Averaging Problem and Backreaction

Thomas Buchert

Marseille 2018

CRAL
CENTRE DE RECHERCHE ASTROPHYSIQUE DE LYON

erc
European Research Council
Established by the European Commission

Lecture Outline

- Averaging scalars for flow-orthogonal foliations
- Averaging of general fluids on general foliations
- Backreaction in (quasi-)Newtonian cosmology ?

- Closure assumptions
- Scalar field analogy and exact scaling solutions
- Dark Energy-free Models and Observational Tests

General Thoughts

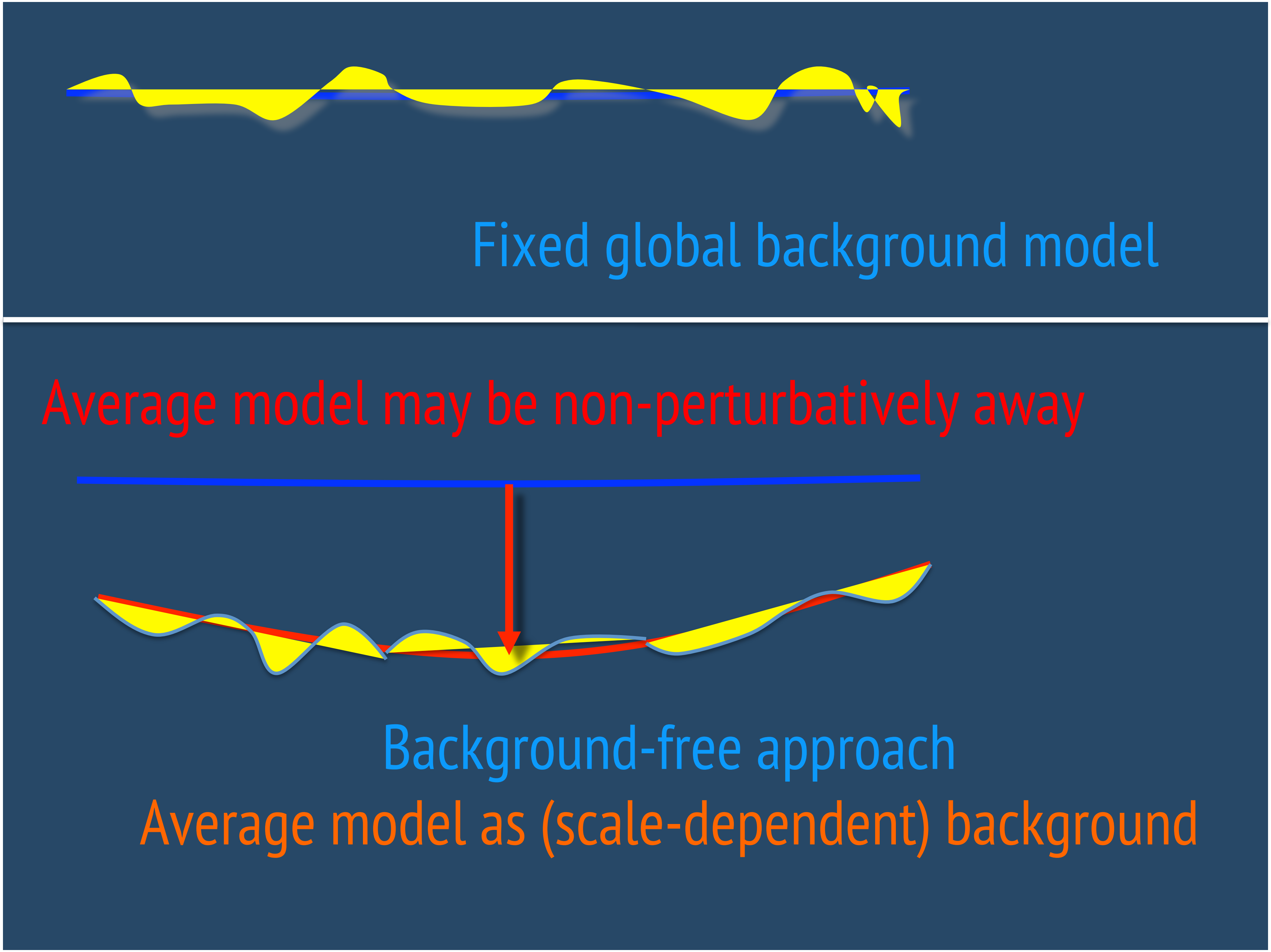
Why averaging ?

We see structures and conceive them as fluctuations with respect to an assumed background

The description of fluctuations makes only sense with respect to their average

The average distribution is homogeneous and large-scale isotropic ...

... but can be dynamically very different from a homogeneous-isotropic solution


Fixed global background model
Average model may be non-perturbatively away
Background-free approach
Average model as (scale-dependent) background

Why scalar averaging ?

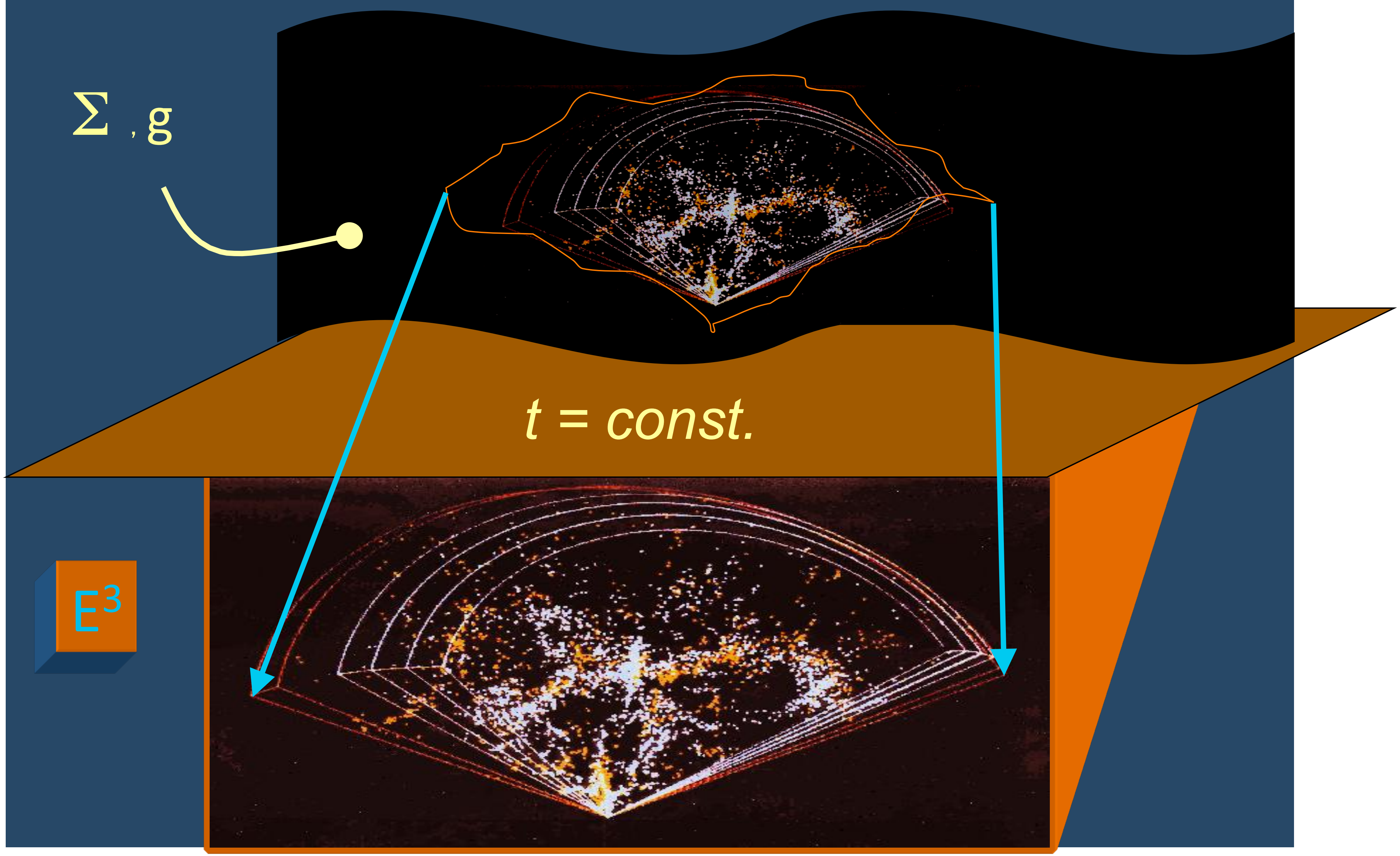
Averaging scalars is well-defined in GR

Averaging of tensors is rather a smoothing operation

Averaging of scalars captures integral properties within the inhomogeneous geometry …

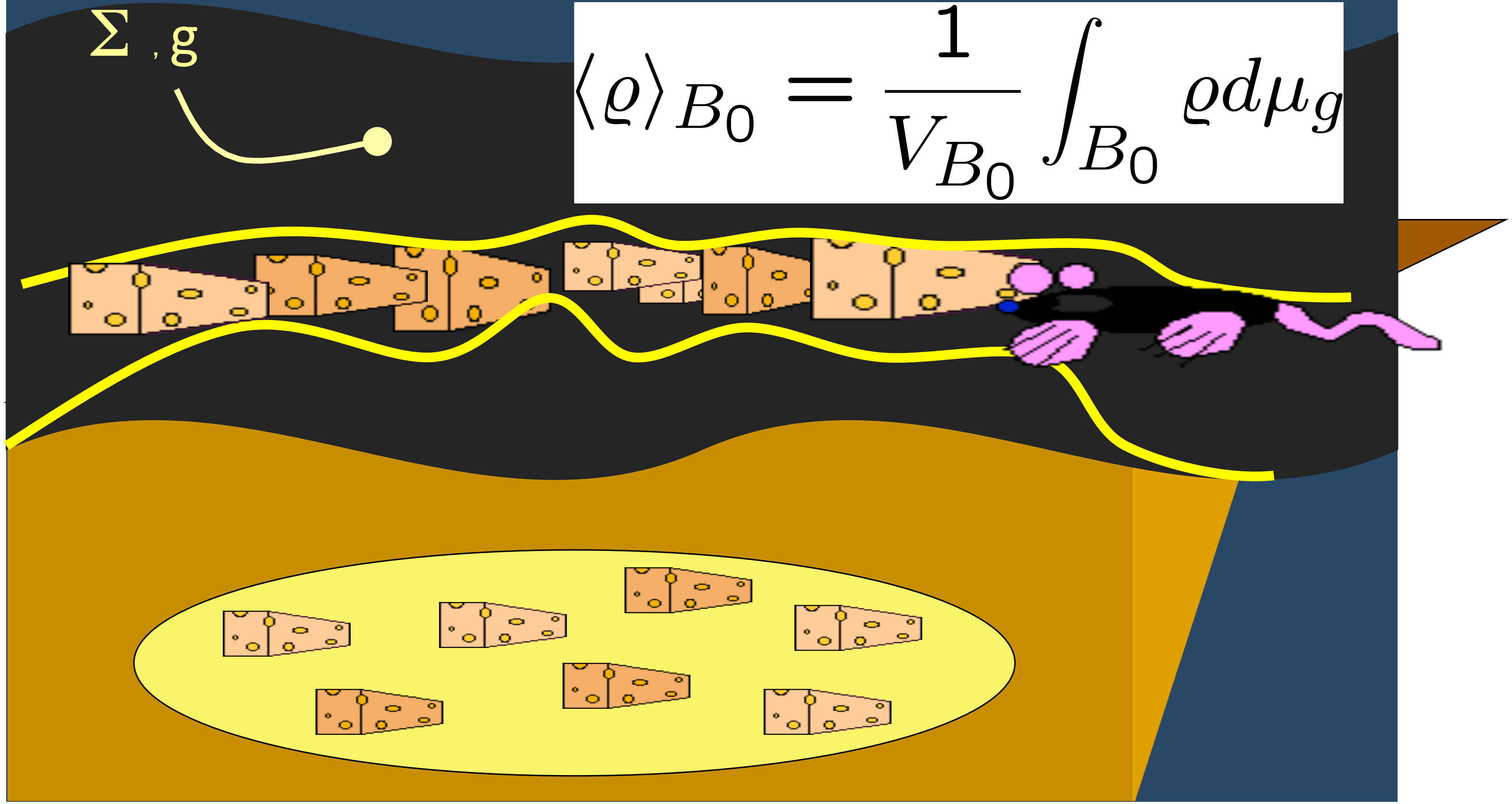
… while smoothing of tensors results in ‘dressed‘ integral properties on a homogeneous geometry

