$\boxed{\text{Lectures on Beyond } \Lambda}$

I Λ - CDM

data in very good agreement with Λ - CDM

- Baryons
- neutrinos
- photons
- CDM $\to$ non-interacting dark matter

\+

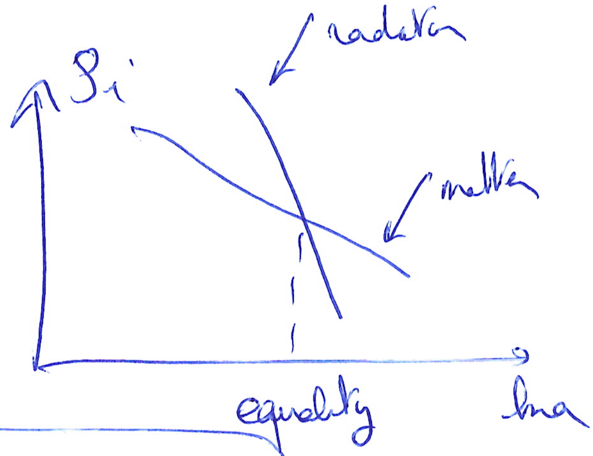
$\Lambda \equiv$ | cosmological constant
or
constant vacuum energy

$$\left\{ \begin{array}{l} \boxed{H^2 = \dfrac{1}{3 m_{Pl}^2} \left(\displaystyle\sum_{i=1}^{4} \rho_i + \underbrace{\rho_\Lambda}_{\text{constant}} \right)} \\[2ex] H = \dfrac{\dot{a}}{a} \\[2ex] ds^2 = -dt^2 + a^2 dx^2 \end{array} \right.$$

$k = 0$ no spatial curvature

(preferred by data ...)

$$\rho_i = \rho_{0i} \left(\frac{a}{a_0} \right)^{-3(1+w_i)}$$



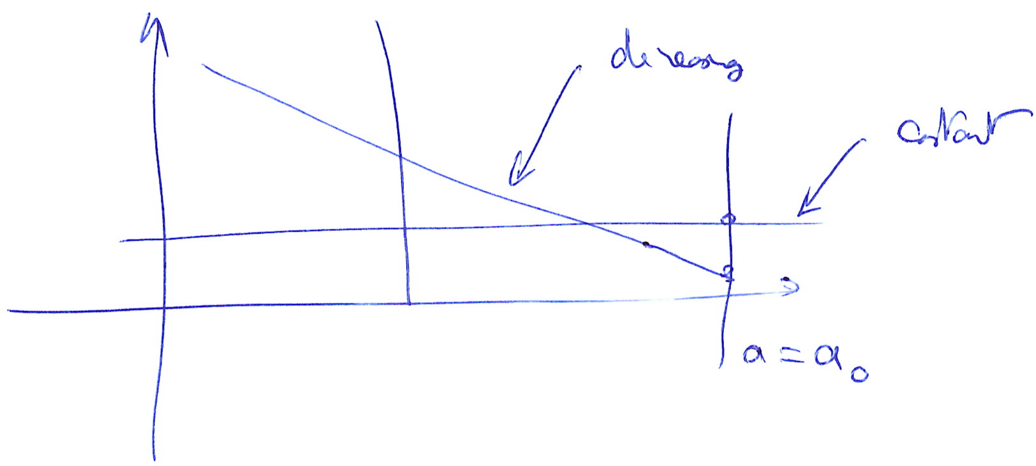
$$\begin{cases} w_i = 0 & \text{CDM Baryons} \\ w_i = \frac{1}{3} & \text{radiation} \end{cases}$$

$$\Omega_{i0} = \frac{\rho_{0i}}{3 H_0^2 m_{pl}^2} \qquad \boxed{\sum_{i=1}^{4} \Omega_{i0} + \Omega_\Lambda = 1}$$

$\Omega_{b0} \sim 5\%$ $\Omega_{CDM0} \sim 25\%$ $\Omega_\Lambda \sim 70\%$

Coincidence problem: $\boxed{\Omega_{CDM0} \sim \Omega_\Lambda}$

why now?



no reason for the two curves to cross around now and not in the past or future.

II Cosmological constant problem

ρ_Λ is a constant energy density and the data are such that (in agreement with)

$$\boxed{\frac{p_\Lambda}{\rho_\Lambda} \simeq -1}$$

repulsive fluid

$$\rho_\Lambda \sim 10^{-29}\ g/cm^3 \sim 10^{-48}\ GeV^4$$

The standard model based on GR + ρ_Λ

$$S_{gravity} = \int \sqrt{-g}\, d^4x \left(\frac{R}{16\pi G_N} - \rho_\Lambda \right)$$

$\rho_\Lambda \equiv$ constant energy density in vacuum

Can we calculate it? Yes and No ....

3 contributions to ρ_Λ

$$\rho_\Lambda \equiv \text{constant} + \text{"latent heat"} + \text{quantum corrections}$$

put by hand by Einstein in his equation

from all the phase transitions in the Universe

the constant is such that

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu}}$$

introduced by Einstein to get <u>static</u> + <u>spherical</u> universe

(not supported by data)

$$\boxed{\rho_\Lambda = \frac{\Lambda}{8\pi G_N}}$$

<u>phase transitions</u>: they release energy like a cooling process ..

<u>two</u> well known transitions

- electroweak (Higgs mechanism)

$$\delta\rho_\Lambda \sim M_W^4 \qquad M_W \sim 100\ \mathrm{GeV}$$

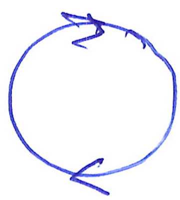
- QCD (protons from quarks)

$$\delta\rho_\Lambda \sim \Lambda_{QCD}^4 \qquad \Lambda_{QCD} \sim 250\ \mathrm{MeV}$$

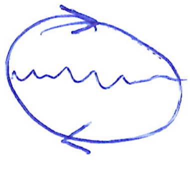
and all the other ones which could have happened in the early Universe.

quantum corrections: the vacuum is <u>not</u> empty

quantum mechanics $\longrightarrow$ creation of particles-antiparticles from nothing

