-redshift gauge*) the photon geodesics are given by (in conformal coordinates)

$$x(z) = [\tau_0 - \chi(z), \chi(z)\,\mathbf{n}]$$

χ: comoving distance



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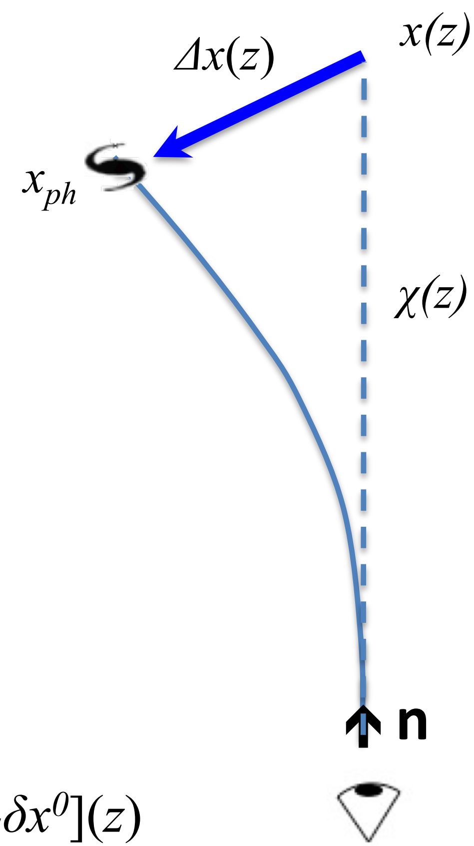

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χ: comoving distance

In a generic perturbed Universe we have:

$$a_{ph} = a\,[1+\Delta \ln z]$$
$$x_{ph} = x + \Delta x$$

$$\Delta \ln z\,(z) = a\mathcal{H}\,\Delta x^0(z) = a\mathcal{H}\,[\delta\chi - \delta x^0](z)$$
$$\Delta x^i(z) = n^i\,\delta\chi(z) + \delta x^i(z)$$

COSMIC LABORATORY

(cosmic rulers)

Jeong, Hirata & Schmidt 2011
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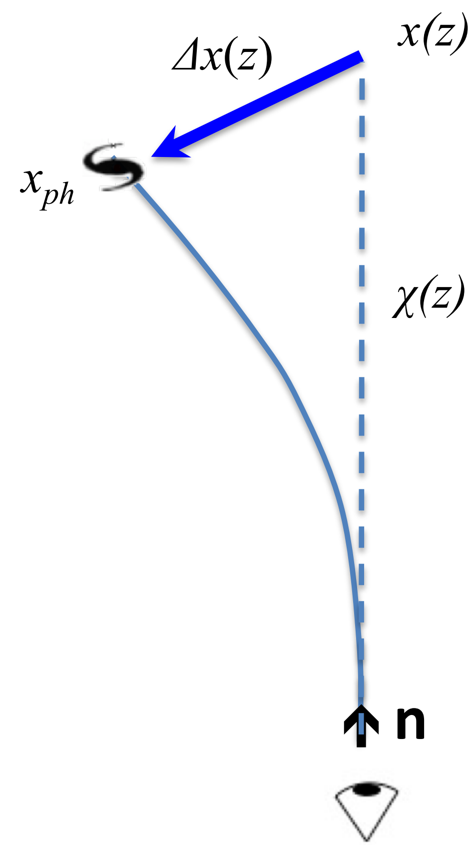
$$\delta\chi = -\left(\chi_{\rm s} + \frac{1}{\mathcal{H}}\right)\left[\Psi_{\rm o} - \left(n_{\rm s}^i v_i\right)_{\rm o}\right] + \frac{1}{\mathcal{H}}\left[\Psi_{\rm e} - \left(n_{\rm s}^i v_i\right)_{\rm e}\right] + \int_0^{\chi_{\rm s}}\left[2\Psi + (\chi_{\rm s} - \chi)\partial_0\left(\Phi + \Psi\right)\right] {\rm d}\chi + \frac{1}{\mathcal{H}}\int_0^{\chi_{\rm s}} \partial_0\left(\Phi + \Psi\right) {\rm d}\chi\,,$$

$$\delta x^0 = -\chi_{\rm s}\left[\Psi_{\rm o} - \left(n_{\rm s}^i v_i\right)_{\rm o}\right] + 2\int_0^{\chi_{\rm s}} \Psi\, {\rm d}\chi + \int_0^{\chi_{\rm s}} (\chi_{\rm s} - \chi)\,\partial_0\left(\Phi + \Psi\right) {\rm d}\chi\,,$$

$$\delta x^i = -\left(v_{\rm o}^i + \Phi_{\rm o} n_{\rm s}^i\right)\chi_{\rm s} + 2n_{\rm s}^i\int_0^{\chi_{\rm s}} \Phi\, {\rm d}\chi - \int_0^{\chi_{\rm s}} (\chi_{\rm s} - \chi)\delta^{ij}\partial_j\left(\Phi + \Psi\right) {\rm d}\chi\,,$$



- Local corrections express the Sachs-Wolfe and the Doppler effects
- Integrated along the line of sight terms derive from gravitational lensing, the Shapiro time-delay and the integrated Sachs-Wolfe effect

Observed galaxy density perturbation Δ_g

see also Ruth talk

$$N_{\rm tot} = \int \sqrt{-g}\, n_{\rm phy}\, \varepsilon_{abcd}\, u^d\, \frac{\partial x^a}{\partial z_{\rm obs}} \frac{\partial x^b}{\partial \theta_{\rm obs}} \frac{\partial x^c}{\partial \phi_{\rm obs}}\, dz_{\rm obs}\, d\theta_{\rm obs}\, d\phi_{\rm obs}$$

manifestly gauge-invariant!

for example, see Yoo 2008, Yoo et al 2009, Bonvin et al 2011, Challinor et al 1011 and Jeong et al 2011

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manifestly gauge-invariant!

$$\Delta(\mathbf{n}, z) = \delta(\mathbf{n}, z) - 3\frac{\delta z}{1+\bar{z}} + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

GR effects: redshift + volume distortions

for example, see Yoo 2008, Yoo et al 2009, Bonvin et al 2011, Challinor et al 1011 and Jeong et al 2011

Observed galaxy density perturbation Δ_g

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$$\Delta_g(\boldsymbol{n}, z) = \Delta_{\text{local}}(\boldsymbol{n}, z) + \Delta_\kappa(\boldsymbol{n}, z) + \Delta_I(\boldsymbol{n}, z).$$

Observed galaxy density perturbation Δ_g

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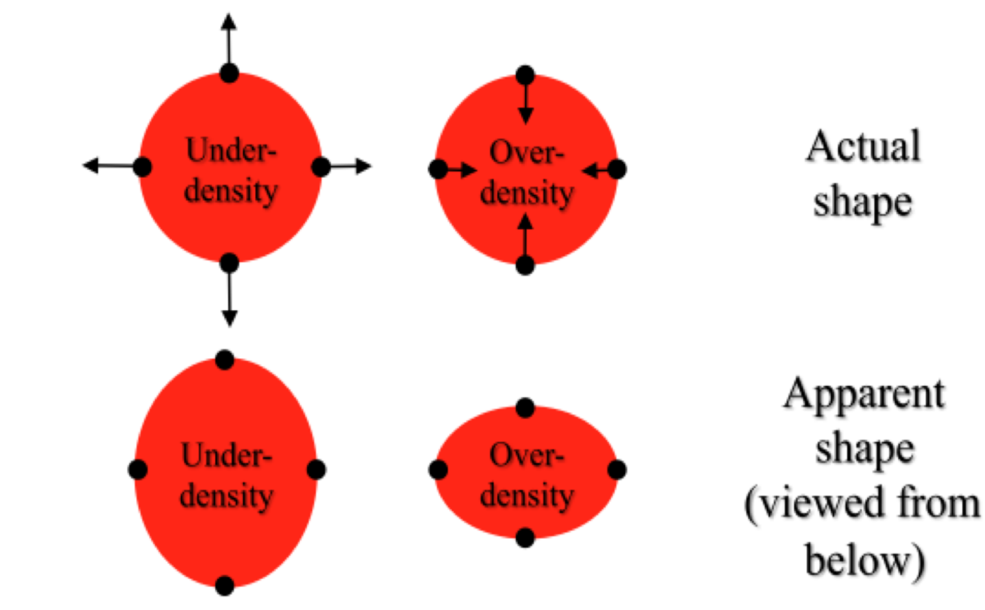
$$\Delta_g(\boldsymbol{n}, z) = \Delta_{\rm local}(\boldsymbol{n}, z) + \Delta_\kappa(\boldsymbol{n}, z) + \Delta_I(\boldsymbol{n}, z).$$

Local term which includes:

- galaxy density perturbation,
- redshift distortion $\propto \partial \boldsymbol{n} \cdot \boldsymbol{v}/\partial\chi$
- velocity term $\propto \boldsymbol{n} \cdot \boldsymbol{v}$
- potential terms Φ, Ψ



Motion of galaxies carries an imprint of the rate of growth of large-scale structure.