Smoothing the metric

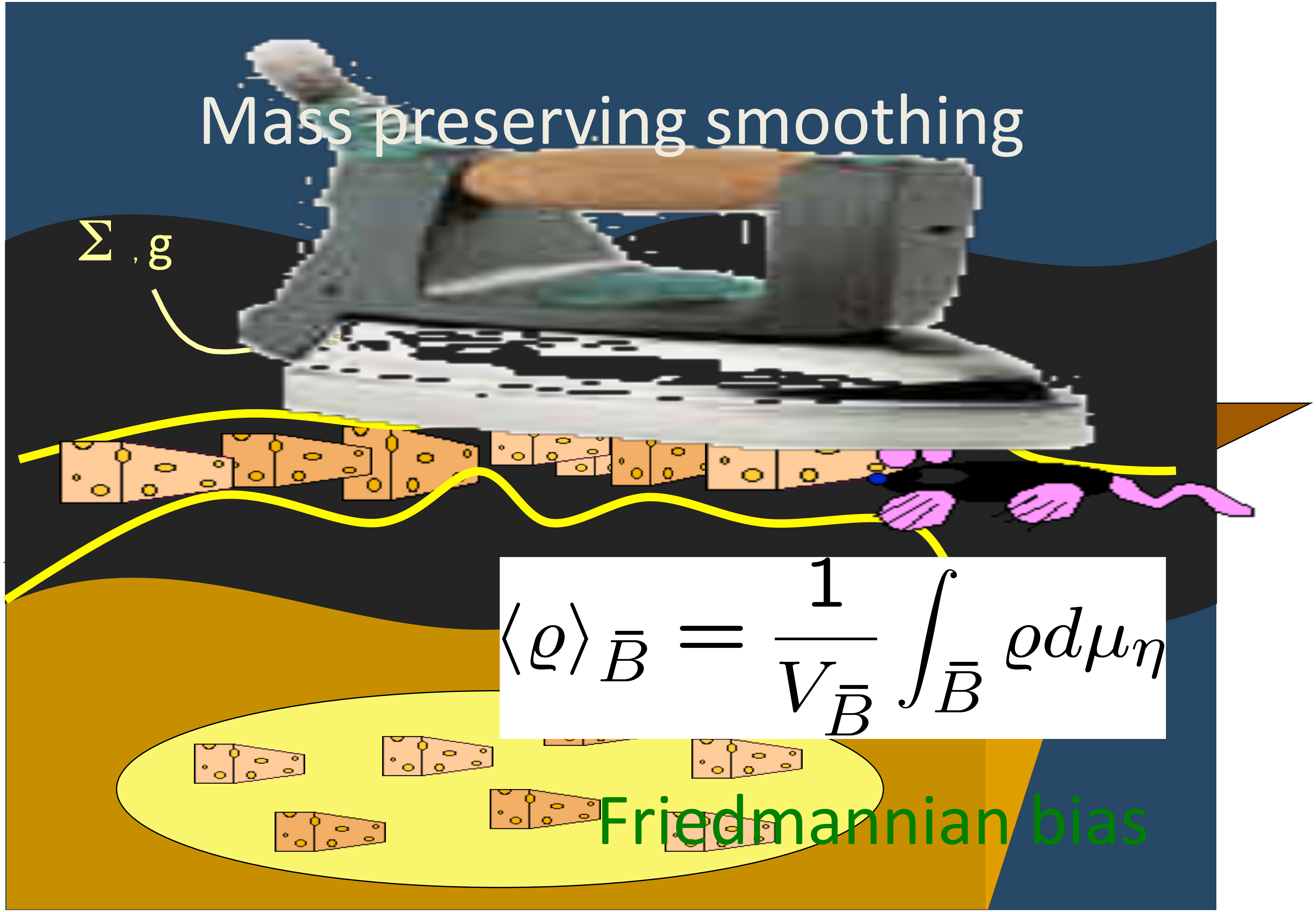


Mass preserving smoothing

$$\langle \varrho \rangle_{B_0} = \frac{1}{V_{B_0}} \int_{B_0} \varrho \, d\mu_g$$



Mass preserving smoothing



$$\langle \varrho \rangle_{\bar{B}} = \frac{1}{V_{\bar{B}}} \int_{\bar{B}} \varrho \, d\mu_\eta$$

Mass preserving smoothing

- ‘Bare’ Average Restmass Density :

$$\langle \varrho \rangle_{\mathcal{B}_0} := \frac{\mathrm{M}}{V_{\mathcal{B}_0}}$$

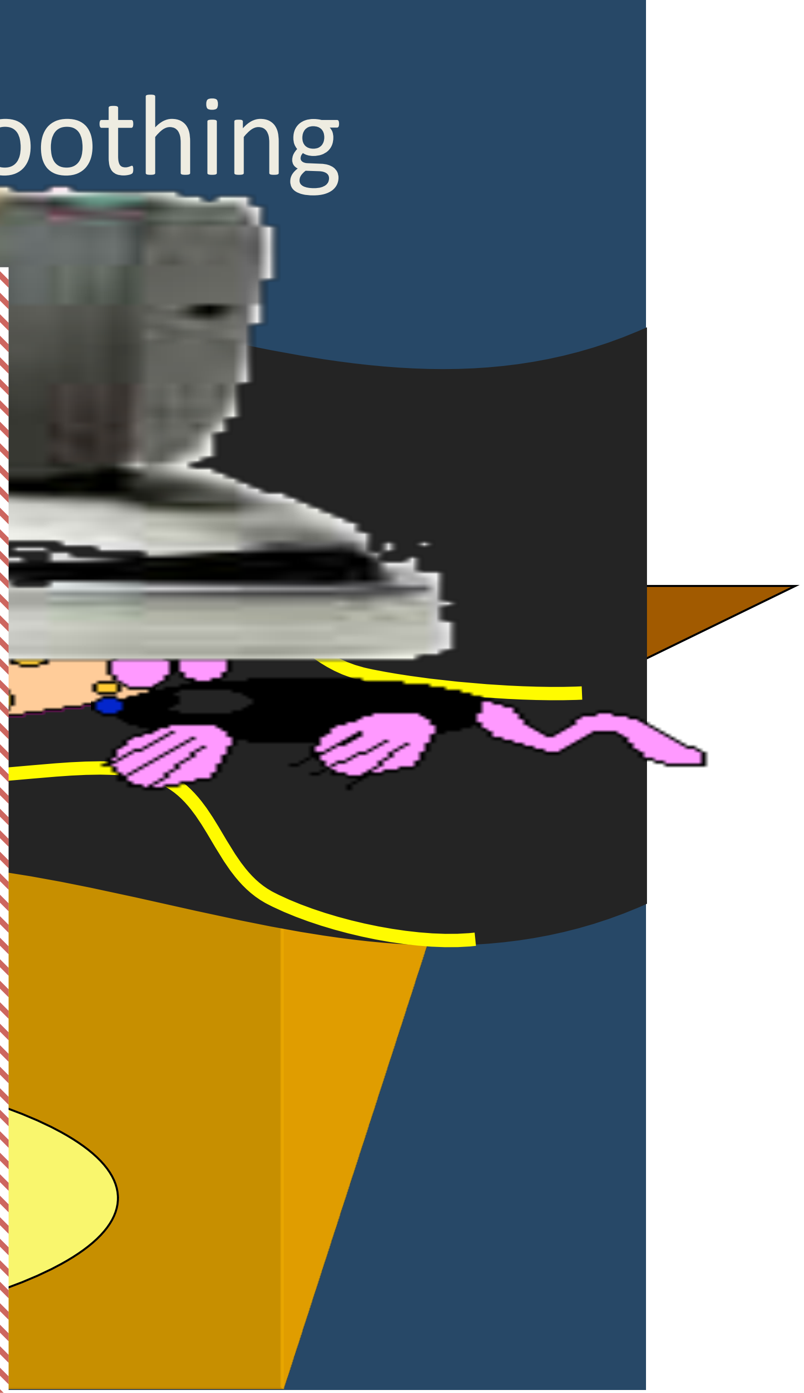
- ‘Dressed’ Average Restmass Density :

$$\langle \varrho \rangle_{\overline{\mathcal{B}}} := \frac{\mathrm{M}}{V_{\overline{\mathcal{B}}}}$$

- Volume Fraction :

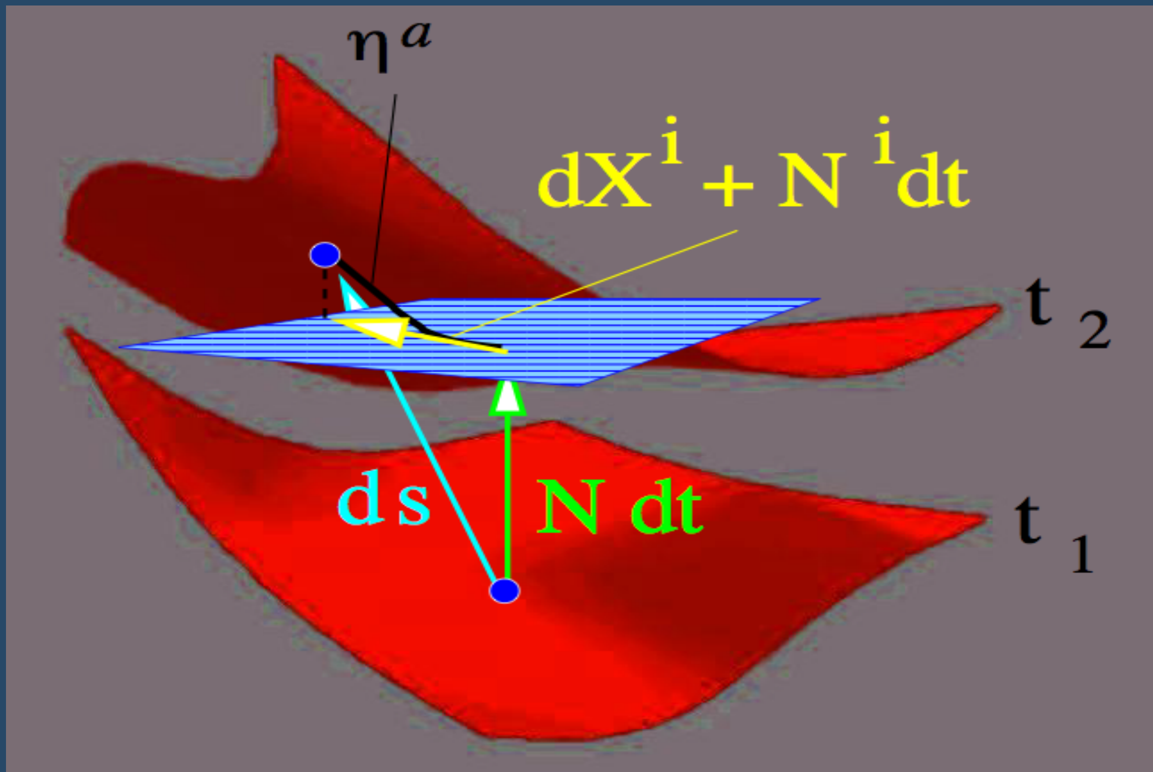
$$\nu := \frac{V_{\overline{\mathcal{B}}}}{V_{\mathcal{B}_0}}$$

T. Buchert, M. Carfora
arXiv:gr-qc/0210045
arXiv:gr-qc/0210037



Why spatial averaging ?