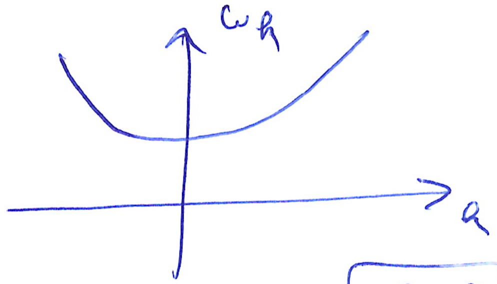
 $\longrightarrow$ one loop diagram

+

 etc...

$$\rho_{\text{quantum}} \equiv \frac{1}{2} \sum_{\text{excitations}} \hbar \omega_k$$

particles $\equiv$ excitations



$\omega_k = \sqrt{k^2+m^2}$

$\hbar \equiv 1$ in the following

$$\begin{cases} \rho_Q = \dfrac{1}{2} \displaystyle\int \frac{d^3k}{(2\pi)^3} \sqrt{k^2+m^2} \\ P_Q = \dfrac{1}{6} \displaystyle\int \frac{d^3k}{(2\pi)^3} \frac{k^2}{\omega_k} \end{cases}$$

relativistic fluid

$M \sim m_{pl}$

<u>math better</u>

$$\rho_Q = \frac{1}{2} \int^{M} \frac{4\pi k^2 dk}{(2\pi)^3} \sqrt{k^2+m^2}$$

$$\sim \frac{1}{4\pi^2} \int^{M} k^3 dk$$

$$\sim \frac{M^4}{16\pi^2}$$

$$P_Q \sim \frac{M^4}{64\pi^2}$$

like radiation

$$\frac{P_Q}{\rho_Q} \sim \frac{1}{3}$$

not compatible with

$\rho_Q \to$ energy of vacuum

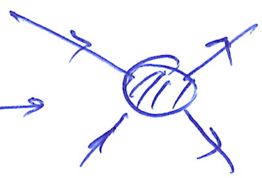
why? because a hard cut-off Λ breaks Lorentz invariance ($k \leq \Lambda$ breaks the boost invariance)

$\rightarrow$ need to regularise in a Lorentz invariant way

$$\boxed{\rho_\Lambda = \frac{1}{64\pi^2} m^4 \ln \frac{m^2}{\mu^2}}$$

μ unknown scale.... (renormalisation scale)

In particle physics μ = energy of the collision

$\mu \longrightarrow$ 

in cosmology ... not clear ... maybe H ?

as taking into account all the fluctuations inside the Horizon $d \lesssim H^{-1}$??

$$\boxed{\rho_\Lambda = \frac{\Lambda}{8\pi G_N} + \rho_{\text{transtion}} + \sum_i (-1)^{2j+1}(2j+1) \times m_j^4 \ln \frac{m_j^2}{\mu^2}}$$

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$$(-1)^{2j+1} = \begin{cases} +1 & \text{bosons} \\ -1 & \text{fermions} \end{cases}$$

$(2j+1) \rightarrow$ quantum degnacy.

$\Lambda \rightarrow$ unknown

$\rho_{\text{transition}} \rightarrow (100\ GeV)^4 + (250\ MeV)^4 + \dots$

$m_j^4 \rightarrow m^4_{\text{electrons}} + m^4_{\text{quarks}}$ etc

$m_{top} \sim 175\ GeV$

need to fine-tune Λ to get

$$(10^{-12})^4 = (100)^4 + (0{,}25)^4 + (175)^4 + \dots + \frac{\Lambda}{8\pi G_N}$$

two problems : (9)

- The "old" cosmological constant problem

why fine-tune Λ ?

- The dark energy problem :

need to fine tune Λ and leave a small

$$\frac{\rho_\Lambda}{(100)^4} \sim 10^{-56} \quad \text{why?}$$

People hoped $\exists$ mechanism /

- ρ_Λ dynamically generated

$\downarrow$

$\boxed{\text{Self tuning}}$

- Another possibility is :

misinterpreting data $\rightarrow$ modification of GR at large scale would solve the dark energy problem

but not the "old" problem as fine-tuning

Λ still needed....

II Modifying gravity

oldest example $\longrightarrow$ massive gravity (Fierz-Pauli 1939)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

in GR gravitons are massless

$$\boxed{\mathcal{L}_{massive} = -\frac{m_G^2 \, m_{Pl}^2}{8} \left(h_{\mu\nu} h^{\mu\nu} - (h^\mu_\mu)^2 \right)}$$

no ghost in Minkowski background

$$h_{\mu\nu} = \partial_\mu \phi_\nu + \partial_\nu \phi_\mu$$

(ϕ_μ Stuckelberg fields
||
goldstone bosons of
broken covariance)

$$\mathcal{L}_{massive} \sim -\frac{m_G^2 m_{pl}^2}{8} F_{\mu\nu} F^{\mu\nu}$$

like in QED

no ghost.

$$F_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

massive graviton $\Rightarrow$ gravity falls off like

$$\frac{1}{r} e^{-m_{GR} r}$$

$r \gtrsim \frac{1}{m_{GR}}$ gravity is weaker $\rightarrow$ link with acceleration ?

not clear ....

problems of massive gravity

- $j = 2$ $\qquad 2j+1 = 5$

$$5 = \underset{\text{graviton}}{2} + \underset{\text{vector}}{2} + \underset{\text{scalar}}{1}$$

Scalar → problems in the presence of matter

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \partial_\nu \phi$$

↑ scalar

$\bar{h}_{\mu\nu}$ and ϕ are coupled → diagonalize the action (quadratic order)

$$\begin{cases} \hat{h}_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{m_G^2}{2} \eta_{\mu\nu} \phi \\ \hat{\phi} = \sqrt{\frac{3}{8}}\, m_G^2 m_{Pl}\, \phi \end{cases}$$

After gauge fixing (like in electrodynamics)

- $\partial_\mu \psi^{\mu\nu} = 0$ de Donder

$$\psi^{\mu\nu} = \hat{h}_{\mu\nu} - \frac{1}{2} \hat{h}\, \eta_{\mu\nu}$$

(gauge invariance)

- $\sqrt{\frac{3}{2}}\, \hat{\phi} + \frac{\hat{h}}{4} m_{Pl} = 0$ ($\hat{h}$ and $\hat{\phi}$ degenerate)

The equations of motion are

$$\boxed{(\Box - m_G^2)\,\hat{\phi} = \frac{T}{\sqrt{6}\, m_{Pl}}}$$ Klein-Gordon