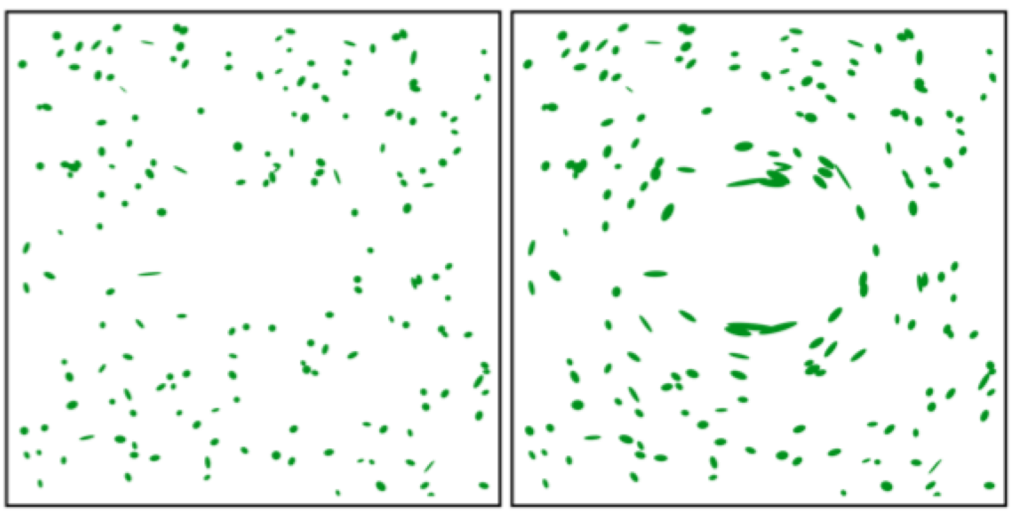
Observed galaxy density perturbation Δ_g

see also Ruth talk

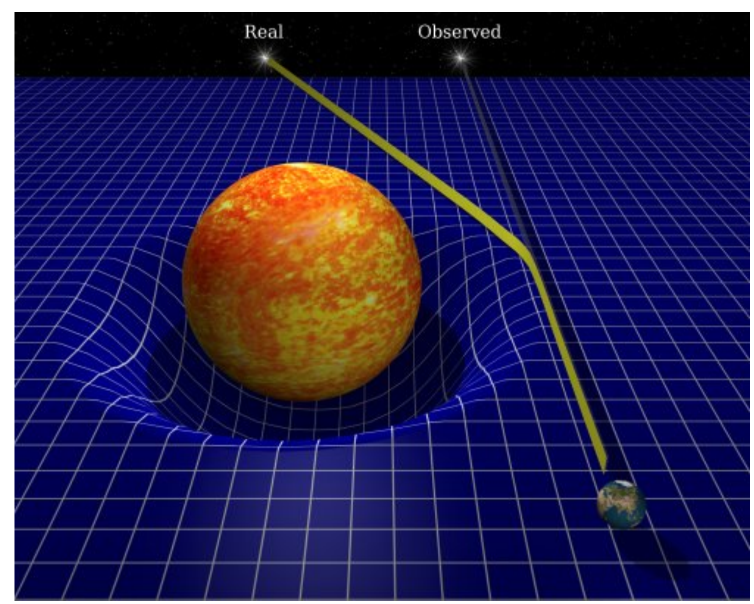
$$\Delta_g(\boldsymbol{n}, z) = \Delta_{\mathrm{local}}(\boldsymbol{n}, z) + \Delta_\kappa(\boldsymbol{n}, z) + \Delta_I(\boldsymbol{n}, z).$$

Weak lensing convergence integral

$$\propto \nabla_\perp^2 \int_0^\chi d\tilde{\chi}\, (\chi - \tilde{\chi}) \frac{\chi}{\tilde{\chi}} (\Phi + \Psi)$$



Photons from a distant galaxy are bent by the matter between the galaxy and us.



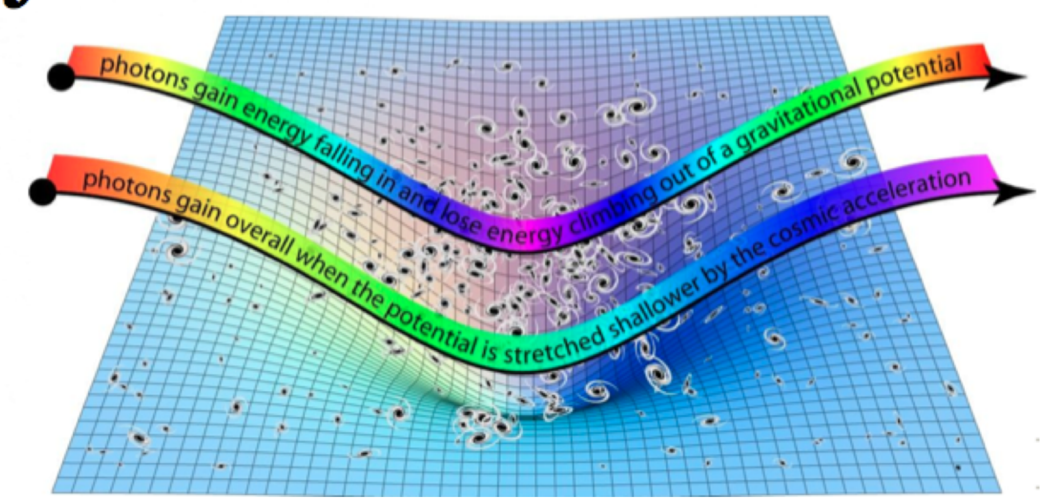
Observed galaxy density perturbation Δ_g

see also Ruth talk

$$\Delta_g(\boldsymbol{n}, z) = \Delta_{\rm local}(\boldsymbol{n}, z) + \Delta_\kappa(\boldsymbol{n}, z) + \Delta_I(\boldsymbol{n}, z).$$

Time delay integrals along the line sight:

- ISW: $\propto \int d\chi \, (\Phi' + \Psi')$
- Time (Shapiro) delay $\propto \int d\chi \, (\Phi + \Psi)$



(Caldwell)

GR corrections at large scales

- Multiple efforts have been made in the literature to investigate the detectability of subtle relativistic effects with Euclid and other forthcoming surveys.

- Generally these studies are based on the Fisher-information matrix, use idealised survey characteristics and neglect systematics.

- The ultimate test to discern what relativistic effects will be observable is to apply the very same estimators that are used for the data to mock catalogs that include all the physics.