Cosmology is conceived as an
evolving space / hypersurface (3+1)
with a synchronous time (vs. local proper time)



Take home summary I

Backreaction describes the deviations of the average from an assumed homogeneous-isotropic FLRW solution

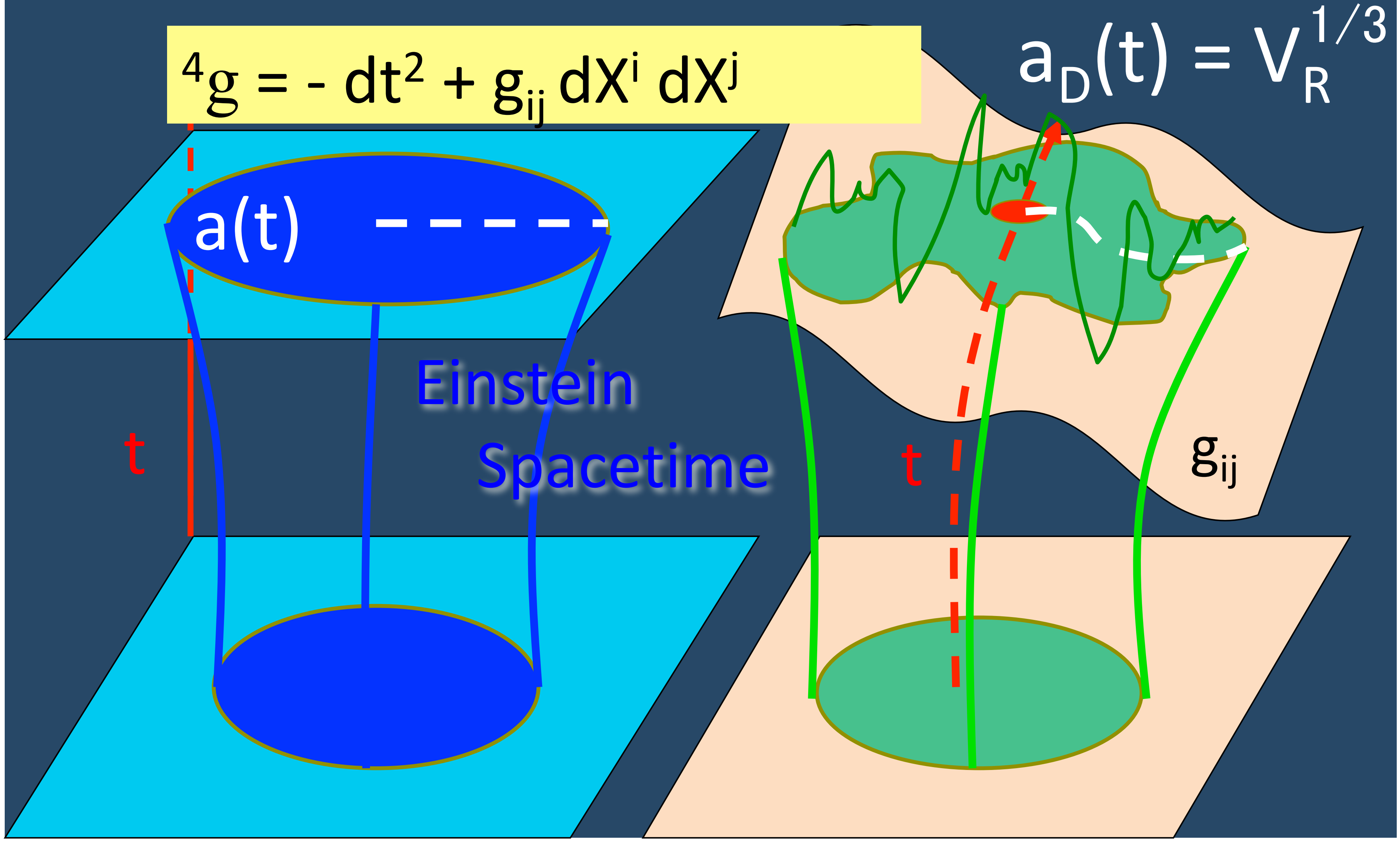
Backreaction arises when the fluctuations are allowed to determine the dynamics of the average model
Structures ' talk ' to the ' background '

Backreaction arises from inhomogeneities in geometry

Backreaction depends on the choice of foliation of space-time

Averaging in a flow-orthogonal foliation
Irrotational Dust

Averaging dust fluids in free fall



Averaging Operator

Spatial average of scalars on a compact domain :

$$\langle \mathcal{A} \rangle_{\mathcal{D}} := \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \mathcal{A} \, d\mu_g$$

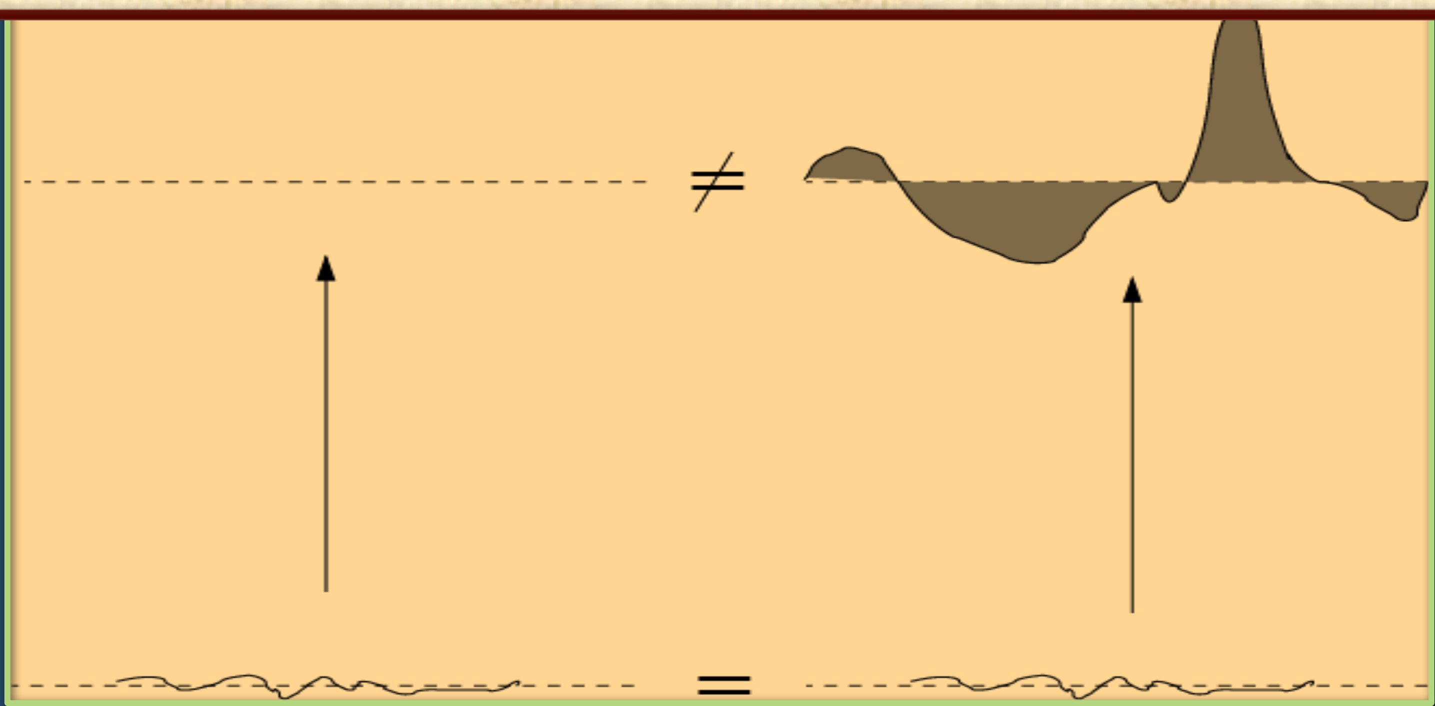
↑

Restmass conservation on the domain

important to compare averages at different times

Non-Commutativity

$$\frac{\partial}{\partial t}\langle \mathcal{A} \rangle - \langle \frac{\partial}{\partial t}\mathcal{A} \rangle = \langle \theta \mathcal{A} \rangle - \langle \theta \rangle \langle \mathcal{A} \rangle$$



Non-Commutativity

$$\frac{\partial}{\partial t}\langle \mathcal{A} \rangle \; - \; \langle \frac{\partial}{\partial t} \mathcal{A} \rangle \; = \; \langle \theta \mathcal{A} \rangle \; - \; \langle \theta \rangle \langle \mathcal{A} \rangle$$

$$\frac{\partial}{\partial t}\langle \theta \rangle \; - \; \langle \frac{\partial}{\partial t} \theta \rangle \; = \; \langle \theta^2 \rangle \; - \; \langle \theta \rangle^2$$

$$= \; \langle (\theta - \langle \theta \rangle)^2 \rangle$$

Relative Information Entropy increases

Kullback-Leibler distance : arXiv: gr-qc/0402076

$$\mathcal{S} = \int_{\mathcal{D}} \varrho \ln \frac{\varrho}{\langle \varrho \rangle} d\mu_g$$

Deviations from the standard model increase !

$$\langle \partial_t \varrho \rangle - \partial_t \langle \varrho \rangle = \frac{1}{V} \partial_t \mathcal{S}$$

$$\frac{\partial_t \mathcal{S}\{\varrho || \langle \varrho \rangle_{\mathcal{D}}\}}{V_{\mathcal{D}}} = - \langle \delta\varrho \Theta \rangle_{\mathcal{D}} = - \langle \varrho \delta\Theta \rangle_{\mathcal{D}} = - \langle \delta\varrho \delta\Theta \rangle_{\mathcal{D}}$$

Relative Information Entropy increases

Kullback-Leibler distance : arXiv: 1208.3376

$$\frac{\mathcal{S}_{\mathcal{D}}}{V_{\mathcal{D}}} = \frac{9}{32\pi G}\left(\frac{t^2}{8}\langle C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}\right) \quad \frac{\dot{\mathcal{S}}_{\mathcal{D}}}{V_{\mathcal{D}}} = \frac{3}{8\pi G}\left(\frac{t}{8}\langle C_{\mu\nu\lambda\rho}C^{\mu\nu\lambda\rho}\rangle_{\mathcal{D}} + \frac{\mathcal{Q}_{\mathcal{D}}}{t}\right),$$

- Deviations from the standard model increase !

$$\langle \partial_t \varrho \rangle - \partial_t \langle \varrho \rangle = \frac{1}{V}\partial_t \mathcal{S}$$

$$\frac{\partial_t\, \mathcal{S}\{\varrho || \langle \varrho \rangle_{\mathcal{D}}\}}{V_{\mathcal{D}}} = -\langle \delta\varrho\Theta \rangle_{\mathcal{D}} = -\langle \varrho\delta\Theta \rangle_{\mathcal{D}} = -\langle \delta\varrho\delta\Theta \rangle_{\mathcal{D}}$$

Averaging the scalar parts of Einstein's equations

$$\frac{1}{2}R + \frac{1}{3}\Theta^2 - \sigma^2 = 8\pi G\rho + \Lambda \quad ; \quad \sigma^i{}_{j||i} = \frac{2}{3}\Theta_{|j} \quad ;$$

$$\partial_t \rho = -\Theta\rho \quad ; \quad \partial_t g_{ij} = 2\, g_{ik}\sigma^k{}_j + \frac{2}{3}\Theta g_{ik}\delta^k{}_j \quad ;$$

$$\partial_t \Theta + \frac{1}{3}\Theta^2 + 2\sigma^2 + 4\pi G\rho - \Lambda = 0 \quad ;$$

$$\partial_t \sigma^i{}_j + \Theta\sigma^i{}_j = -\left(R^i{}_j - \frac{1}{3}\delta^i{}_j R\right) \quad ,$$