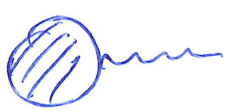
$$\boxed{(\Box - m_G^2)\,\varphi_{\mu\nu} = -16\pi G_N T_{\mu\nu}}$$

GW equation

$$\rightarrow \quad \hat{\phi} = -\frac{1}{\sqrt{6}\, m_{Pl}} \frac{T}{p^2 + m_G^2}$$

$$\rightarrow \quad \hat{h}_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)$$

$$\hookrightarrow \quad \boxed{\bar{h}_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T\right)}$$



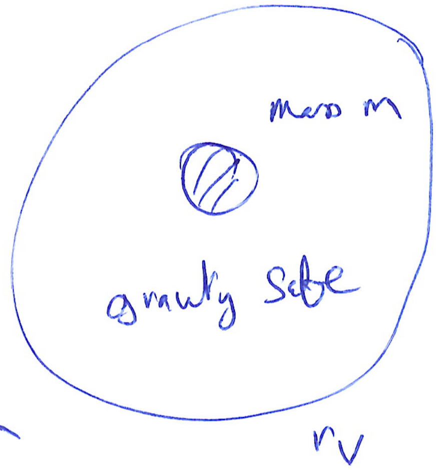
as $m_G^2 \to 0$ $\qquad \frac{1}{3} \neq \frac{1}{2} \Rightarrow$

Van Dam - Veltman - Zakharov discontinuity !

Kills the model .... No ! saved by

non-linearities.

$$\hat{\phi} \sim -\frac{m}{m_{Pl}\, r}$$



non-linear corrections to Lagrangian

$$\int d^4x \; m_{Pl}^2 \, m_G^2 \left(\partial^2 \hat{\phi}\right)^3$$

$$\sim \int d^4x \; \frac{\left(\partial^2 \hat{\phi}\right)^3}{\underbrace{m_{Pl}\, m_G^4}_{\Lambda_5^5}} \qquad \frac{\left(\partial^2 \phi\right)^3}{m_{Pl}\, m_G^2} \gg \left(\partial \phi\right)^2$$

$$\Rightarrow \quad \boxed{r \lesssim r_V = \left(\frac{m}{m_{Pl}^2\, m_G^4}\right)^{1/5}}$$

$\Rightarrow$ gravity + ~~scalar~~

not propagating inside r_V

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$$\Lambda_S = \left(m_{Pl} \, m_G^4 \right)^{1/5}$$ cut off scale

of theory valid for $r \lesssim \frac{1}{\Lambda_S}$

(perturbative expansion)

$$\boxed{\frac{1}{\Lambda_S} \sim 10^{11} \, m}$$

for $m_G = H_0$

gravity modified on cosmological scales.

Non-linear massive gravity

unknown until 2010's ... still problematic....

need two metrics and two dynamical ones!

$$g^{\alpha}_{\mu\nu} = \eta_{ab} \, e^{\alpha a}_{\mu} \, e^{\alpha b}_{\nu}$$

a = flat indices $\quad \alpha = 1, 2 \quad e^{\alpha a}_{\mu}$ vielbein

μ = curved

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$$\boxed{S = \int d^4x \sqrt{-g^1}\, \frac{R_1}{16\pi G_N} + \int d^4x \sqrt{-g^2}\, \frac{R_2}{16\pi G_N}}$$

The coupling to matter is through the composite metric

$$g_{\mu\nu} = ? = \eta_{ab}\, e^a_\mu e^b_\nu$$

$$\boxed{e^a_\mu = \beta_1 e^{1a}_\mu + \beta_2 e^{2a}_\mu}$$

$\beta_{1,2} \rightarrow$ coupling constants normalised such that

$$\beta_1^2 + \beta_2^2 = 1$$

The potential term corresponding to a massive graviton

$$\boxed{S_V = \Lambda^4 \sum_{i,j,k,l} m^{ijkl}\, \varepsilon_{abcd}\, \varepsilon^{\mu\nu\rho\sigma} \int d^4x\, e^a_{i\mu} e^b_{j\nu} e^c_{k\rho} e^d_{l\sigma}}$$

where $\boxed{m_g^2 \sim \dfrac{\Lambda^4}{m_{Pl}^2}}$

link between dark energy Λ^4 and the graviton mass

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Cosmology

$$ds_\alpha^2 = a_\alpha^2\left(-b_\alpha^2 d\eta^2 + dx^2\right)$$

choose $b_1 = 1$ $\quad b_2 = b$

2 Friedman equations

$$\boxed{\begin{aligned} 3H_1^2 m_{P1}^2 &= \beta_1 \frac{a_J^3}{a_1^3}\rho + 24\Lambda^4 m^{1jkl}\frac{a_j a_k a_l}{a_1^3} \\ \frac{3H_2^2}{b^2} m_{P1}^2 &= \beta_2^2 \frac{a_J^3}{a_2^3}\rho + 24\Lambda^4 m^{2jkl}\frac{a_j a_k a_l}{a_2^3} \end{aligned}}$$

$$g_{\mu\nu} = \left(\left(\beta_1 a_1 + \beta_2 a_2 b\right)^2, \left(\beta_1 a_1 + \beta_2 a_2\right), \left(\beta_1 a_1 + \beta_2 a_2\right), \left(\beta_1 a_1 + \beta_2 a_2\right)\right)$$

a_J

Composite metric

Conservation of matter $\Rightarrow$ $\rho_{\text{con}} \sim \rho_0 / a_J^3$ etc...

Compatibility between the (00) and (ic) (18)

Einstein equations (long and tedious!)

$\Downarrow$

$$\boxed{b = \frac{a_2 H_2}{a_1 H_1}}$$

physical interpretation

$\Lambda^4 \rightarrow$ plays the role of cosmological constant

. in fact when $\Lambda^4 \ll \rho$ (matter or radiation dom.)

$$\boxed{\frac{a_2}{a_1} = \frac{\beta_2}{\beta_1}}$$ proportional metrics

. if DE dominates $\Lambda^4 \gg \rho$ $\frac{a_2}{a_1} = $ const

$$\boxed{\frac{a_2}{a_1} = \frac{m^{2jkl} a_j a_k a_l}{m^{1jk} a_j a_k a_l}}$$

GW constraints

- $|b-1| \lesssim 10^{-15} \Rightarrow b = 1$ at late time

if $b=1$ at late time $\Rightarrow$ $b=1$ all the time

(theorem) $\Rightarrow$ constraint between the parameters

m^{2}, $\beta_{1,2}$.