GR corrections at large scales with (Newtonian) N-body simulations

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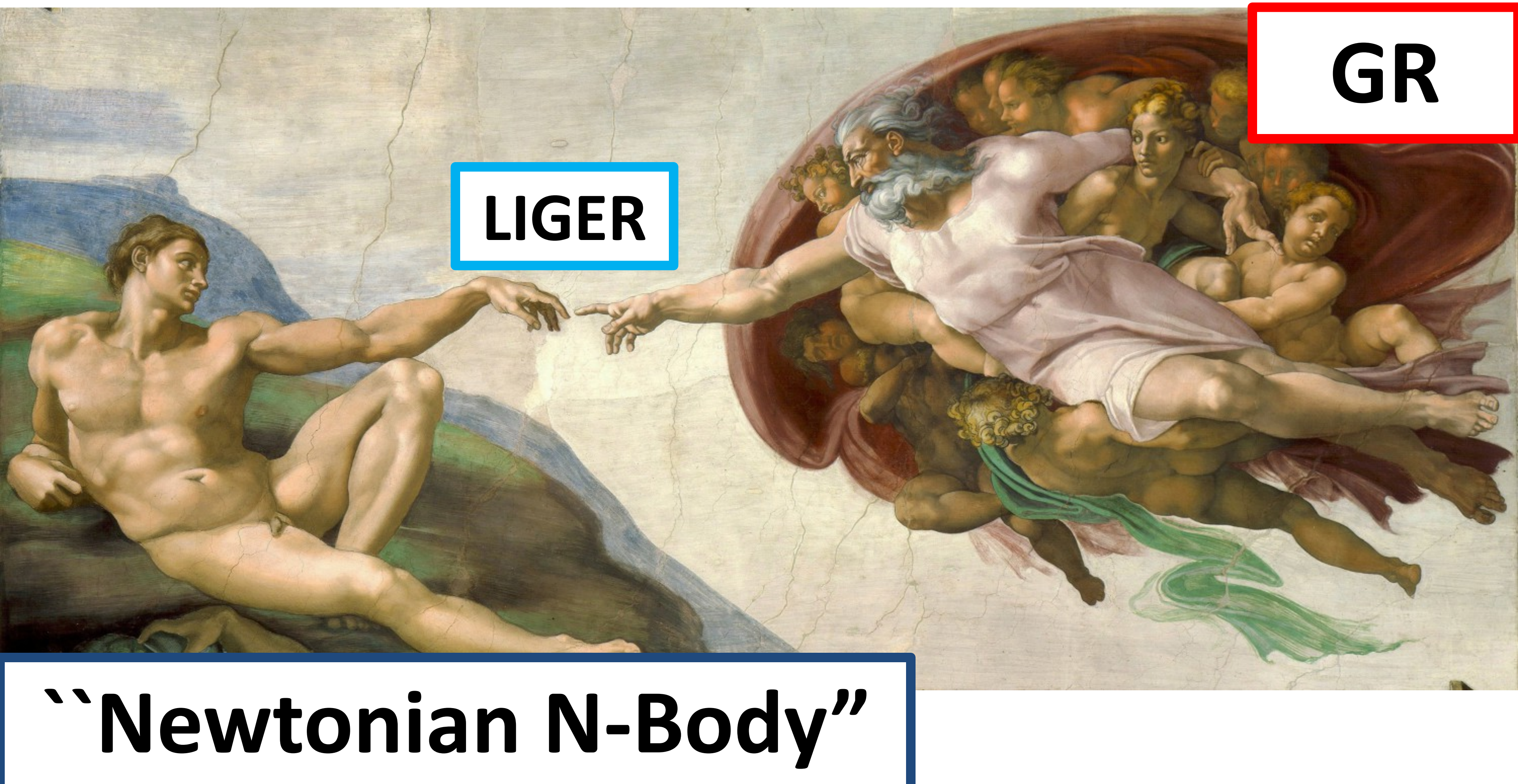
↓

Simulations!

- Raul Abramo and DB 1706.01834
- Borzyszkowski, DB and Porciani,MNRAS (2017) 471, 4, astro-ph:1703.03407

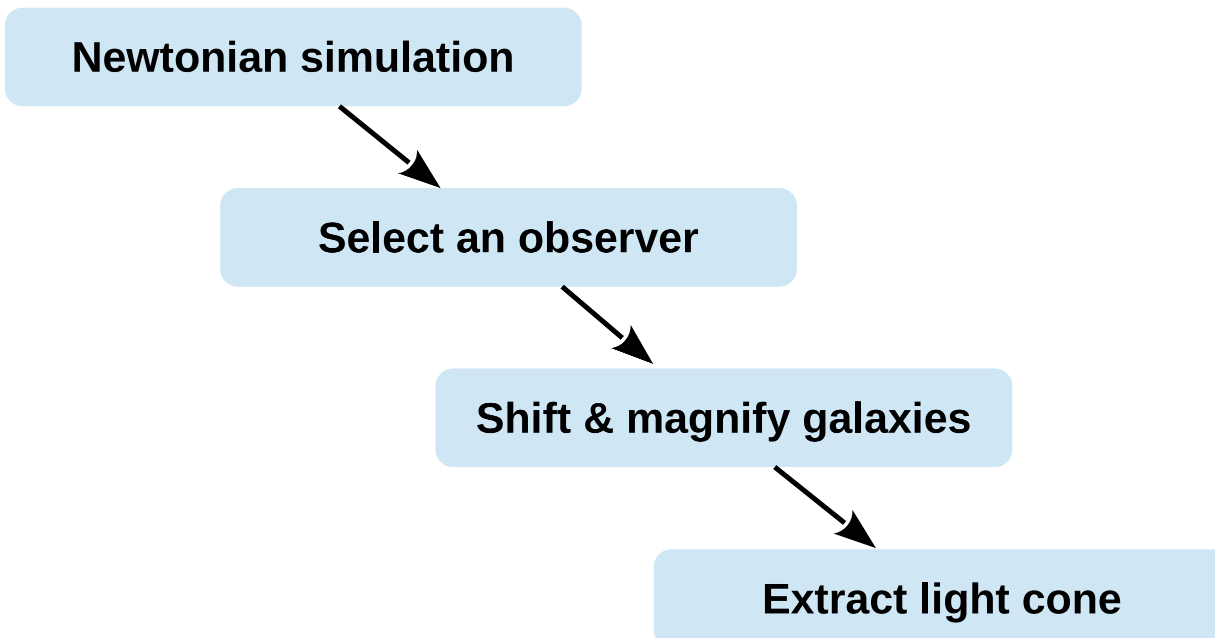
GR corrections at large scales with N-body simulations

Publicly available code: http://www.astro.uni-bonn.de/go/LIGER



LIGER: motivation and philosophy

LIGER is a code that takes a Newtonian simulation (N-body or hydro) as an input and outputs the distribution of galaxies in comoving redshift space (i.e. on the light cone of a perturbed FRW background).

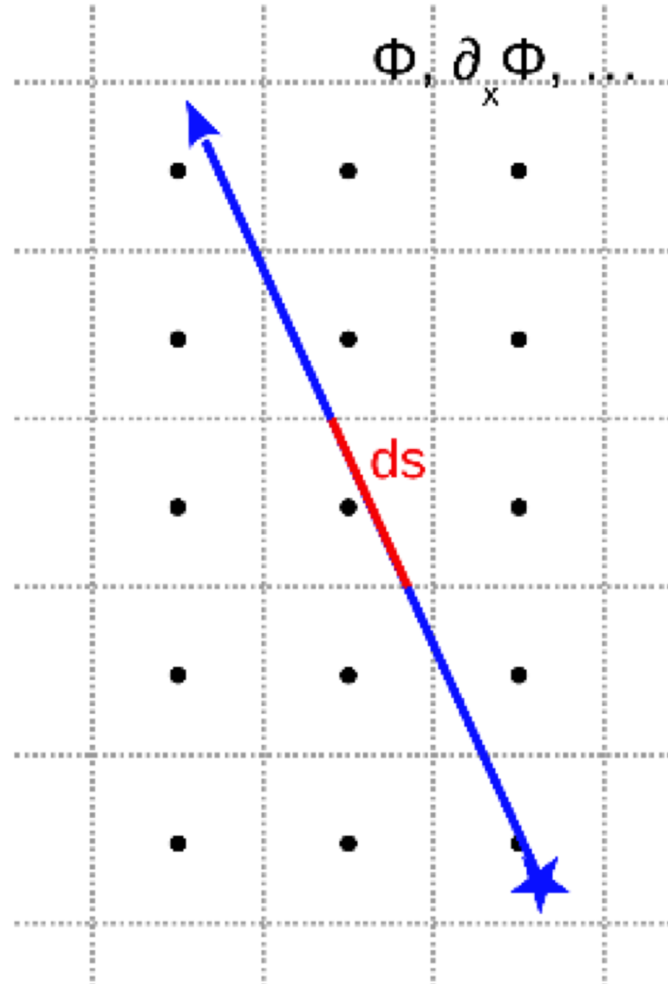


This is achieved by using a coordinate transformation that includes local terms and contributions that are integrated along the line of sight.