→

$$a_{\mathcal{D}}(t) := \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_o}}\right)^{1/3} .$$

$$\langle\theta\rangle_{\mathcal{D}} = \frac{\dot{V}_{\mathcal{D}}}{V_{\mathcal{D}}} = 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} .$$

Volume acceleration despite local deceleration

$$\partial_t \theta = \Lambda - 4\pi G \varrho + 2II - I^2$$

$$\partial_t \langle\theta\rangle = \Lambda - 4\pi G \langle\varrho\rangle + 2\langle II\rangle - \langle I\rangle^2$$

$$\underline{2II - I^2} = -\frac{1}{3}\theta^2 - 2\sigma^2$$

$$\sigma^2 := 1/2\, \sigma_{ij}\sigma_{ij}$$

$$\underline{2\langle II\rangle - \langle I\rangle^2} = \frac{2}{3}\langle(\theta - \langle\theta\rangle)^2\rangle - 2\langle(\sigma - \langle\sigma\rangle)^2\rangle$$

$$-\frac{1}{3}\langle\theta\rangle^2 - 2\langle\sigma\rangle^2$$

Kinematical Backreaction

- Acceleration Law : $$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$

- Expansion Law : $$3\left(\frac{\dot{a}}{a}\right)^2 - 8\pi G \varrho_H - \Lambda = -\frac{3k}{a^2}$$

- Conservation Law : $$\dot{\varrho}_H + 3\left(\frac{\dot{a}}{a}\right)\varrho_H = 0$$

- Integrability :

Kinematical Backreaction

- Acceleration Law :

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda = \mathcal{Q}_{\mathcal{D}}$$

- Expansion Law :

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda = -\frac{\langle \mathcal{R} \rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}}{2}$$

- Conservation Law :

$$\langle \varrho \rangle\dot{}_{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} \langle \varrho \rangle_{\mathcal{D}} = 0$$

- Integrability :

$$\frac{1}{a_{\mathcal{D}}^6}\partial_t \left(\mathcal{Q}_{\mathcal{D}}\, a_{\mathcal{D}}^6 \right) + \frac{1}{a_{\mathcal{D}}^2}\partial_t \left(\langle \mathcal{R} \rangle_{\mathcal{D}}\, a_{\mathcal{D}}^2 \right) = 0$$

Effect of Kinematical Backreaction

$$3\frac{\ddot{a}_{\mathcal{D}_t}}{a_{\mathcal{D}_t}} + 4\pi G\langle\varrho\rangle_{\mathcal{D}_t} - \Lambda = \mathcal{Q}_{\mathcal{D}_t}$$

$$3\frac{\dot{a}^2_{\mathcal{D}_t}}{a^2_{\mathcal{D}_t}} + 3\frac{k_{\mathcal{D}_t}}{a^2_{\mathcal{D}_t}} - 8\pi G\langle\varrho\rangle_{\mathcal{D}_t} - \Lambda = \frac{1}{a^2_{\mathcal{D}_t}}\int_{t_0}^{t} dt'\ \mathcal{Q}_{\mathcal{D}_{t'}}\frac{d}{dt'}a^2_{\mathcal{D}_{t'}}(t') \qquad H_{\mathcal{D}_t} := \frac{\dot{a}_{\mathcal{D}_t}}{a_{\mathcal{D}_t}}$$

Kinematical Dark Energy / Kinematical Dark Matter :

$$\mathcal{Q}_{\mathcal{D}_t} := 2\langle II\rangle_{\mathcal{D}_t} - \frac{2}{3}\langle I\rangle^2_{\mathcal{D}_t}$$

$$\mathcal{Q}_{\mathcal{D}_t} = \frac{2}{3}\left(\langle\theta^2\rangle_{\mathcal{D}_t} - \langle\theta\rangle^2_{\mathcal{D}_t}\right) + 2\langle\omega^2\rangle_{\mathcal{D}_t} - 2\langle\sigma^2\rangle_{\mathcal{D}_t} \ .$$

$$\sigma^2 := 1/2\,\sigma_{ij}\sigma_{ij}$$

$$\omega^2 := 1/2\,\omega_{ij}\omega_{ij}$$

Effective Form

Recall : Standard Models for Dark Sources

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_h}{3} + \frac{\Lambda}{3} - \frac{k}{a^2};$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G(\rho_h + 3p_h)}{3} + \frac{\Lambda}{3};$$

$$\dot{\rho}_h + 3\left(\frac{\dot{a}}{a}\right)(\rho_h + p_h) = 0.$$

$$\rho(t) = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$P(t) = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0.$$

Quintessence

Scalar Dark Matter

Inflation

Effective Equations – Friedmannian Form

$$\varrho^{\mathcal{D}}_{\mathrm{eff}} = \langle\varrho\rangle_{\mathcal{D}} - \frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} - \frac{1}{16\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad ; \quad p^{\mathcal{D}}_{\mathrm{eff}} = -\frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} + \frac{1}{48\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad .$$

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G\varrho^{\mathcal{D}}_{\mathrm{eff}} - \Lambda = 0 \;;$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G(\varrho^{\mathcal{D}}_{\mathrm{eff}} + 3p^{\mathcal{D}}_{\mathrm{eff}}) - \Lambda = 0 \;;$$

$$\dot{\varrho}^{\mathcal{D}}_{\mathrm{eff}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\left(\varrho^{\mathcal{D}}_{\mathrm{eff}} + p^{\mathcal{D}}_{\mathrm{eff}}\right) = 0 \;.$$

$$-\frac{1}{8\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} = 3U_{\mathcal{D}}$$

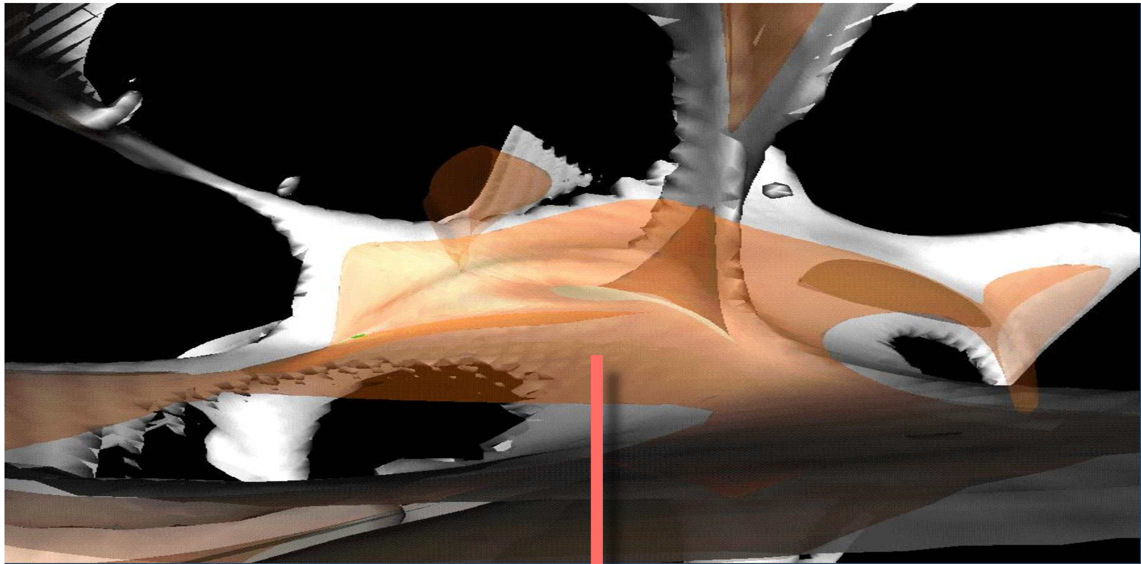
$$-\frac{1}{8\pi G}\mathcal{Q}_{\mathcal{D}} = \epsilon\dot{\Phi}^2_{\mathcal{D}} - U_{\mathcal{D}}$$

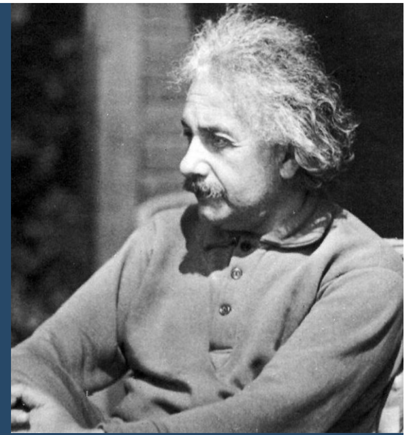
Effective Scalar Field : ‘Morphon‘

$$\varrho^{\mathcal{D}}_{\Phi} = \epsilon\frac{1}{2}\dot{\Phi}^2_{\mathcal{D}} + U_{\mathcal{D}} \quad ; \quad p^{\mathcal{D}}_{\Phi} = \epsilon\frac{1}{2}\dot{\Phi}^2_{\mathcal{D}} - U_{\mathcal{D}}$$

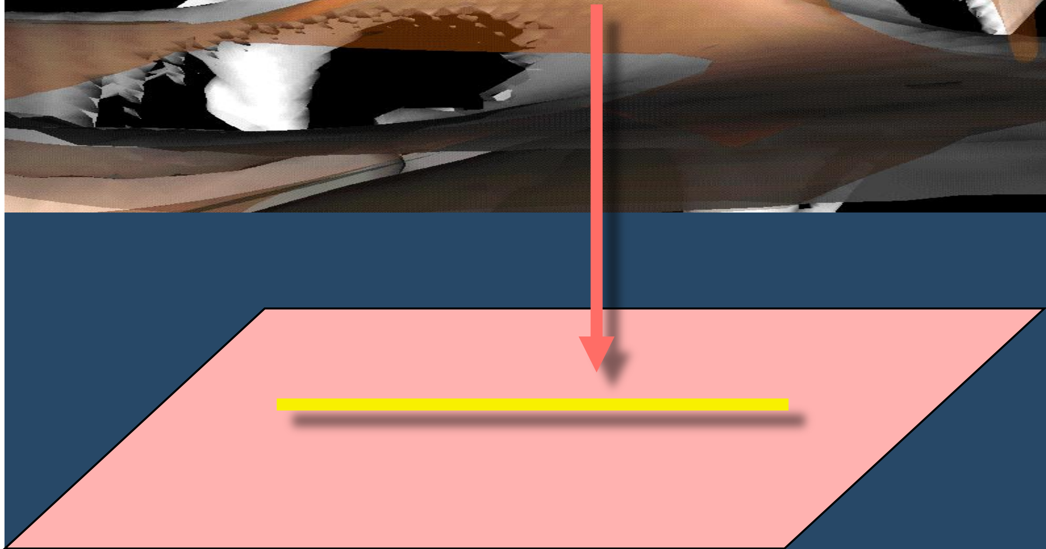
$$\ddot{\Phi}_{\mathcal{D}} + 3H_{\mathcal{D}}\dot{\Phi}_{\mathcal{D}} + \epsilon\frac{\partial}{\partial\Phi_{\mathcal{D}}}U(\Phi_{\mathcal{D}}, \langle\varrho\rangle_{\mathcal{D}}) = 0$$