- if $b=1$ the solution is

proportional metrics

$$\frac{a_2}{a_1} = \frac{\beta_2}{\beta_1} \quad \text{all the time}$$

- At the background level, identical

to Λ-CDM

perturbation theory (complicated!)

Separate into two sectors :

$$ds^2_\alpha = a^2_\alpha \left(\eta_{\mu\nu} + h^\alpha_{\mu\nu} \right) dx^\mu dx^\nu$$

perturbations

$$\boxed{\begin{cases} h_+ = a_1 h^1_{\mu\nu} + a_2 h^2_{\mu\nu} \\ h_- = h^1_{\mu\nu} - h^2_{\mu\nu} \end{cases}}$$

- h_+ ↙ cosmological perturbations coupled to matter, identical to GR

- h_- ↙ cosmological perturbations decoupled from matter

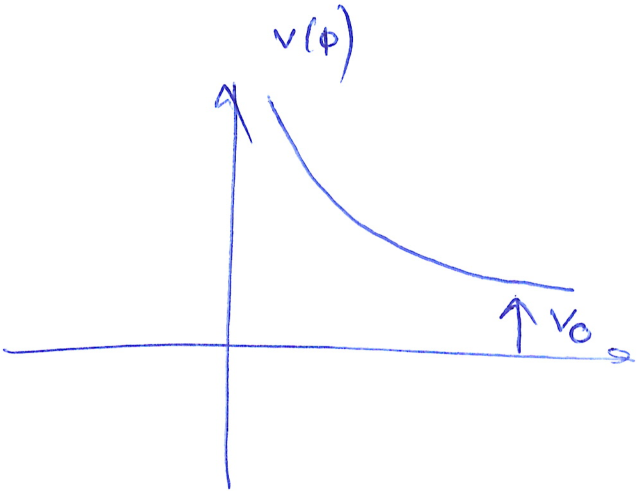
problem $h_{-\mu\nu}$ contains vectors with an instability (exponential growth) in the radiation era → physical effects need further study

II Scalar models

- need to add some matter to generate acceleration and/or vacuum energy.

Nothing to do with "old" cosmological constant problem

as



V_0 not determined apriori

→ general obstruction "Weinberg's no-go theorem"

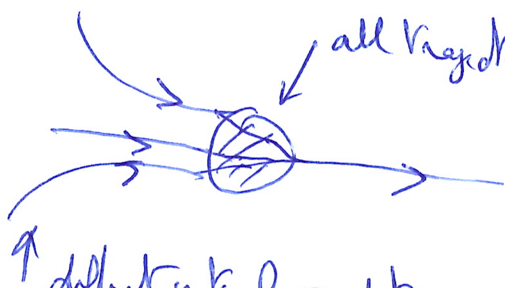
↓

"No Poincaré invariant vacuum with self tuning"

- dependence on the initial conditions?

→ need attractors

all trajectories converging



different initial conditions

- Singlet model → scalar field

Klein-Gordon

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

energy - pressure

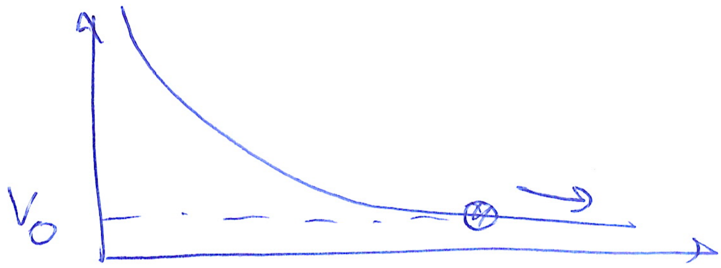
$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$w_\phi \approx -1 \Rightarrow \frac{\dot{\phi}^2}{2} \ll V(\phi)$ slow roll

two types of models

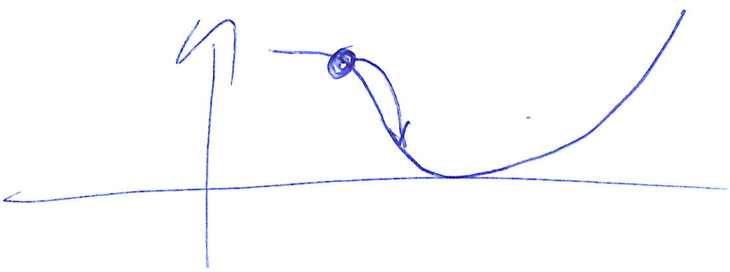
1) freezing



field going down and slowing down at late time

$V_0 \approx 3\Omega_{\Lambda 0} m_{Pl}^2 H_0^2$

2) Thawing



field moving at late time only and near bottom of potential now

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freezing

$$\begin{cases} \lambda_\phi = -\dfrac{d\ln V}{d x_4 \phi} \\ \dfrac{d\lambda_\phi}{dt} = -\sqrt{6}\,\lambda_\phi^2 (\Gamma - 1) H x \end{cases}$$

$x_4 = \dfrac{1}{m_{pl}}$

$x = \dfrac{x_4 \dot{\phi}}{\sqrt{6}\, H}$

$x \neq 0$ (violation of Weinberg's no-go theorem)

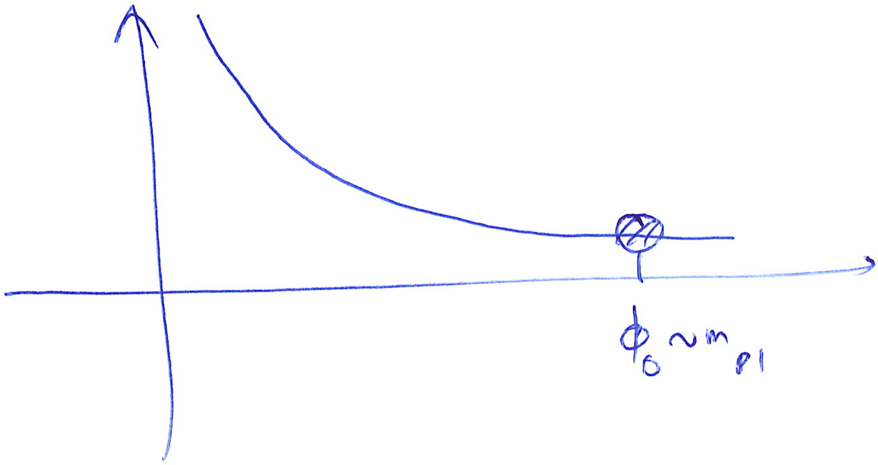
$\Gamma > 1 \Rightarrow \boxed{\lambda_\phi = 0 \text{ attractor}}$

eg Ratra - peebles

$$\begin{cases} V(\phi) = \dfrac{\Lambda^{4+n}}{\phi^n} \\ \Gamma = 1 + \dfrac{1}{n} > 1 \end{cases}$$