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

Particle Shift

- All fields are sampled on a grid.
- Integration is performed as sum of the light path intersections with the grid.
- Field values are redshifted in time by interpolating the snapshots.
- Use Born approximation



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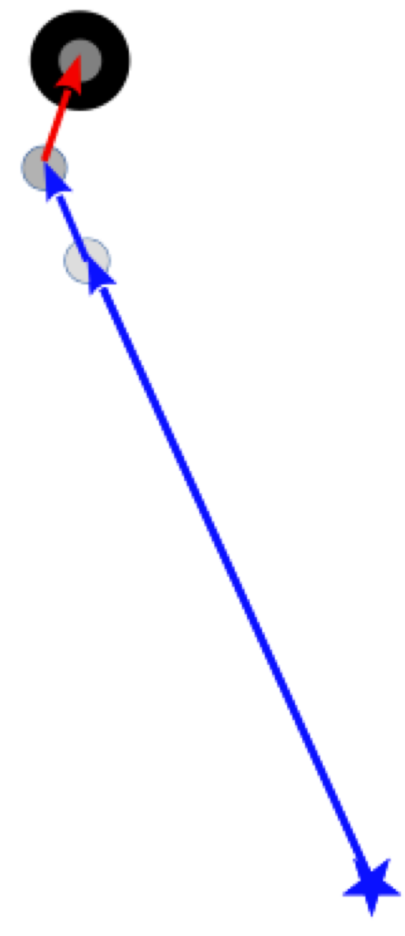
Particle Shift

- Add v-RSD due to particle velocity.
  - Kaiser term $\sim \partial \boldsymbol{n} \cdot \boldsymbol{v} / \partial \chi$
  - Doppler term $\sim \boldsymbol{n} \cdot \boldsymbol{v}$
- GR effects
  - Grav. Lensing
  - Volume Distortions, ...
- Shift also „time“.
- Compute magnification.
  - Convergence and Doppler



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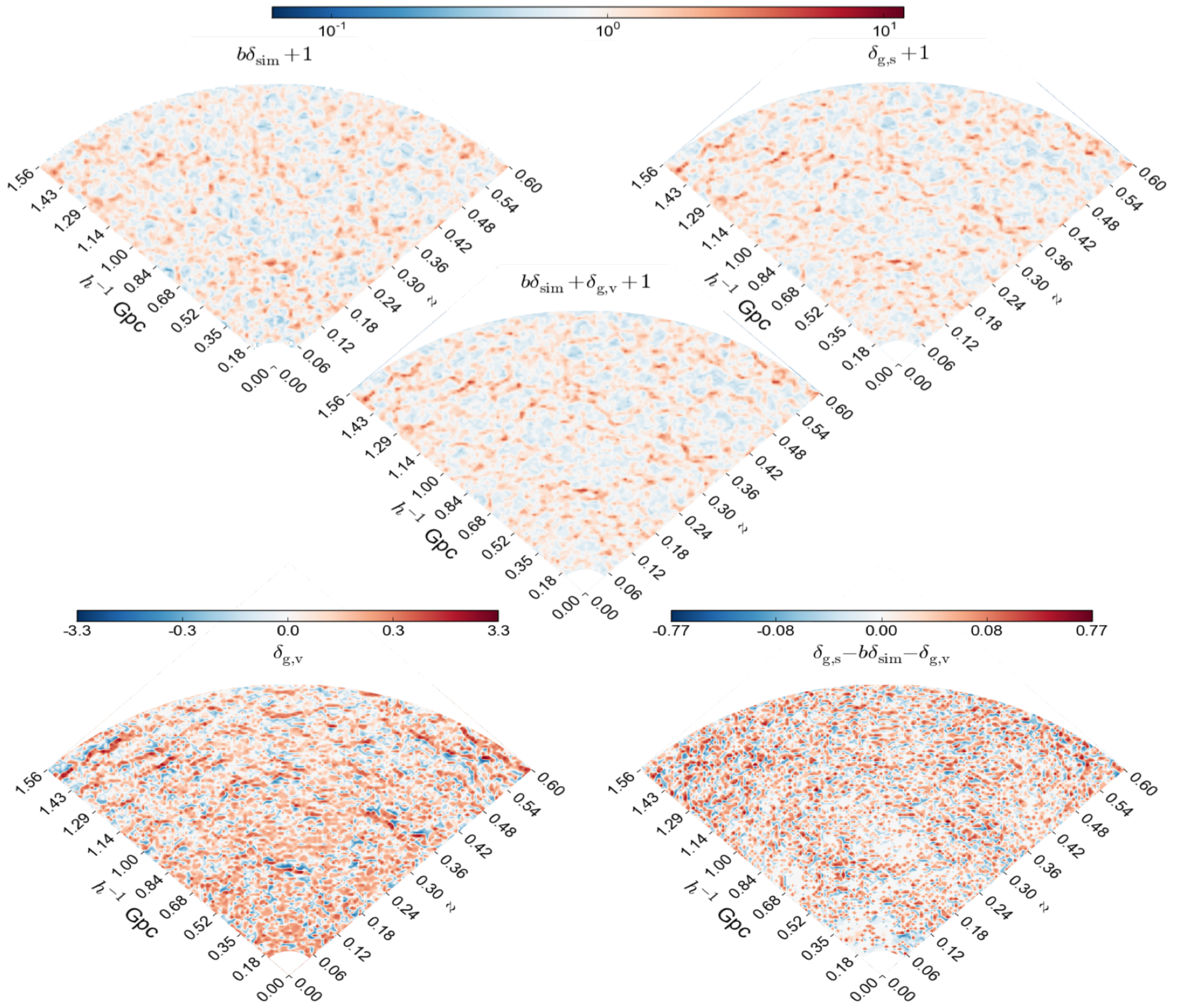
Particle Shift with N-Body simulations



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LIGER's functionality

- We have reanalyzed results which have already been widely discussed in the literature:

1) The impact of magnification bias in the observed cross-correlation of galaxy samples at substantially different redshifts.
2) We discuss the more challenging detection of Doppler terms in the galaxy angular power spectrum at low redshift.

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

1) The impact of magnification bias

- Assume Euclid like survey.

In order to evaluate the relative importance of the velocity-induced shift, we build two other new Euclid mock catalogues

1) GR mock includes relativistic effects

2) KD model (Kaiser and Doppler):
where $\delta\chi = -\mathbf{n}\,\mathbf{v}/\mathcal{H}$, $\delta\mathbf{x} = 0$ and $\mathcal{M} = 1$, respectively
(this is the standard way to implement redshift-space distortions in simulations and omits the terms proportional to Q in α).

3) κKD model (WL + KD):
the redshift-space distortions + weak lensing assuming that the convergence is the only source of magnification, i.e. $\mathcal{M}_\kappa = 1 + 2\kappa$
(i.e. we do not consider doppler magnification!)