Buchert, Larena, Alimi
arXiv: gr-qc / 0606020

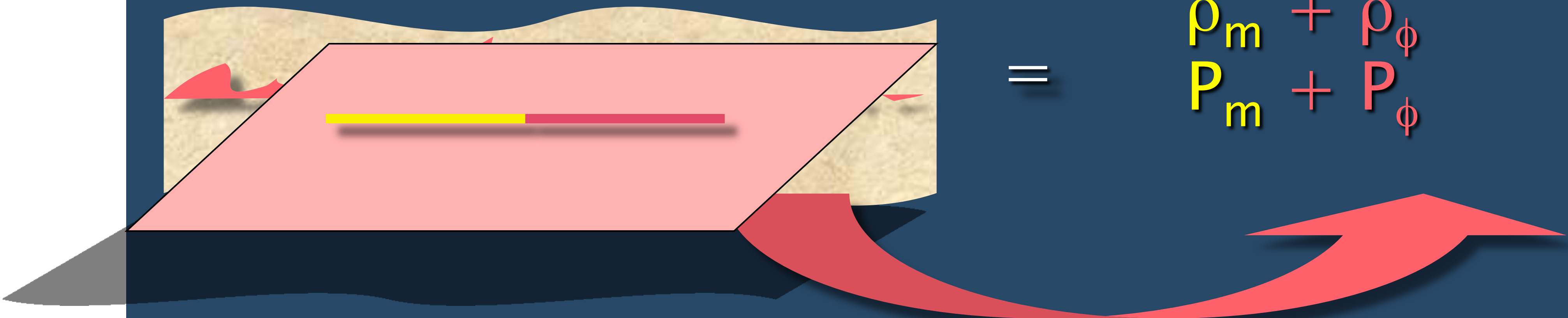




$$G_{\mu\nu} = \kappa\, T_{\mu\nu}$$



$$= \begin{matrix} \rho_m + \rho_\phi \\ P_m + P_\phi \end{matrix}$$



$$= \begin{matrix} \rho_m + \rho_\phi \\ P_m + P_\phi \end{matrix}$$

Take home summary II

Backreaction can act accelerating or decelarating
as a function of scale

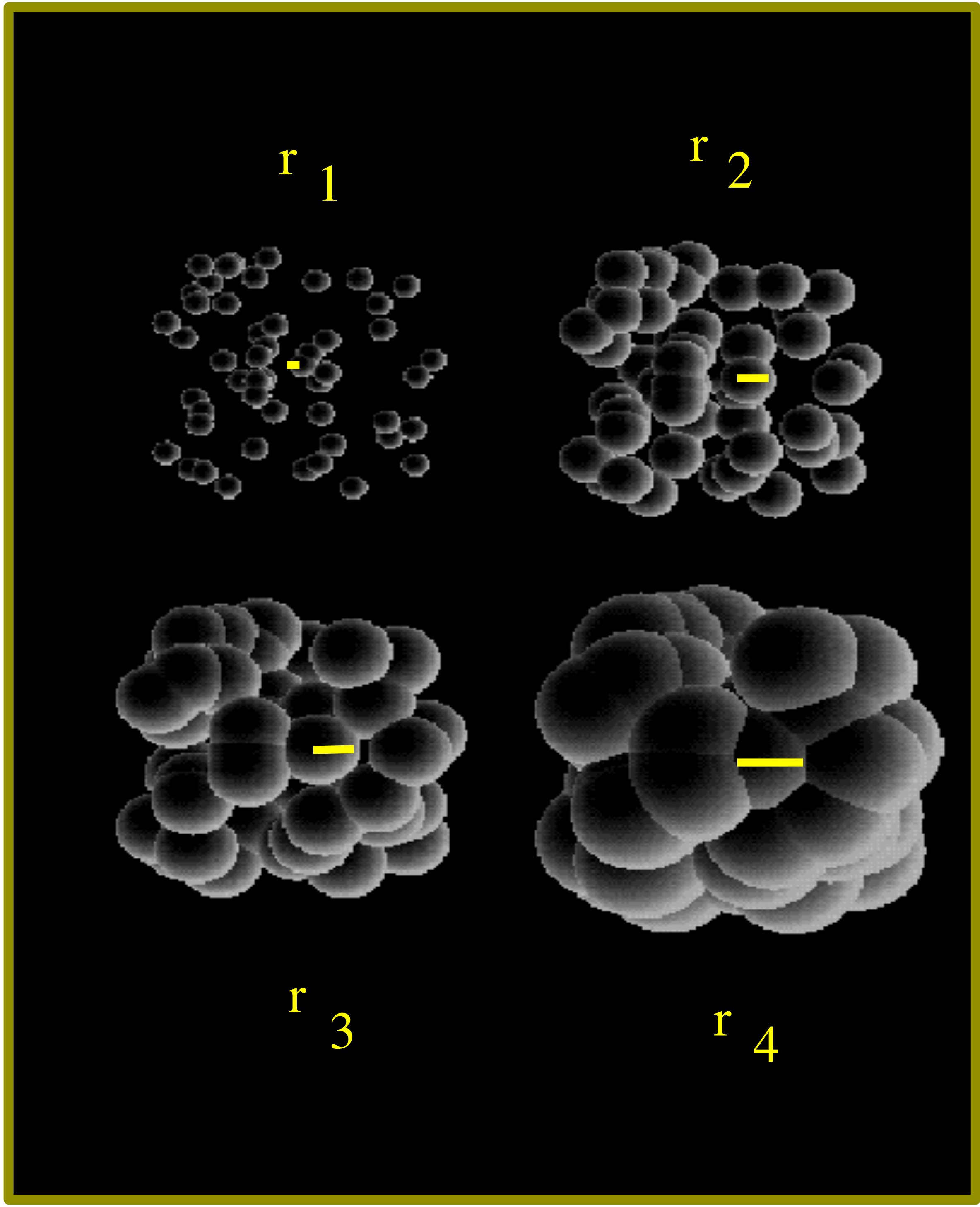
Backreaction is due to non-local fluctuation terms

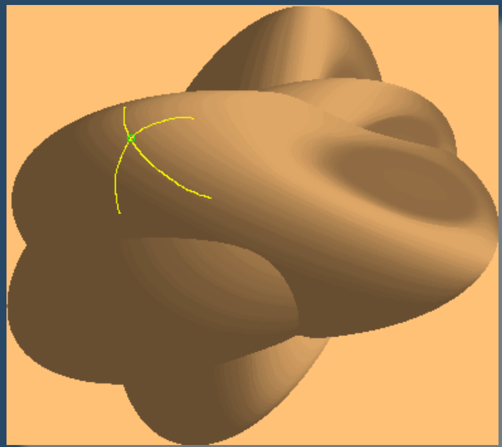
Backreaction couples to the average scalar curvature
Structures ‘talk‘ to the ‘background‘

Backreaction can be described as an effective scalar field

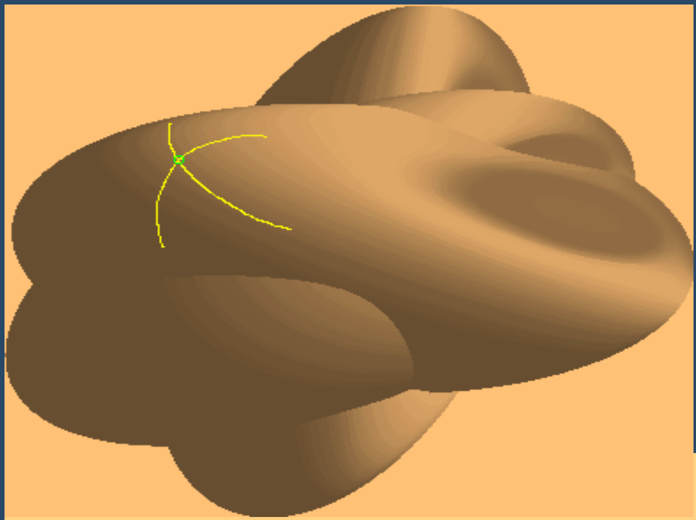
Newtonian Excursion

Morphometry as a function of scale





Morphological interpretation of backreaction



Integral Properties
of an averaging domain

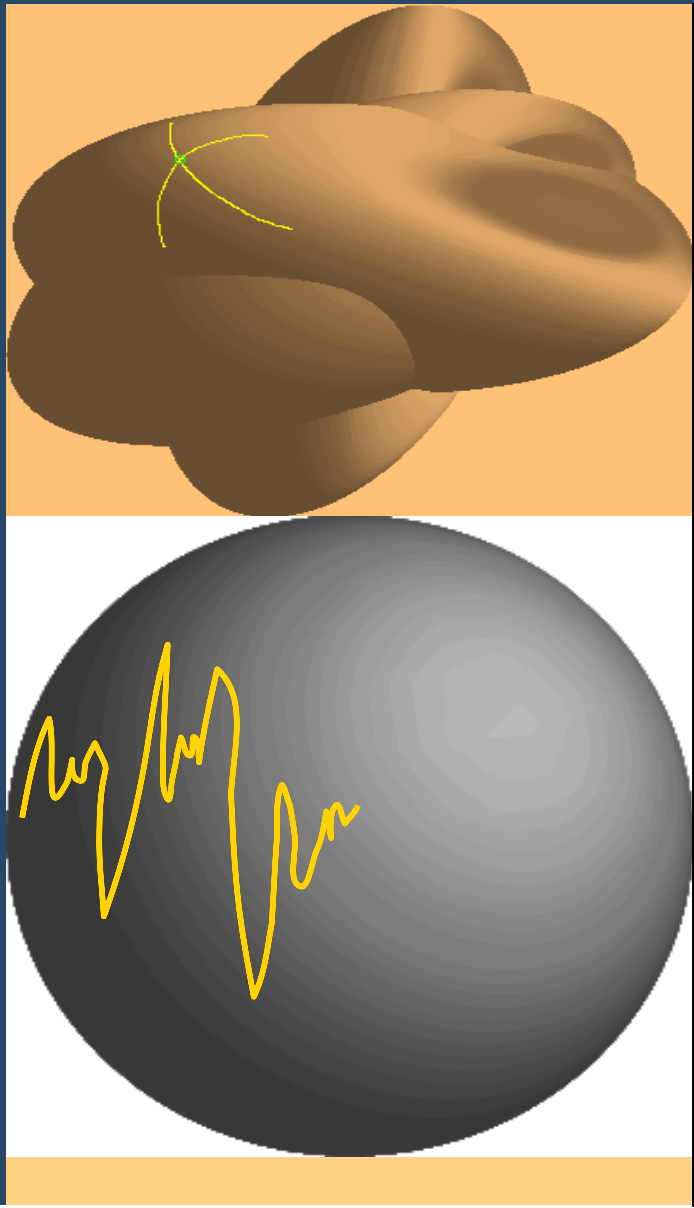
Minkowski Functionals

$$2H = \frac{1}{R_1} + \frac{1}{R_2} \quad ; \quad G = \frac{1}{R_1 \cdot R_2}$$

Minkowski Functionals $\mathbf{W}_\alpha$:

$$\mathbf{W}_0 = V(\mathcal{D})$$
$$\mathbf{W}_1 = \frac{1}{3}\int_{\partial\mathcal{D}} dA$$
$$\mathbf{W}_2 = \frac{1}{3}\int_{\partial\mathcal{D}} 2H\, dA$$
$$\mathbf{W}_3 = \frac{1}{3}\int_{\partial\mathcal{D}} G\, dA = \frac{4\pi}{3}\chi(\mathcal{D})$$

Morphological interpretation of backreaction



Generalized Friedmann Equation :

$$H_{\mathcal{D}}^2 - \frac{8\pi G M_{\mathcal{D}}}{3a_{\mathcal{D}}^3} + \frac{k_{\mathcal{D}}}{a_{\mathcal{D}}^2} - \frac{\Lambda}{3} = \frac{2}{3a_{\mathcal{D}}^2}\int_{t_0}^{t} dt' \mathcal{Q}_{\mathcal{D}}\, \dot{a}_{\mathcal{D}} a_{\mathcal{D}}$$

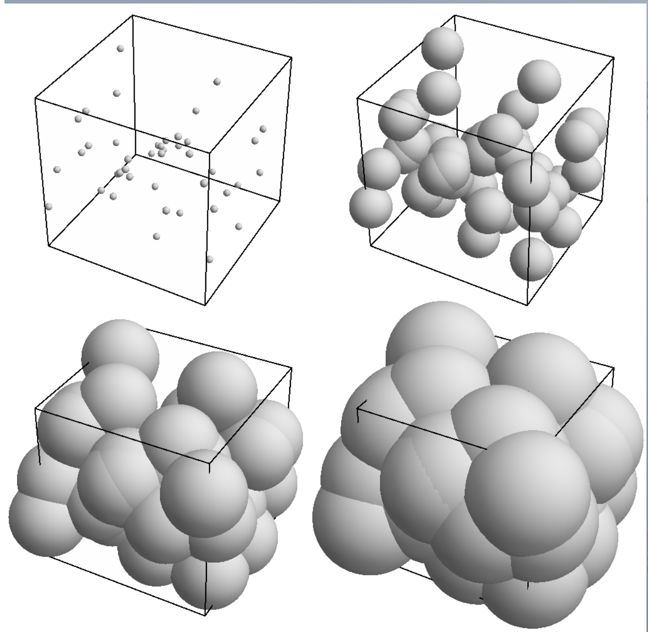
Backreaction Term :

$$\mathcal{Q}_{\mathcal{D}_t} := 2\langle II\rangle_{\mathcal{D}_t} - \frac{2}{3}\langle I\rangle_{\mathcal{D}_t}^2$$