Λ needs to be fine-tuned

slow roll roll $\Rightarrow m^2 \sim O(H^2) \Rightarrow \boxed{\phi_0 \sim m_{pl}}$



$$\Lambda^{4+n} \sim \phi_0^n \, m_{Pl}^2 \, H_0^2 \sim m_{Pl}^{n+2} \, H_0^2$$

$$m_0^2 \sim \frac{V(\phi)}{\phi^2} \sim \frac{m_{Pl}^2 H_0^2}{\phi_0^2} \sim \frac{\Lambda^{4+n}}{\phi_0^{n+2}} \sim H_0^2$$

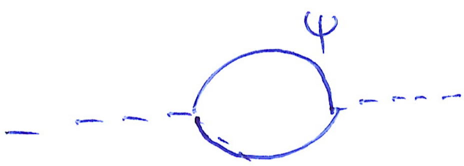
quintessence problems

- very light scalar fields!

$$\delta V_\phi \sim \frac{m_\phi^4}{64\pi^2} \qquad \delta m_\phi^2 \sim \frac{H_0^4}{64\pi^2 m_{Pl}^2} \ll m_0^2$$

$\Rightarrow$ correction to scalar (quantum) negligible

- if ϕ couples to matter (see later)



$$\delta m_\phi^2 \sim \beta^2 \frac{m_\psi^4}{m_{Pl}^2}$$

too large for $\beta \sim 1$ & $m_\varphi \gtrsim 10^{-3}\,eV$

→ requires a very small $\beta \ll 1$ coupling otherwise

extremely large quantum corrections.

→ problem for all scalar models with a non-linear potential

→ can be evaded by models with shift symmetry

$$\phi \to \phi + c$$

(Non-Renormalisation theorems) (see 1607.01129 for a detailed analysis of K-mouflage.

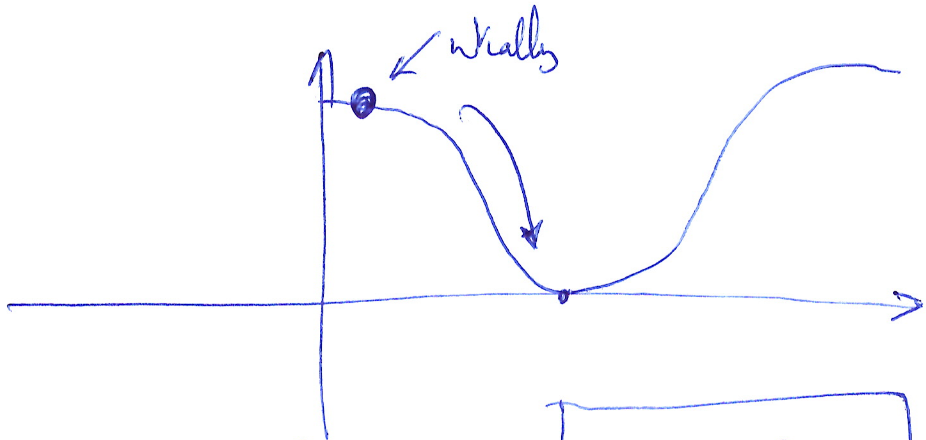
thawing models

typical example Goldstone (pseudo) boson

$$U(1)_{global} \text{ broken at } \langle \phi \rangle = f$$

pseudo ⇒ explicit symmetry breaking and

$$V(\phi) = \mu^4 \left(\cos \frac{\phi}{\sqrt{2} f} + 1 \right)$$



$$\mu^4 \sim H_0^2 m_{Pl}^2 \Rightarrow \boxed{\mu \sim 10^{-3}\,\mathrm{eV}}$$

$$\boxed{m_\phi^2 \sim \frac{\mu^4}{f^2}}$$

The field evolves when $H \lesssim m_\phi$

(Hubble friction prevails at before)

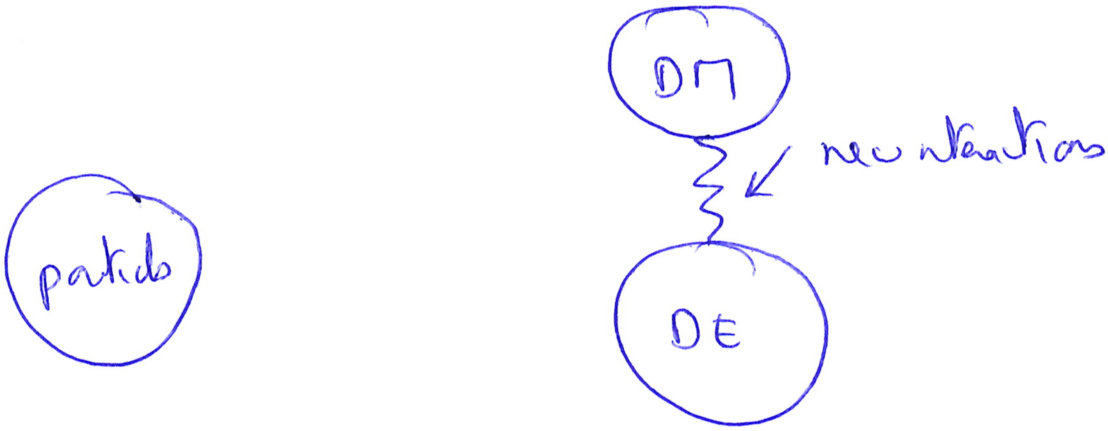
$$\Rightarrow \boxed{f \sim m_{Pl}}$$

Because of the (softly) U(1) symmetry, interaction of ϕ with matter are derivative

$$\mathcal{L}_{int} \sim \frac{(\partial\phi)^2}{f^3} \bar{\Psi}\Psi \Rightarrow \left\{ \begin{array}{l} \text{Tiny corrections} \\ \text{to the} \\ \text{mass} \end{array} \right.$$