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

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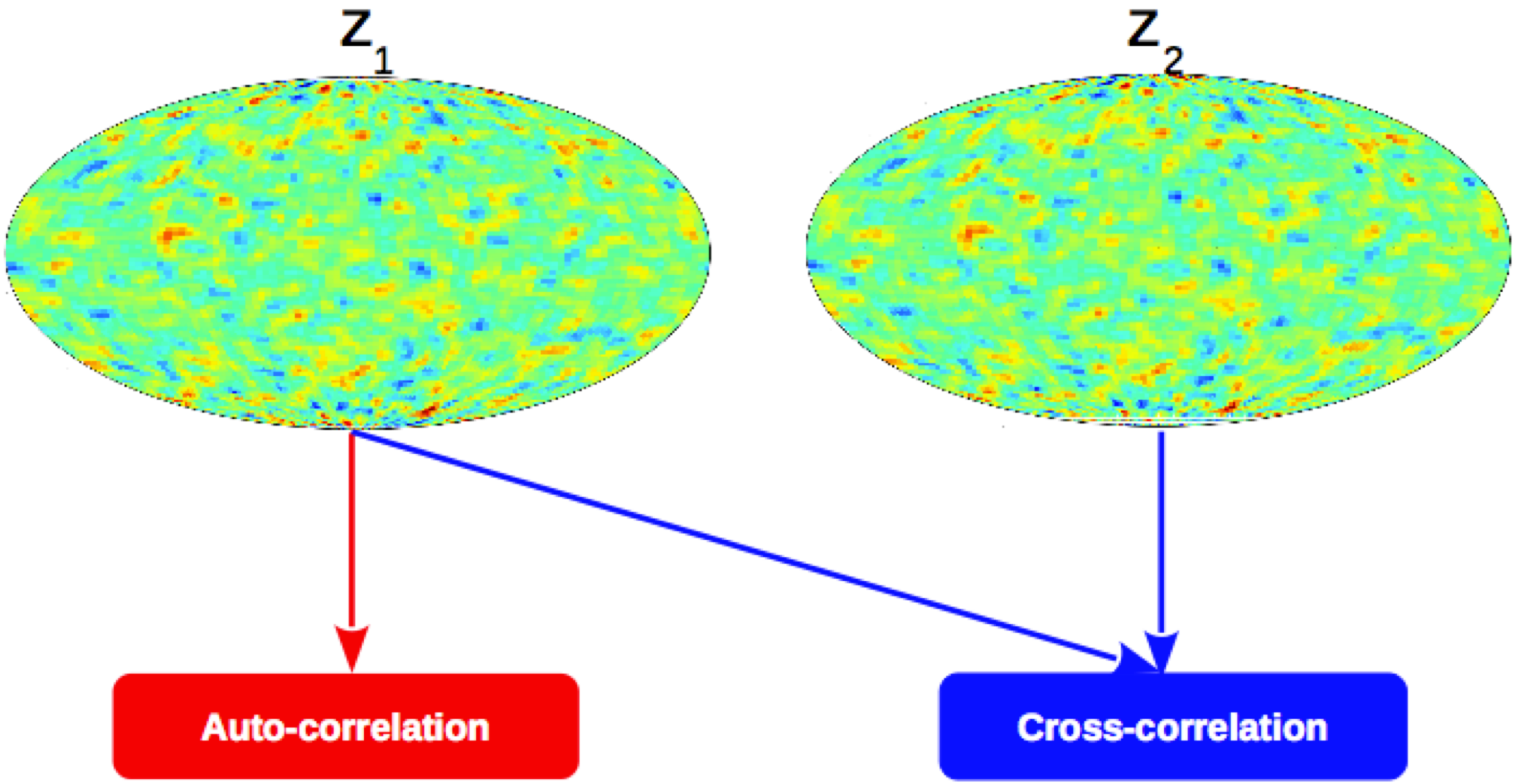

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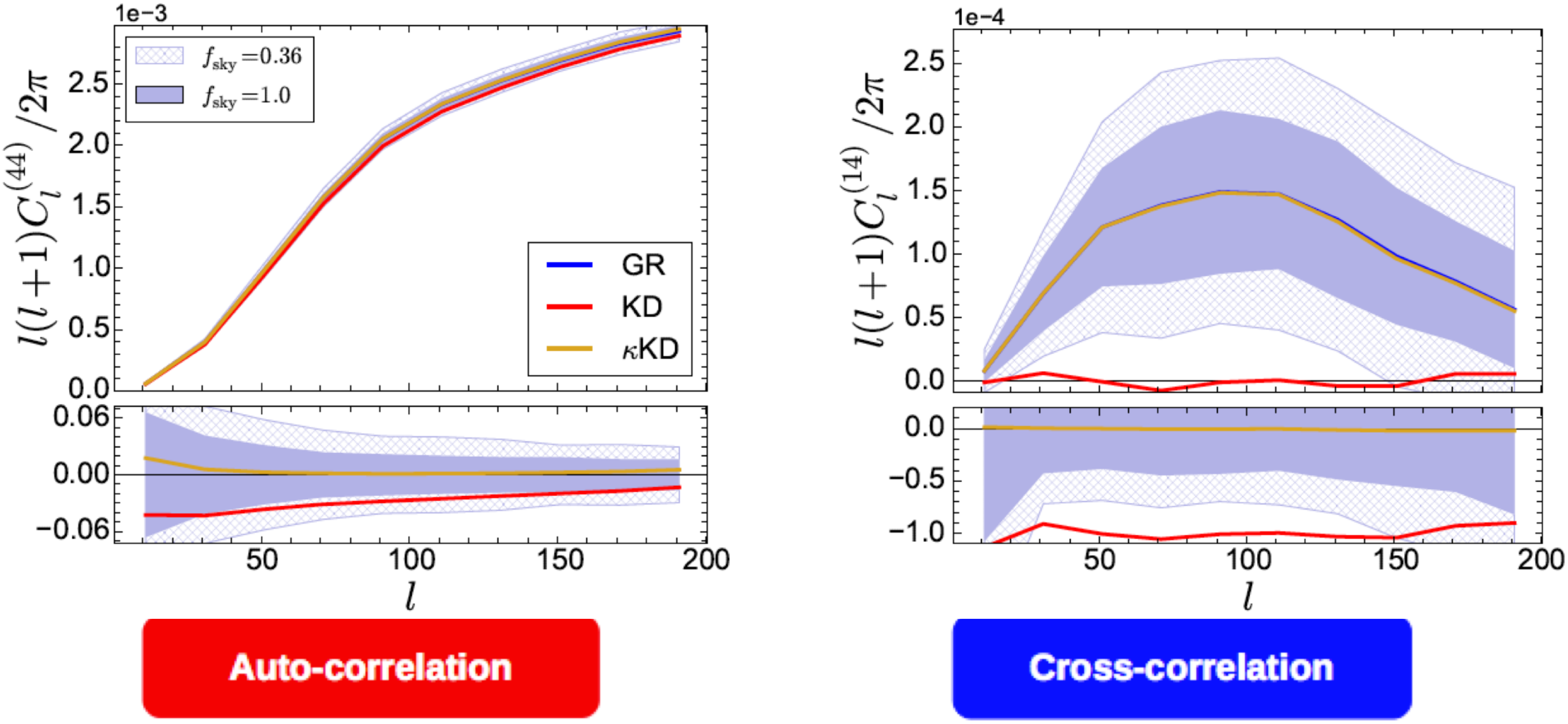

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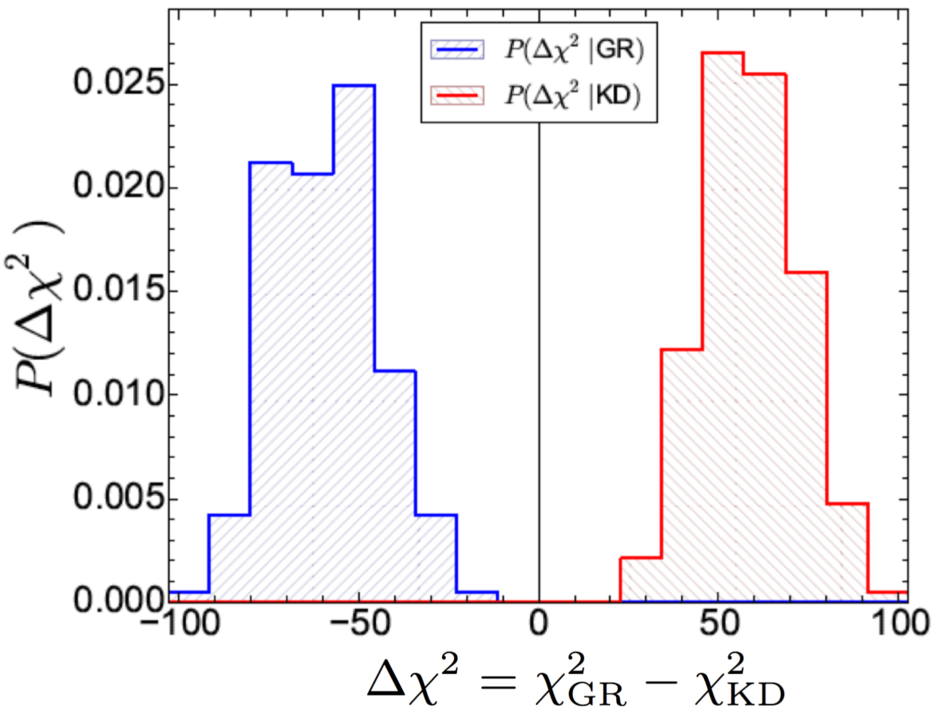