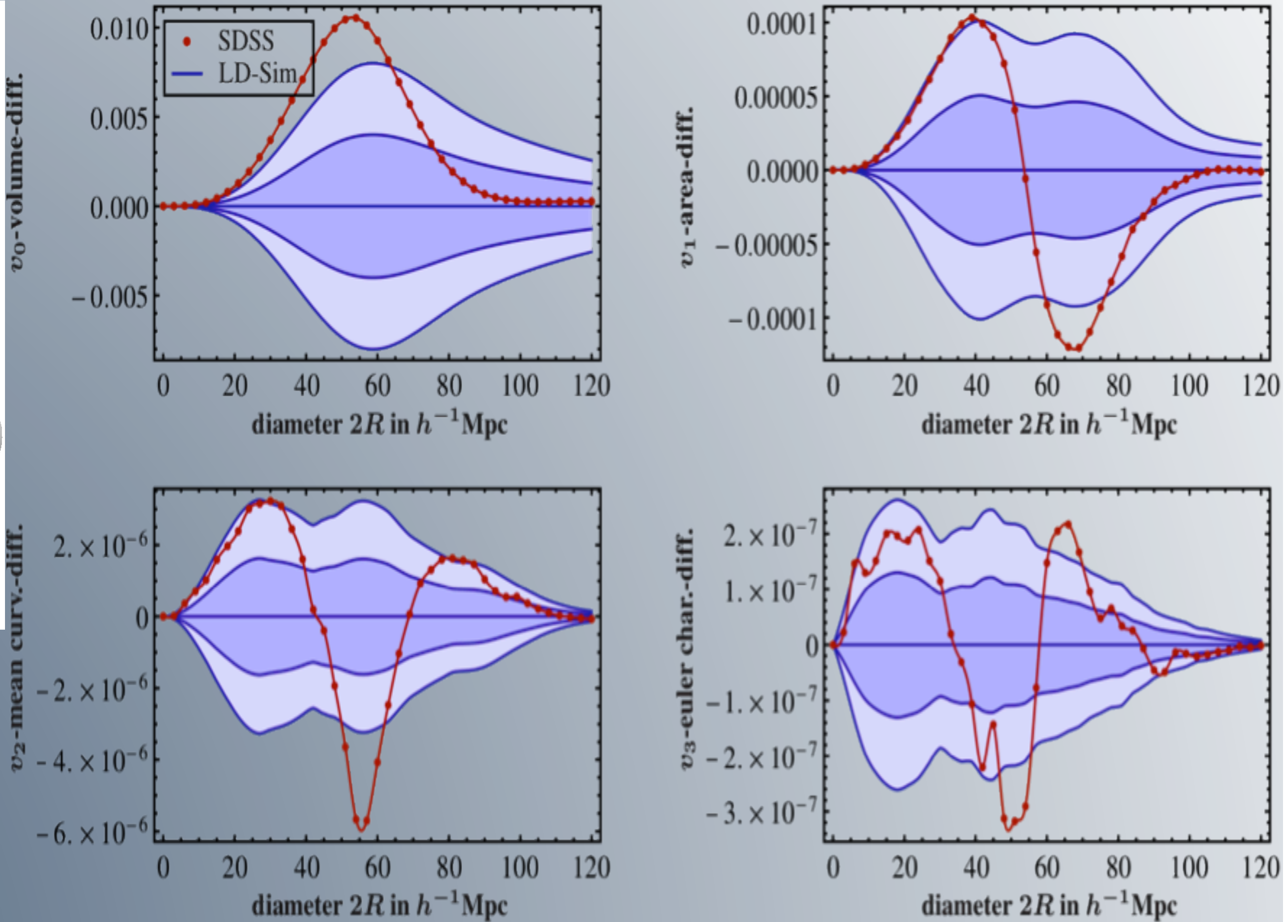
$$\mathcal{Q}_{\mathcal{D}}(s) = 6\left(\frac{\mathbf{W}_2}{\mathbf{W}_0} - \frac{\mathbf{W}_1^2}{\mathbf{W}_0^2}\right)$$

$$\mathcal{Q}_{\mathcal{D}}^{\mathrm{sph}} = 0$$

Statistical Analysis of Galaxy Catalogues



Germ-Grain Model



SDSS - DR7 – LRG Sample 700 Mpc/h :
Wiegand, Buchert, Ostermann arXiv: 1311.3661

Backreaction can be observed !

Closure Assumptions

Closure Assumptions - general

$$\varrho^{\mathcal{D}}_{\text{eff}} = \langle\varrho\rangle_{\mathcal{D}} - \frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} - \frac{1}{16\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad ; \quad p^{\mathcal{D}}_{\text{eff}} = -\frac{1}{16\pi G}\mathcal{Q}_{\mathcal{D}} + \frac{1}{48\pi G}\langle\mathcal{R}\rangle_{\mathcal{D}} \quad .$$

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 - 8\pi G\varrho^{\mathcal{D}}_{\text{eff}} - \Lambda = 0 \; ;$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G(\varrho^{\mathcal{D}}_{\text{eff}} + 3p^{\mathcal{D}}_{\text{eff}}) - \Lambda = 0 \; ;$$

$$\dot{\varrho}^{\mathcal{D}}_{\text{eff}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\left(\varrho^{\mathcal{D}}_{\text{eff}} + p^{\mathcal{D}}_{\text{eff}}\right) = 0 \; .$$

→ Dynamical Equation of State

Exact Scaling and Effective Quintessence

$$\mathcal{Q}_{\mathcal{D}} = r \, \langle R \rangle_{\mathcal{D}} = r \, R_{\mathcal{D}_{\mathrm{i}}} \, a_{\mathcal{D}}^{n} \quad ; \; n = -2\frac{(1+3r)}{(1+r)} \; ; \; r = -\frac{(n+2)}{(n+6)}$$

$$U(\Phi_{\mathcal{D}}, \langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}}) = \frac{2(1+r)}{3} \left((1+r) \frac{\Omega_{R}^{\mathcal{D}_{\mathrm{i}}}}{\Omega_{m}^{\mathcal{D}_{\mathrm{i}}}} \right)^{\frac{3}{n+3}} \langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}} \sinh^{\frac{2n}{n+3}} \left(\frac{(n+3)}{\sqrt{-\epsilon n}} \sqrt{2\pi G} \Phi_{\mathcal{D}} \right)$$

$$\Phi_{\mathcal{D}}(a_{\mathcal{D}}) = \frac{2\sqrt{\epsilon(1+3r)(1+r)}}{(1-3r)\sqrt{\pi G}} \operatorname{arsinh} \left(\sqrt{\frac{-(1+r)\mathcal{R}_{\mathcal{D}_{\mathrm{i}}}}{16\pi G \langle \varrho \rangle_{\mathcal{D}_{\mathrm{i}}}} a_{\mathcal{D}}^{\frac{(1-3r)}{(1+r)}}} \right)$$

$$= \frac{\sqrt{-2\epsilon n}}{(n+3)\sqrt{\pi G}} \operatorname{arsinh} \left(\sqrt{(1+r)\gamma_{\mathcal{R}m}^{\mathcal{D}}} \right) \quad ,$$

$$E_{\mathrm{kin}}^{\mathcal{D}} + \frac{(1+3r)}{2\epsilon} E_{\mathrm{pot}}^{\mathcal{D}} = 0 \; .$$

Closure through explicit models for structure formation – Relativistic Lagrangian perturbation theory

I: arXiv:1203.6263 II. arXiv:1303.6193 III. arXiv:1503.02566 IV. arXiv:1711.01597

$$^{\mathrm{RZA}}\mathcal{Q}_{\mathcal{D}} = \frac{\dot{\xi}^2\left(\gamma_1 + \xi\gamma_2 + \xi^2\gamma_3\right)}{\left(1 + \xi\langle \mathrm{I_i}\rangle_{\mathcal{C_D}} + \xi^2\langle \mathrm{II_i}\rangle_{\mathcal{C_D}} + \xi^3\langle \mathrm{III_i}\rangle_{\mathcal{C_D}}\right)^2}$$

$$\gamma_1 := 2\langle \mathrm{II_i}\rangle_{\mathcal{C_D}} - \frac{2}{3}\langle \mathrm{I_i}\rangle^2_{\mathcal{C_D}} = \mathcal{Q}^{\mathrm{initial}}_{\mathcal{C_D}} \ ;$$

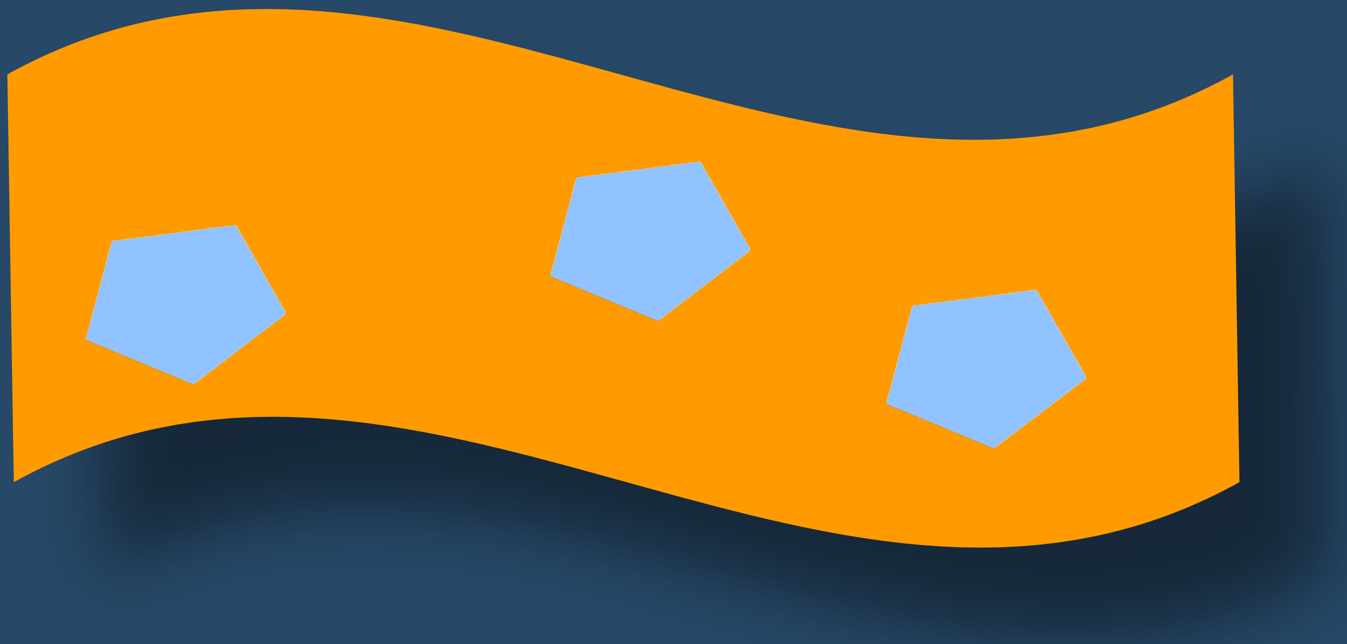
$$\gamma_2 := 6\langle \mathrm{III_i}\rangle_{\mathcal{C_D}} - \frac{2}{3}\langle \mathrm{II_i}\rangle_{\mathcal{C_D}}\langle \mathrm{I_i}\rangle_{\mathcal{C_D}} \ ;$$

$$\gamma_3 := 2\langle \mathrm{I_i}\rangle_{\mathcal{C_D}}\langle \mathrm{III_i}\rangle_{\mathcal{C_D}} - \frac{2}{3}\langle \mathrm{II_i}\rangle^2_{\mathcal{C_D}} \ .$$