$\uparrow$ fermions

Coupled dark energy



The simplest way of introducing interaction

$$m_0 \longrightarrow m = A(\phi)\, m_0$$

mass of DM particles

function of DE

Conservation of # of particles n $\Rightarrow$

$$\frac{d(a^3 n)}{dt} = 0$$

Conserved matter density $\boxed{\rho = m_0 n}$

The density of DM $\neq \rho$ $\boxed{\rho_E = m n}$

non-conservation equation

$$\boxed{\dot{\rho}_E + 3H\rho_E = \frac{\beta_\phi}{m_{Pl}}\rho_m\dot{\phi}}$$

$$\boxed{\beta_\phi = \frac{d\ln A}{d\phi} m_{Pl}}$$

Now for the scalar

$$\begin{cases} \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

Friedman

$$H^2 = \frac{1}{3m_{Pl}^2}(\rho_E + \rho_\phi)$$

$$\rho_E + \rho_\phi = \rho + \rho_{eff}$$

$$\boxed{\rho_{eff} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + (A(\phi) - 1)\rho}$$

$$\boxed{V_{eff}(\phi) = V(\phi) + (A - 1)\rho}$$

$\rho_T = \rho_E + \rho_\phi$ conserved $\Rightarrow$

$$\boxed{\frac{d\rho_T}{dt} + 3H(\rho_T + p_T) = 0}$$

$p_T = p_\phi$

$$\Rightarrow \quad \frac{d\rho_\phi}{d\phi} + 3H(\rho_\phi + p_\phi) = -\frac{\beta\phi}{m_{Pl}}\rho_E\dot{\phi}$$

$\Rightarrow$ Klein-Gordon

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_{eff}}{\partial\phi} = 0}$$

together with

$$\boxed{H^2 = \frac{1}{3m_{Pl}^2}(\rho + \rho_{eff})}$$

Moreover $\quad \dfrac{d\rho_{eff}}{dt} + 3H(\rho_{eff} + p_\phi) = 0 \quad \Rightarrow$

$$\boxed{w_\phi = \frac{P_\phi}{\rho_{eff}}}$$

$$w_\phi = \frac{\frac{1}{2}\dot\phi^2 - V}{\frac{1}{2}\dot\phi^2 + V + \rho(A-1)} \Rightarrow$$

Can be $w_\phi < -1$

if $\dot\phi \sim 0$

$$w_\phi \sim \frac{-V}{V + \rho(A-1)} < -1$$

if $A < 1$

$\Rightarrow$ Can cross the "phantom divide"

Constraints from CMB : $\beta_\phi \lesssim 0,05$

$\rightarrow$ modification of the position of the CMB peaks.

eg $\quad A(\phi) = e^{-\beta\phi/m_{Pl}} \qquad V = V_0 e^{-\lambda\phi/m_{Pl}}$

$$a \sim a_{eq}\left(\frac{t}{t_{eq}}\right)^{\frac{2}{3} - \frac{4}{9}\beta^2}$$

modification of the distance by β^2 effects.

IV Scalar tensor - theory

all matter coupled to

$$\boxed{g^J_{\mu\nu} = A^2(\phi)\, g^E_{\mu\nu}}$$

Jordan — Einstein

$$\boxed{S = \int d^4x \sqrt{-g_E}\left(\frac{R_E}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_m(\psi, g^J_{\mu\nu})}$$

Same cosmology as coupled dark energy, baryons also coupled.

↳ Weak equivalence principle respected

Matter non conserved (in Einstein frame)

$$\boxed{D_\mu T^{\mu\nu} = -\frac{\beta\phi}{m_{Pl}} T^{\mu\nu}\,\partial_\nu \phi}$$

Important consequence: fifth forces!

take matter

$$T_{\mu\nu} = \rho_E u^m_\mu u^n_\nu \qquad u^\mu = \frac{dx^\nu}{d\tau}$$

τ proper time

$$\dot\phi = u^\mu D_\mu \phi \text{ etc..}$$

- non-conservation $\Rightarrow$

$$\boxed{\dot\rho_E + 3h\,\rho_E = \frac{\beta\phi}{m_{Pl}}\,\rho_E\,\dot\phi} \qquad \underbrace{3h = D_\mu u^\mu}_{\text{local Hubble rate}}$$

$$\Rightarrow \quad \boxed{\rho_E = A\rho}$$

ρ conserved even with perturbations etc...

$$\dot\rho + 3h\rho = 0$$

and the geodesics :

$$\boxed{\dot u_\mu = -\frac{\beta\phi}{m_{Pl}}\left(\partial_\mu\phi + \dot\phi\, u_\mu\right)}$$

Non-Relativistic limit:

$$\boxed{\partial_t v^i = \underbrace{-Hv^i}_{\text{Hubble friction}} \underbrace{-\partial^i\Phi_N}_{\text{Newton}} \underbrace{-\partial^i \ln A}_{\text{fifth force}}}$$

The scalar induces an acceleration

$$a^i = -\partial^i \ln A$$

why?

$$\begin{cases} ds^2_E = a^2\left((-1-2\phi_N)\,d\eta^2 + (1-2\phi_N)\,dx^2\right) \\ ds^2_J = a^2 A^2\left((-1-2\phi_N)\,d\eta^2 + (1-2\phi_N)\,dx^2\right) \end{cases}$$

$$= \underbrace{a^2 A_0^2}_{\text{Background Cosmology}} \left((-1-2\Phi)\,d\eta^2 + (1-2\Psi)\,dx^2\right)$$

$$\boxed{\begin{aligned} \Phi &= \Phi_N + \ln\frac{A}{A_0} \\ \Psi &= \Phi_N - \ln\frac{A}{A_0} \end{aligned}}$$

In Jordan frame acceleration

origin of the scalar force

$$-\vec\nabla\Phi \equiv \hat a = -\vec\nabla\left(\Phi_N + \ln\frac{A}{A_0}\right)$$

Consequences :