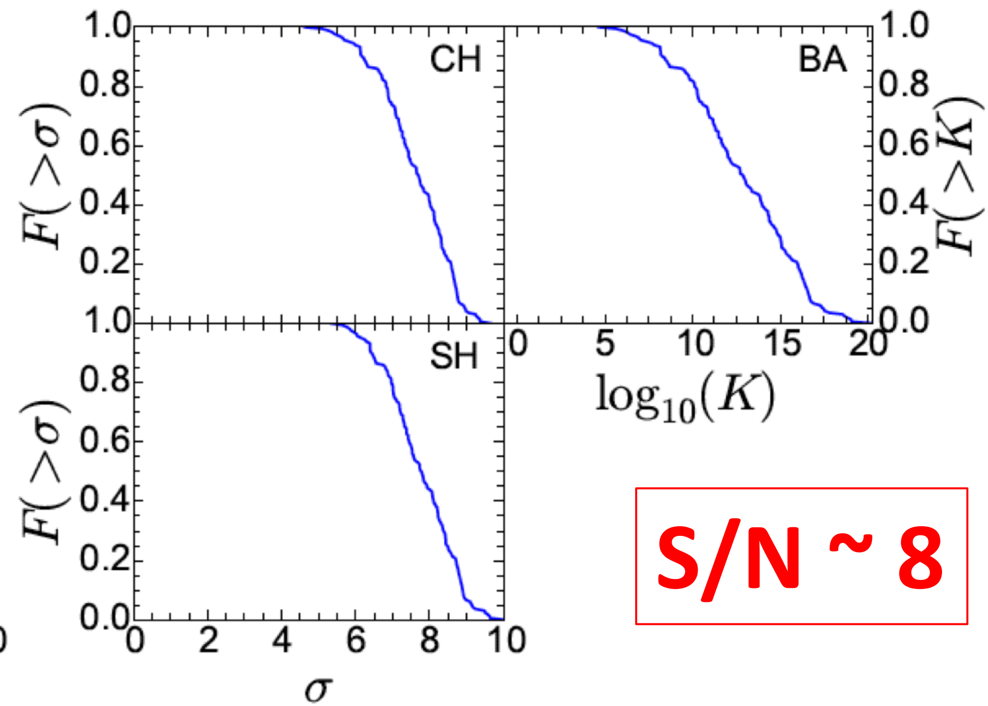

S/N ~ 8

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

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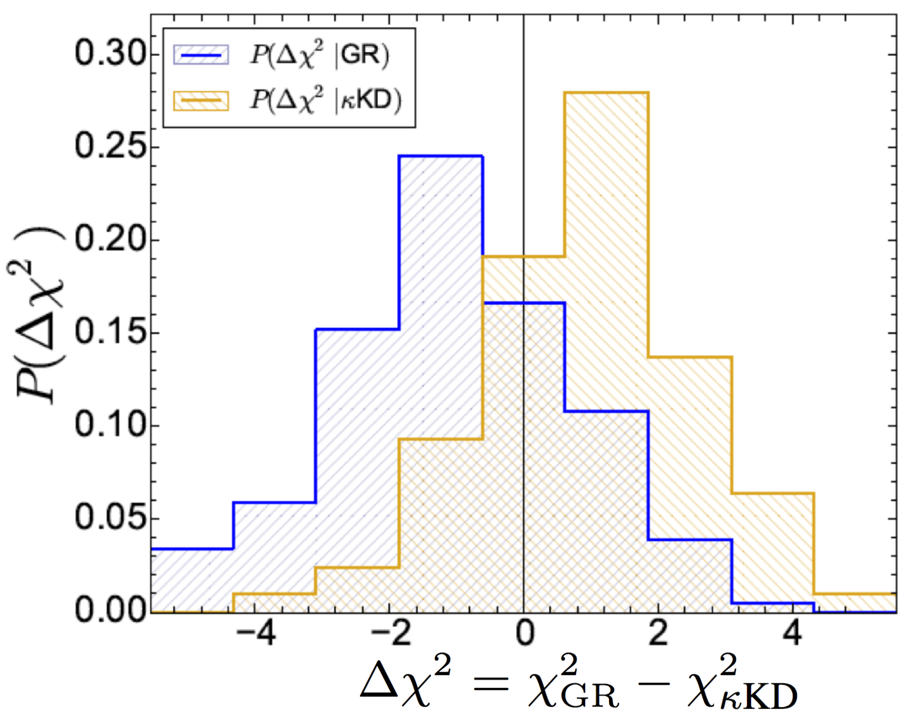
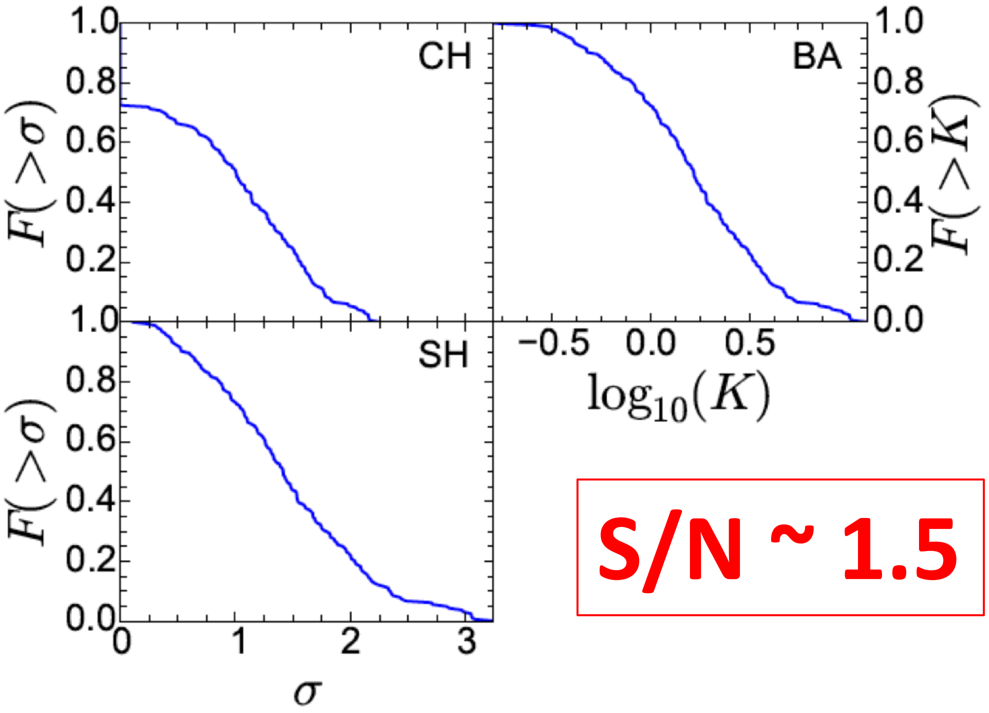




S/N ~ 1.5

due to the doppler magnification!