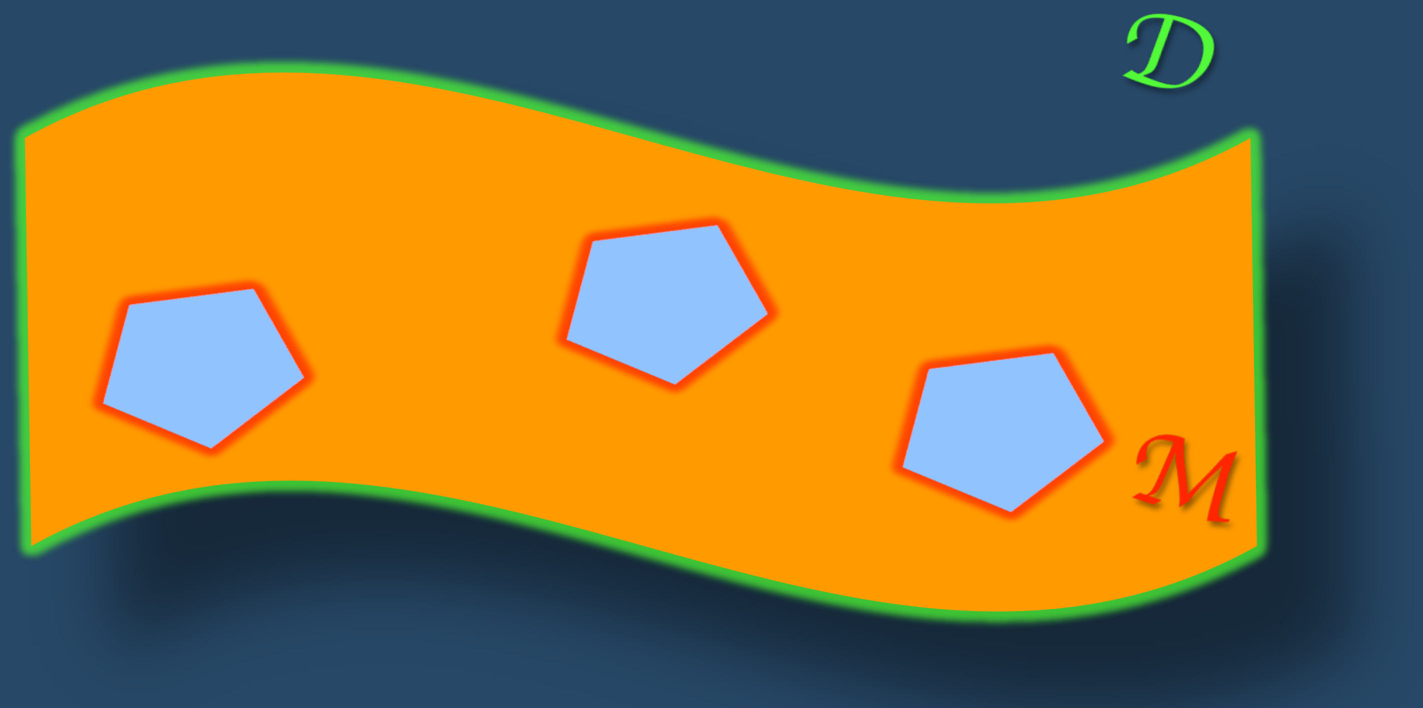
Dark Energy-free models

Background-free Modeling

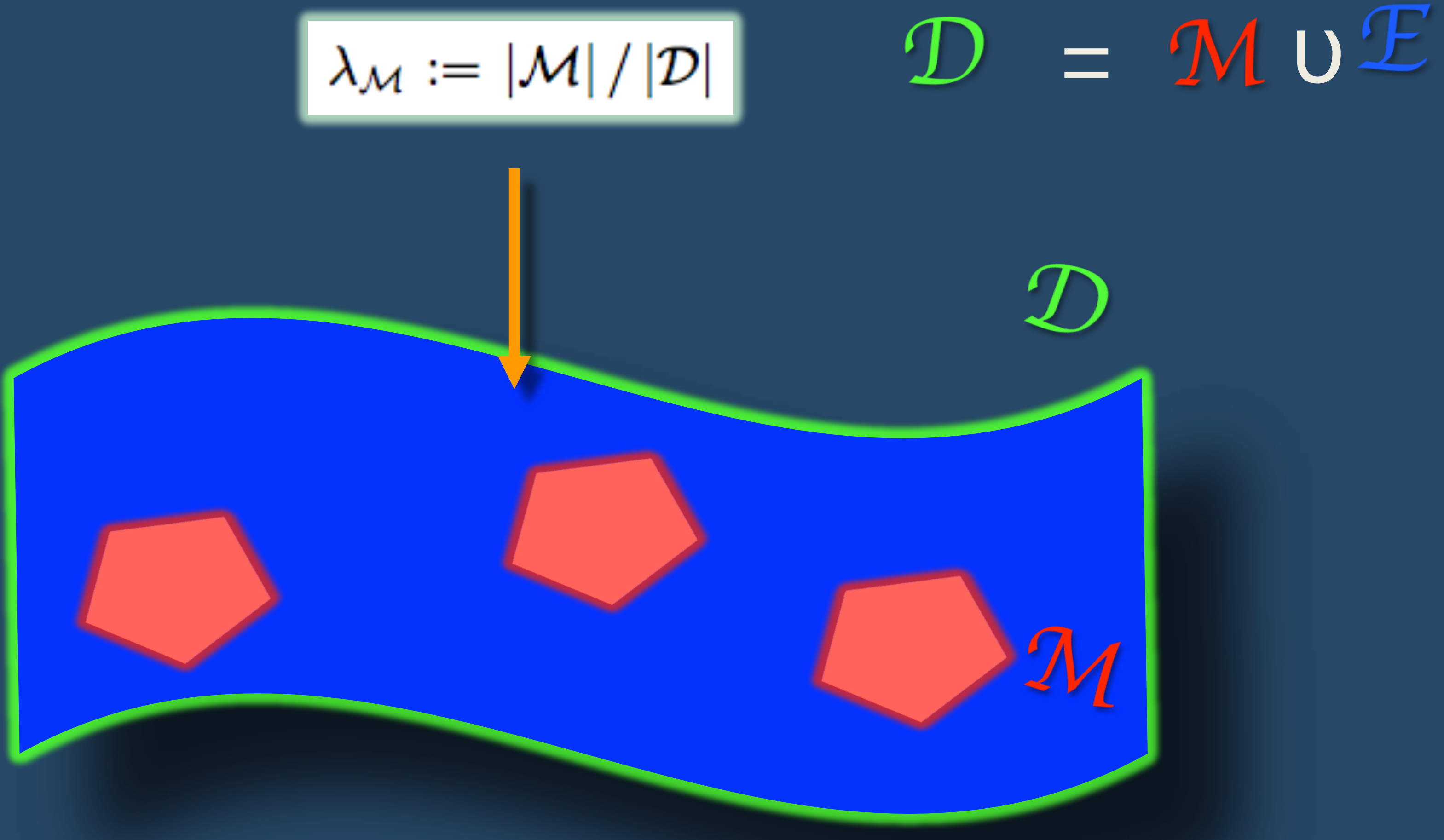
Example : exact average dynamics
for a volume partitioning of spatial slices