- modification of the growth of structure

$$\frac{\partial \vec{v}}{\partial t} + H \vec{v} = - \frac{\vec{\nabla}}{a}\left(\Phi_N + \ln \frac{A}{A_0}\right) \qquad \frac{1}{a} \swarrow \text{comoving!}$$

$$\theta = \vec{\nabla}\cdot\vec{v}$$

$$\frac{\partial \theta}{\partial t} + H\theta = -\frac{\Delta}{a^2}\left(\phi_N + \ln\frac{A}{A_0}\right)$$

$$= \ ?$$

Poisson

$$\Delta \phi_N = 4\pi G_N \, \delta\rho_E \, a^2$$

$$\boxed{\Delta \phi_N = 4\pi G_N A_0 \rho \, \delta a^2}$$

$$\Delta \ln \frac{A}{A_0} = \frac{\beta_\phi}{m_{Pl}} \Delta\phi$$

$$= \frac{\beta_\phi}{m_{Pl}} \Delta \delta\phi$$

Cosmological evolution

$$\delta\rho_E \sim A_0 \,\delta\rho$$

$$\delta = \frac{\delta\rho}{\rho}$$

$$\Delta\phi = \Delta\delta\phi$$

$$\phi = \phi_0 + \delta\phi$$

Cosmo (only t dependence)

Klein Gordon

$$\ddot{\delta\phi} + \frac{k^2}{a^2}\delta\phi + m_\phi^2 \,\delta\phi = -\frac{\beta_\phi a^2}{m_{Pl}} A_0 \,\delta\rho = -\frac{\beta_\phi}{m_{Pl}} A_0 \rho \,\delta a^2$$

$$+ 3H\dot{\delta\phi}$$

quasi-static approximation

$$\boxed{\frac{k^2}{a^2}\,\delta\phi \gg \delta\dot{\phi}}$$

Deep inside the Horizon

$$\boxed{\frac{k}{a} \gg H} \quad \Rightarrow \quad \frac{k^2}{a^2}\,\delta\phi \gg H\,\delta\dot{\phi}$$

$$\left(\frac{k^2}{a^2} + m_\phi^2\right)\delta\phi = -\frac{\beta_\phi A_0}{m_{Pl}}\,\rho\,\delta\,a^2$$

$$\Rightarrow \quad \delta\phi = -\frac{\beta_\phi A_0}{m_{Pl}}\,\frac{\rho\,\delta\,a^2}{\frac{k^2}{a^2}+m_\phi^2} \quad \Rightarrow$$

$$\boxed{\Delta\,\delta\phi = \frac{\beta_\phi}{m_{Pl}}\,\frac{A_0\,\rho\,\delta}{1+\frac{m_\phi^2 a^2}{k^2}}}$$

Conservation of matter

In the Jordan frame matter conserved

$$\theta + \frac{\dot{\delta\rho_J}}{\rho_J} = 0 \quad \Rightarrow \quad \boxed{\theta = -\dot{\delta} - \frac{d\ln A_0}{dt}\,\delta}$$

$\Rightarrow$ (36)

$$\boxed{\ddot{\delta} + \left(H + \frac{d\ln A_0}{dt} \right) \dot{\delta} - \frac{3}{2} \Omega_m H^2 A_0 \left(1 + \frac{2\beta^2 \phi}{1 + \frac{m_\phi^2 a^2}{k^2}} \right) \delta = 0}$$

modification of gravity

- $A_0 \neq 1 \Rightarrow \begin{cases} \text{friction } \frac{d\ln A_0}{dt} \\ \text{time dependence of Newton's} \end{cases}$

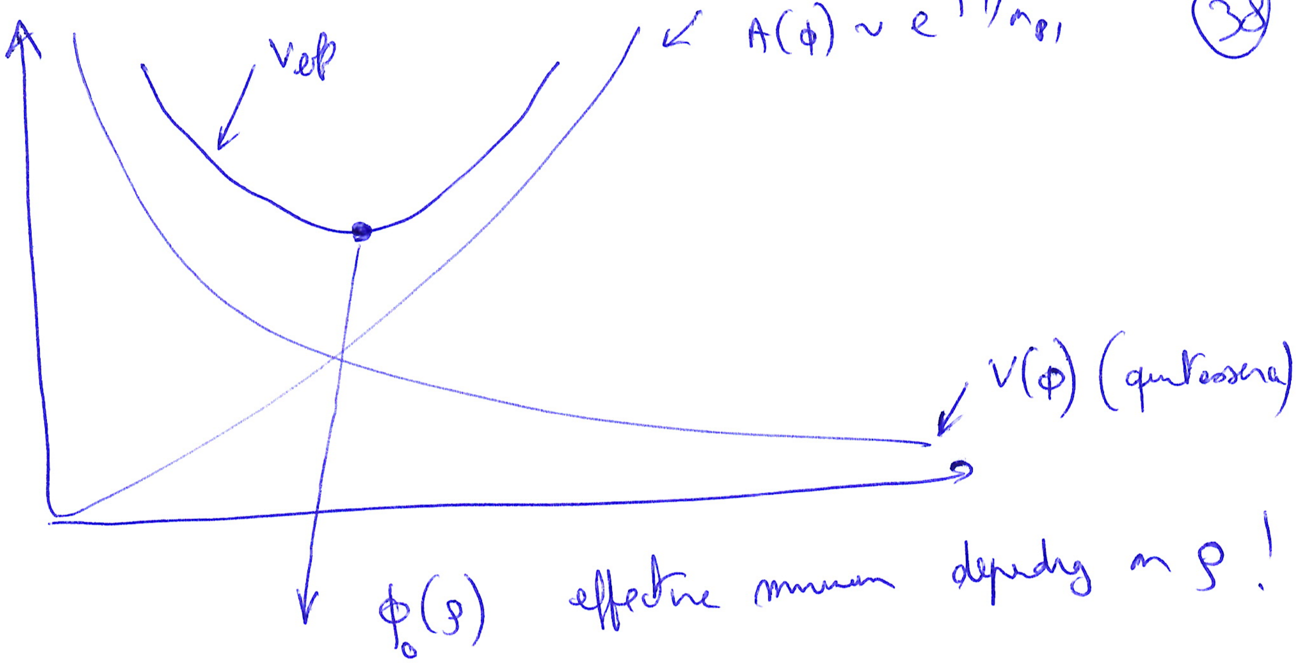
  constant $\Omega_m H^2 A_0 \frac{3}{2}$

  $\neq 1$

- scale dependent effect.