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

2) Detection of Doppler terms

We build two sets of mock catalogues:

1) GR mock includes relativistic effects

2) DS (Doppler suppressed):
we drop the Doppler terms that are proportional to b_e and $\mathcal{Q}$.
$\alpha_{DS} = 2 + [1 - (3/2)\Omega_m(z)]\mathcal{H}\chi$ (in ΛCDM model)

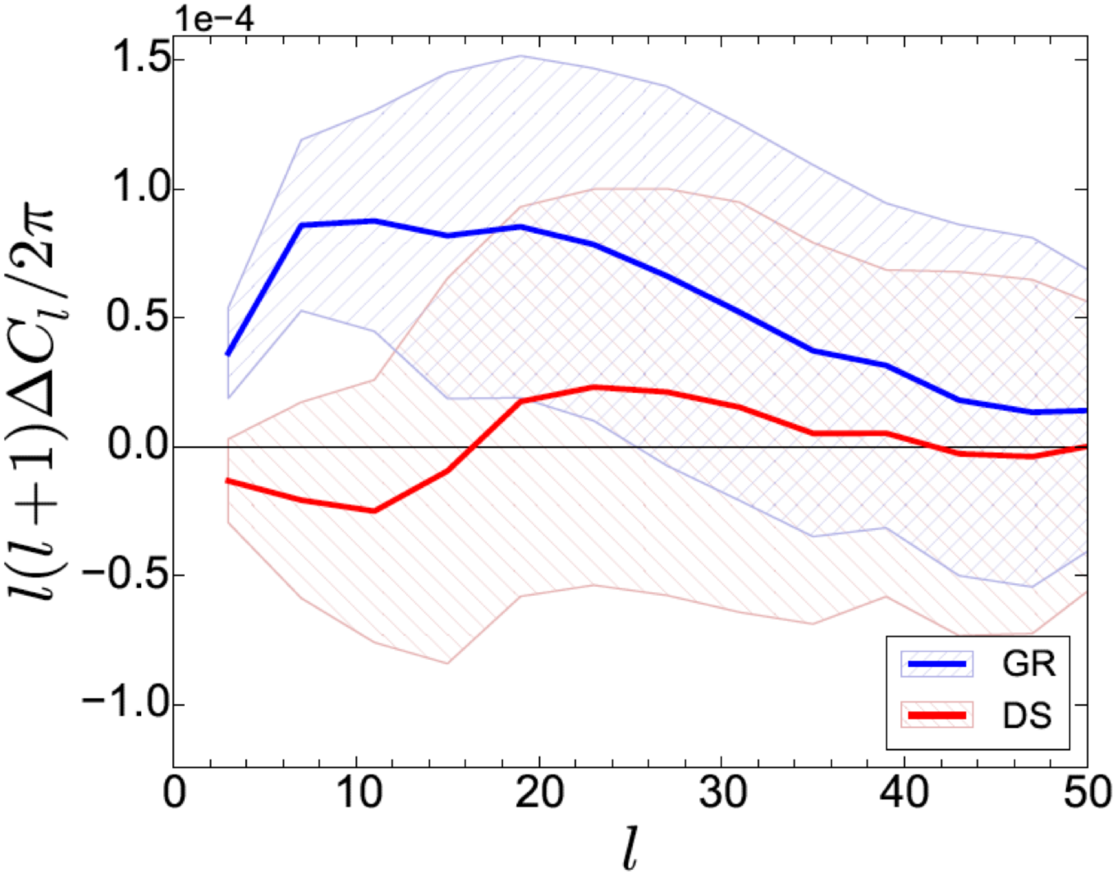
- We consider the interval $0.15 < z < 0.25$ which we further divide into bins I: $0.15 < z < 0.2$ and II: $0.2 < z < 0.25$.
- For SKA II (Yahya et al. 2015):
  Bright sample: with fluxes above 60μJy.
  Total sample with flux above 23 μJy

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

2) Detection of Doppler terms

$$\Delta\hat{C}_l = \hat{C}_l^{(\mathrm{T_I\,B_{II}})} - \hat{C}_l^{(\mathrm{B_I\,T_{II}})}$$

While the relative error on the single cross-spectra is very large, **ΔC_l can be measured!!** (especially for l < 25).

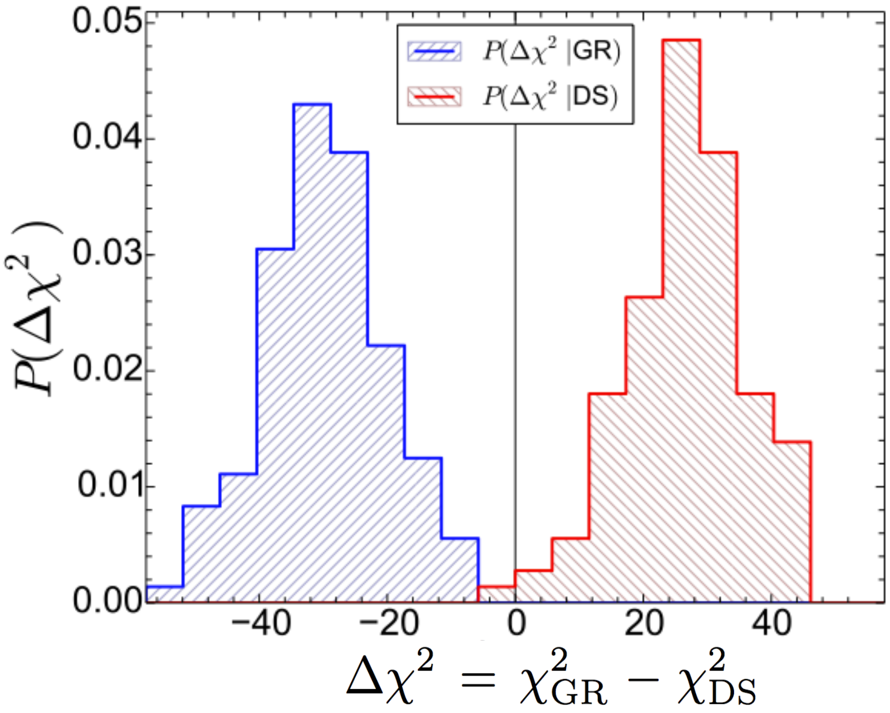


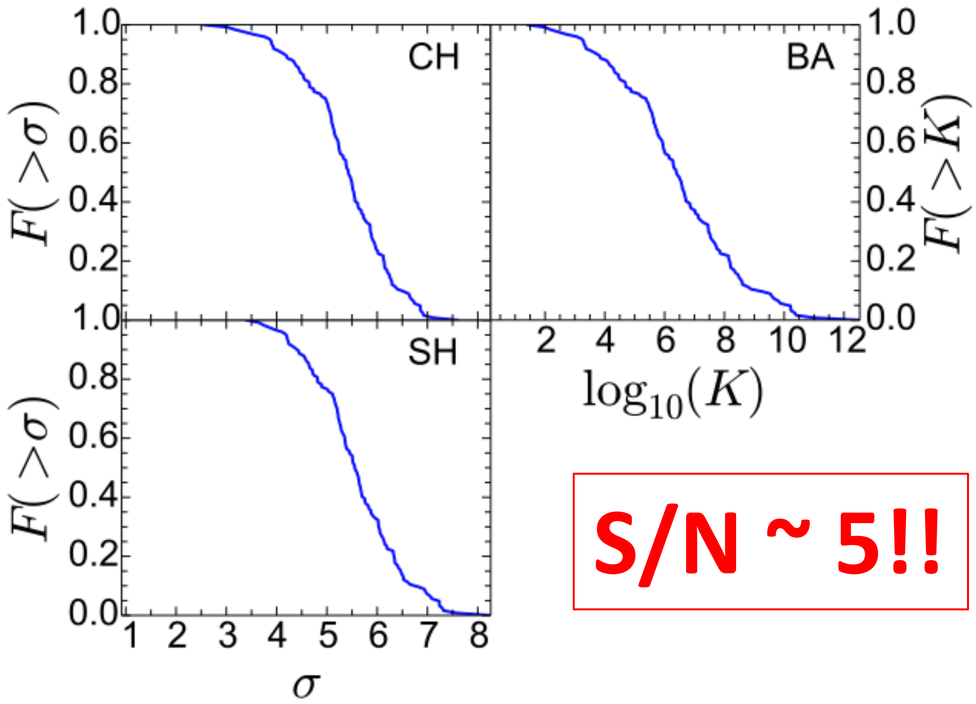
- Both galaxy populations trace the same large-scale structure → most of the noise in the cross-spectra is correlated and thus does not appear in the difference.