$$\varepsilon = 1 + \frac{2\beta^2 \phi}{1 + \frac{m_\phi^2 a^2}{k^2}}$$

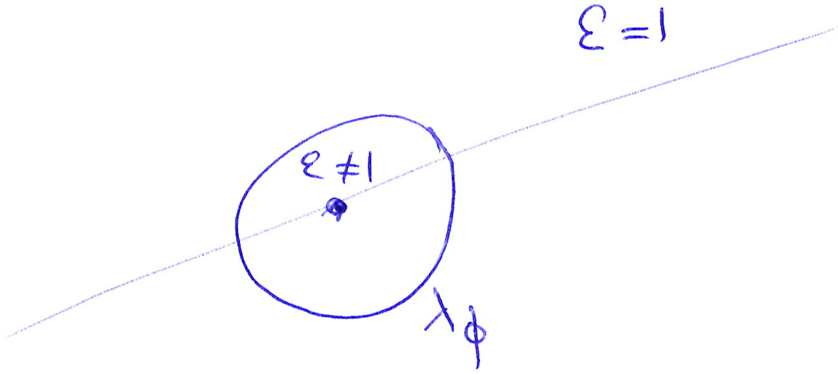
$$m_\phi^2 = \left. \frac{d^2 V_{eff}}{d\phi^2} \right|_{\phi_0} = \text{time dependent}$$



$m^2_\phi \nearrow$ when $\rho \nearrow$ chameleon property

(necessary to achieve screening in solar system)

$$\frac{1}{m_\phi a} = \lambda_\phi = \text{comoving Compton wavelength}$$



$\varepsilon \neq 1$ modification of gravity $\Rightarrow$ more growth of δ inside the Compton wavelength.

Lensing

$$\Phi_L = \frac{\Phi + \Psi}{2} = \Phi_N$$

$\Rightarrow$ $\Sigma = 1$ no new lensing parameter

$$\boxed{\Delta \Phi_L = 4\pi G_N \Sigma a^2 \rho \delta}$$

But effect still as $\delta \neq \delta_{CDM}$ and k

dependent $\Rightarrow$ $\Phi_L \neq \Phi_{LCDM}$

μ parameter

$$\Delta \Phi = 4\pi G_N \mu a^2 \rho \delta$$

$$\mu = 1 + \frac{2\beta^2 \phi}{1 + \frac{m^2_\phi a^2}{k^2}}$$

more growth

Archetypical Example

$f(R)$ theories

$$S = \int d^4x \sqrt{-g}\, \frac{f(R)}{16\pi G_N} + S_m(\psi, g_{\mu\nu})$$

$\updownarrow$ equivalent

tensor-scalar theory with

$$\begin{cases} e^{-2\beta\phi/m_{Pl}} = \dfrac{df}{dR} \equiv f_R \\[2ex] V(\phi) = \dfrac{m_{Pl}^2}{2} \dfrac{R f_R - f}{(f_R)^2} \end{cases}$$

and the coupling is universal $\boxed{\beta = \frac{1}{\sqrt{6}}}$ like in massive gravity!

prime example $\boxed{f(R) = R + 2\Lambda - \frac{f_{R_0}}{n+1} \frac{R_0^{n+1}}{R^n}}$

R_0 = curvature universe now

cosmological constant

$n=1$ $\quad f_{R_0} \lesssim 10^{-6}$ local tests.

V Horndeski

Scalar-tensor theories are not the most general theories with gravity + scalar.

In fact one can include higher derivatives in the Lagrangian $\rightarrow$ problems with extra degrees of freedom which are ghosts (Ostrogradski theorem)

In classical mechanics

$$\mathcal{L}(q, \dot{q}, \ddot{q}) \nrightarrow \mathcal{L}(q, \dot{q}) \quad \text{more variables}$$

$$Q = \dot{q} \longrightarrow \text{new field}$$

Euler Lagrange

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial \mathcal{L}}{\partial \ddot{q}} = 0$$

$\rightarrow$ involves up to $q^{(4)}$ $\nearrow$ necessitates initial condition

$q, q^{(1)}, q^{(2)}, q^{(3)}$

2 degrees of freedom $\longleftarrow$

The Hamiltonian in terms of the 2 variables (42)

is unbounded from below → unstable

this can be avoided if Lagrangian is

degenerate :

$$\begin{cases} \dfrac{\partial^2 L}{\partial \ddot{q}} \neq 0 & \text{non degenerate} \\ \\ \dfrac{\partial^2 L}{\partial \ddot{q}} = 0 & \text{degenerate} \end{cases}$$

in field theory → Horndeski Lagrange

are the ones with <u>2nd</u> order Klein-Gordon

equation ⇒ no new ghost like field

(not the most general case ∃ theories with equations

of motion of higher order than 2 <u>but</u> propagating

only <u>one</u> scalar DHOST (beyond these lectures))

Define $X = -\frac{1}{2}(\partial\phi)^2$

k-essence

$$L_1 = K(X,\phi)$$

cubic

$$L_2 = -\,G_3(X,\phi)\,\Box\phi$$

$$L_3 = G_4(\phi,X)\,R + G_{4X}\left((\Box\phi)^2 - (D_\mu D_\nu\phi)^2\right)$$

$$L_4 = G_5(\phi,X)\,G^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{6}G_{5X}\Big[(\Box\phi)^3 - 3\Box\phi\,(D_\mu D_\nu\phi)^2 + 2(D_\mu D_\nu\phi)^3\Big]$$

Einstein

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}$$

$$S_{\text{Horndeski}} = \int d^4x\,\sqrt{-g}\,\left(L_1 + L_2 + L_3 + L_4\right)$$

Self-tuning

we have seen that theories with $V(\phi)$ non-linear

$\Rightarrow$ fine-tuning of the vacuum energy

Horndeski theories allow self-tuning

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$$\begin{cases} \dot{\phi} \neq 0 \\ \text{attractor with } H \neq 0 \text{ and independent} \end{cases}$$

of the vacuum energy

Unfortunately, this requires $G_4, G_5 \neq 0$

problem: $G_4, G_5 \longrightarrow$ mixing between

gravity and scalars

$$\Downarrow$$

modification of speed of GW

$$\boxed{c_T^2 = \frac{G_4}{G_4 - 2X G_{4,X}}}$$

need $G_4 = \frac{1}{16\pi G_N} = \text{constant}$

Self-tuning not viable with Horndeski (45)

.... even the cubic term generically

leads to too much ISW effect....

Conclusion

- Massive gravity is not self tuning

(need explicit vacuum energy linked to graviton mass)

- Horndeski not self-tuning

→ Beyond Horndeski?

(ambulance chasing?)

- Just a cc?

to be continued....