- This exemplifies the advantage of using **a multi-tracer approach** (McDonald & Seljak 2009)

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

2) Detection of Doppler terms





S/N ~ 5!!

Borzyszkowski, DB and Porciani, MNRAS (2017) 471, 4, astro-ph:1703.03407

Conclusions

- Using **LIGER** we have shown that SKA II should be able to detect Doppler effects in the angular galaxy clustering with S/N ≈ 5 (Borzyszkowski, Bertacca & Porciani 2017)

- Using **LIGER** we have shown that Euclid should be able to detect the impact of magnification bias in the observed cross-correlation of galaxy samples with S/N ≈ 8 (Borzyszkowski, Bertacca & Porciani 2017)

- **LIGER** can be used to post-process any Newtonian simulation independently of the code used to run it

- **LIGER** is being applied to the Flagship simulation of the Euclid Consortium

- Publicly available code: http://www.astro.uni-bonn.de/go/LIGER

Thank You!

Is LIGER missing a term?

LIGER (LIght cones using General Relativity) method

-To evaluate $\Delta x^i(z)$ and $\mathcal{M}$, we need to compute the gravitational potentials.

- In simulations, we need to derive the potentials starting from the particle distribution:
This corresponds to using the matter density contrast in the comoving gauge, i.e. $\delta_{\rm sim} \equiv \delta_{\rm C}$ (comoving gauge).

- At linear order in the perturbations and for a pressureless fluid in a universe with ΛCDM background, the source equation for Ψ in the Poisson gauge can be re-written in terms of $\delta_{\rm C}$ as the standard Poisson equation (e.g. Chisari & Zaldarriaga 2011; Green & Wald 2012):

$$\Phi = \Psi = \phi \,, \quad \nabla^2\phi = 4\pi G a^2 \bar{\rho}_{\rm m}\delta_{\rm sim} \,, \text{ and } v^i = v^i_{\rm sim}$$

For details, see also Adamek and Fidler Talk

LIGER (LIght cones using General Relativity) method

By perturbing the photon geodesic around the FRW solution, Borzyszkowski et al. (2017), we derive the equation for δx_μ (in the Poisson gauge) which is composed of gauge invariant terms only,

$$\delta\chi = -\left(\chi_\mathrm{s} + \frac{1}{\mathcal{H}}\right)\left[\Psi_\mathrm{o} - \left(n_\mathrm{s}^i v_i\right)_\mathrm{o}\right] + \frac{1}{\mathcal{H}}\left[\Psi_\mathrm{e} - \left(n_\mathrm{s}^i v_i\right)_\mathrm{e}\right] + \int_0^{\chi_\mathrm{s}} \left[2\Psi + (\chi_\mathrm{s} - \chi)\partial_0\left(\Phi + \Psi\right)\right] \mathrm{d}\chi + \frac{1}{\mathcal{H}}\int_0^{\chi_\mathrm{s}} \partial_0\left(\Phi + \Psi\right)\,\mathrm{d}\chi\ ,$$

$$\delta x^0 = -\chi_\mathrm{s}\left[\Psi_\mathrm{o} - \left(n_\mathrm{s}^i v_i\right)_\mathrm{o}\right] + 2\int_0^{\chi_\mathrm{s}} \Psi\,\mathrm{d}\chi + \int_0^{\chi_\mathrm{s}} (\chi_\mathrm{s} - \chi)\,\partial_0\left(\Phi + \Psi\right)\,\mathrm{d}\chi\ ,$$

$$\delta x^i = -\left(v_\mathrm{o}^i + \Phi_\mathrm{o} n_\mathrm{s}^i\right)\chi_\mathrm{s} + 2n_\mathrm{s}^i \int_0^{\chi_\mathrm{s}} \Phi\,\mathrm{d}\chi - \int_0^{\chi_\mathrm{s}} (\chi_\mathrm{s} - \chi)\delta^{ij}\partial_j\left(\Phi + \Psi\right)\,\mathrm{d}\chi\ ,$$

Finally, we determine the event at which the perturbed worldline of the tracer crosses the (straight, i.e. unperturbed) light-cone of the observer and record it.

This defines the observed position of the tracer.

Newtonian motion gauges

- Fidler et al. (2017) introduce the class **of Newtonian motion gauges (NmG)** which provide space-time coordinates designed so that matter follows Newtonian trajectories.
- They match the perturbations in the simulation with the relativistic ones in the All relativistic perturbations are constructed combining the output of the Newtonian simulation and a linear Boltzmann code (for the relativistic species).
- Fidler et al. (2017) point out that Newtonian simulations implicitly make use of coordinates x^μ defined in the NmG.
- Therefore, the displacement δx^μ due to the bending photon trajectory needs to be computed in the same gauge.
- In fact, both x^μ and δx^μ are gauge dependent quantities while the final direction from which the observer detects the light rays and the observed redshift are not.
- Alternatively, a coordinate transformation should be first applied to express the particle positions in the Poisson gauge and then the light-ray path can be evaluated in this gauge.

Integrated Coordinate Shift

Based on this reasoning, they conclude that the correction δx^μ currently implemented in LIGER misses a term (which is small within the horizon) that they dub the Integrated Coordinate Shift (ICS) for the photon trajectories. Specifically, the spatial components of δxi should include the additive term,

$$(\hat{\nabla}^i \mathfrak{K}^{-1} H_{\mathrm{T}})_{\mathrm{e}} - (\hat{\nabla}^i \mathfrak{K}^{-1} H_{\mathrm{T}})_{\mathrm{o}} \subset \delta x^i \ ,$$

where

$$\hat{\nabla}^i = -(-\nabla^2)^{-1/2}\nabla_i,\ \mathfrak{K} = (-\nabla^2)^{1/2} \text{ and } H_{\mathrm{T}} = 3\zeta$$

Here ζ denotes the comoving curvature perturbation and we have used the subscripts ‘e’ and ‘o’ to label quantities evaluated at the position of the light source when the photons are emitted and at the location of the observer when the photons are received.

Is LIGER missing a term?

The matter density contrast in redshift space can be written in the following way:

$\delta_s = \delta_{sim} + \delta_{RSD}$

It is important to notice that δ_{RSD} receives contributions from three terms:

1) The determinant $(-g)^{1/2}$

2) The spatial Jacobian determinant of the mapping from real to redshift space ;

3) Through the space-time dependence of $a^3 n_g$.

We note that

$$\nabla_i \delta x^i \quad \subset \quad \delta \left| \frac{\mathrm{d}V}{\mathrm{d}\bar{V}} \right|$$

$$\delta\sqrt{-g} + \nabla_i \delta x^i \quad \subset \quad \delta_{\mathrm{RSD}} \, .$$

Is LIGER missing a term? NO!!

Using the metric defined in Fidler et al. (2017) and the dictionary defined in their Section 4.1 , we find that, at linear order,

$$-3\zeta \quad \subset \quad \delta^{ij}\delta g_{ij}/2 = \delta\sqrt{-g}$$

and

$$+3\zeta \quad \subset \quad \nabla_i \delta x^i$$

Note that $\nabla_i(\hat{\nabla}^i \mathfrak{K}^{-1} H_{\mathrm{T}})_{\mathrm{o}} = 0$ because it is evaluated at the observer!
Thus

$$\nabla_i \delta x^i + \delta\sqrt{-g} \quad \supset \quad +3\zeta - 3\zeta = 0\ .$$

This shows that, even in the NmG, the perturbation δ_{RSD} does not depend on the ICS. Therefore, the redshift-space overdensity produced by LIGER is correct in the ΛCDM model.

LIGER misses a term? NO!!

Note also that

The same conclusion can be drawn following a different line of reasoning based on the gauge invariance of $\delta_{\rm s}$. In Appendix A of Yoo (2010) (see also Appendix A of Yoo & Durrer (2017)), $\delta_{\rm s}$ has been expressed in terms of gauge invariant quantities. The scalar part of the pure gauge term $\mathcal{G}^i$ introduced in equation (B3) is equal to $-\hat{\nabla}^i \mathfrak{K}^{-1} H_{\rm T}$. It follows that this pure gauge term cancels out with the ICS in the gauge invariant equation for δx^i.

Thank You