Background-free Two-scale Model

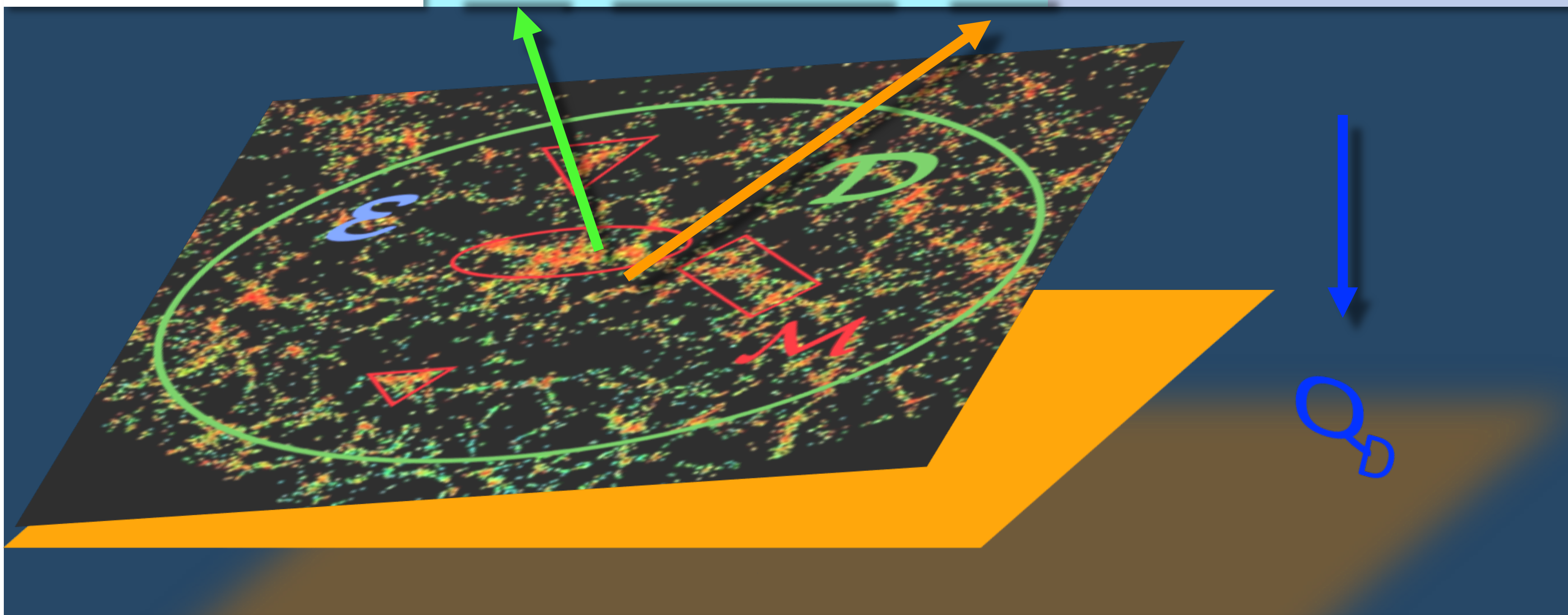


Background-free Two-scale Model



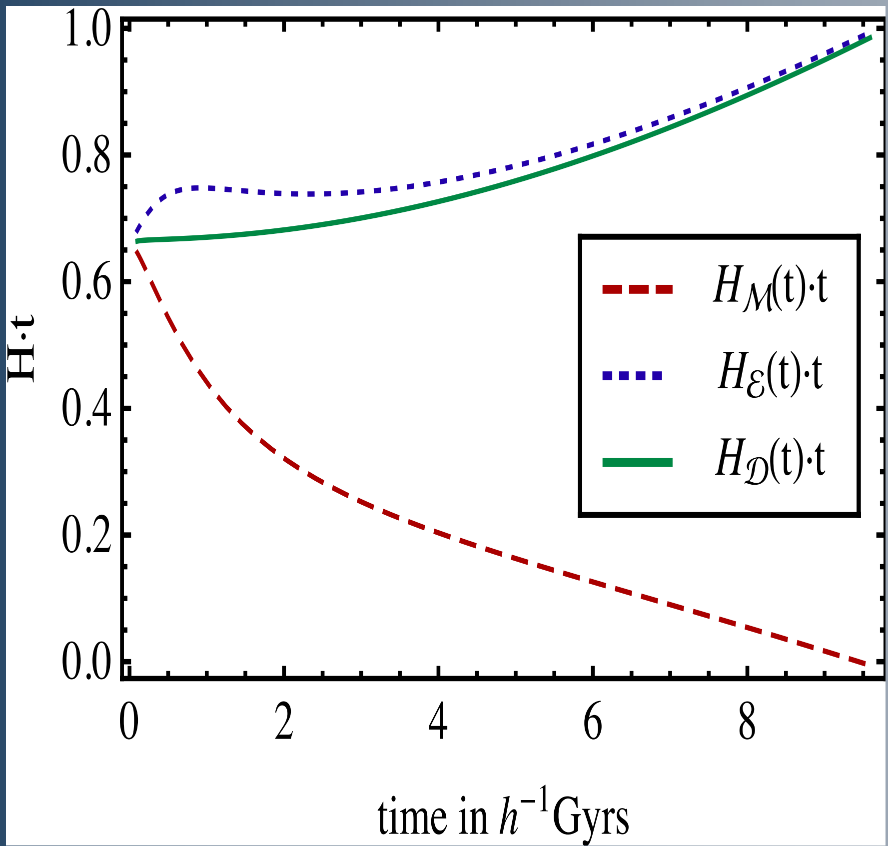
Acceleration in the Two-scale Model

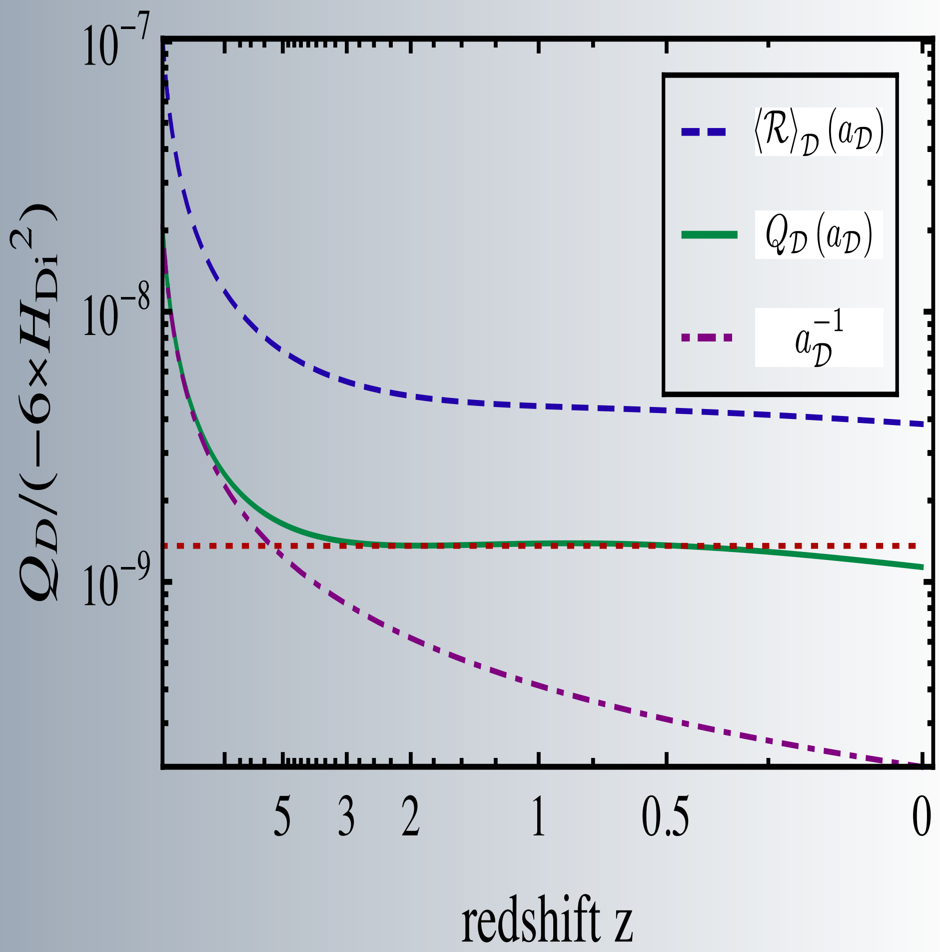
$$a_{\mathcal{D}}(t) := \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_i}}\right)^{1/3} \qquad 3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \varrho \rangle_{\mathcal{D}} - \Lambda = \mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left\langle \left(\theta - \langle\theta\rangle_{\mathcal{D}}\right)^2 \right\rangle_{\mathcal{D}} - 2\left\langle \sigma^2 \right\rangle_{\mathcal{D}}$$



$$\mathcal{Q}_{\mathcal{D}} = \lambda_{\mathcal{M}} \mathcal{Q}_{\mathcal{M}} + (1-\lambda_{\mathcal{M}})\,\mathcal{Q}_{\mathcal{E}} + 6\lambda_{\mathcal{M}}(1-\lambda_{\mathcal{M}})(H_{\mathcal{M}} - H_{\mathcal{E}})^2$$

Acceleration in the Two-scale Model





Wiegand, Buchert
arXiv: 1002.3912

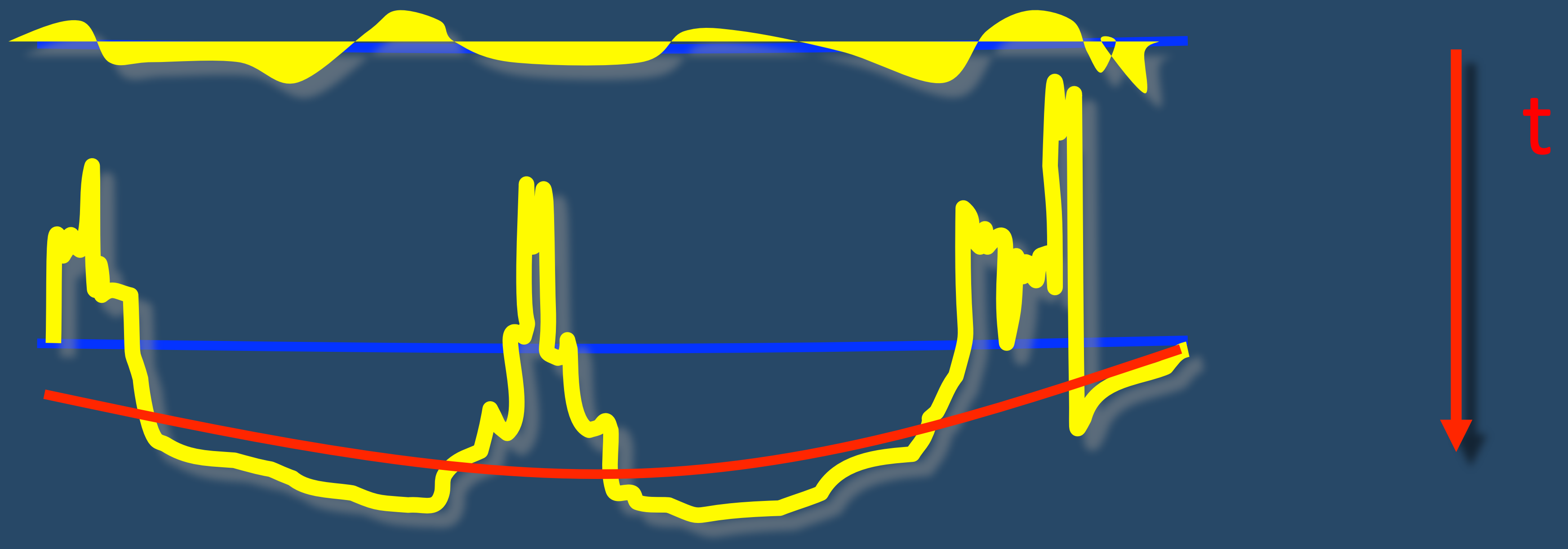
Present-day Universe Estimate

$$Q_D \approx 0 \quad \langle\rho\rangle \approx 0 :$$

$$\langle R\rangle - 2\,\cancel{\Lambda} \approx -6\,H^2$$

Averaged Energy Constraint

All models give negative average curvature for a void-dominated present-day Universe



Curvature is not conserved
while Restmass is conserved

Take home summary III

Backreaction arises due to differential expansion

Structures ‘talk‘ to the ‘background‘

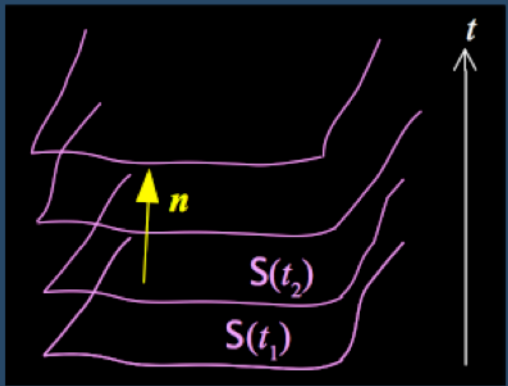
Backreaction arises from the non-conservation of curvature

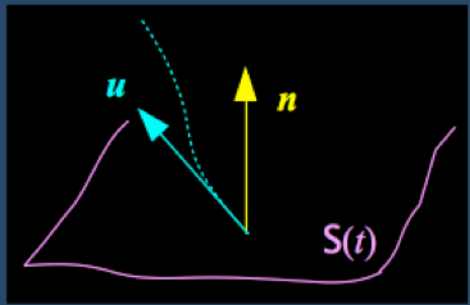
Models that feature conserved curvature
do not describe backreaction

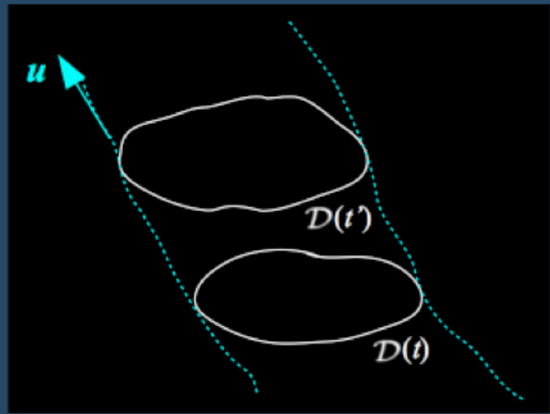
Backreaction leads to emerging negative curvature
in a void-dominated Universe

Foliation Dependence

Foliation Dependence







$$T_{\mu\nu} = \epsilon\, u_\mu u_\nu + 2\, q_{(\mu} u_{\nu)} + p\, b_{\mu\nu} + \pi_{\mu\nu}$$

$$n^\mu = \frac{1}{N}\left(1, -N^i\right) , \quad u^\mu = \gamma(n^\mu + v^\mu) \quad \gamma = \frac{1}{\sqrt{1 - g_{\mu\nu} v^\mu v^\nu}}$$

Averaged Equations :

Buchert, Mourier, Roy
arXiv: 1805.10455

$$3\left(\frac{1}{a_{\mathcal{D}}}\frac{\mathrm{d}a_{\mathcal{D}}}{\mathrm{d}t}\right)^2 = 8\pi G\,\epsilon_{\mathrm{eff}} - 3\frac{k_{\mathcal{D}}}{(a_{\mathcal{D}})^2} + \Lambda\;;$$

$$3\,\frac{1}{a_{\mathcal{D}}}\frac{\mathrm{d}^2 a_{\mathcal{D}}}{\mathrm{d}t^2} = -4\pi G\left(\epsilon_{\mathrm{eff}} + 3\,p_{\mathrm{eff}}\right) + \Lambda\;;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\epsilon_{\mathrm{eff}} + 3\,H_{\mathcal{D}}\left(\epsilon_{\mathrm{eff}} + p_{\mathrm{eff}}\right) = 0\;,$$

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t} = \left.\frac{\partial\mathcal{F}}{\partial t}\right|_{X^i}$$

$$\epsilon_{\mathrm{eff}} \equiv \langle\tilde{\epsilon}\rangle_{\mathcal{D}} - \frac{\tilde{\mathcal{Q}}_{\mathcal{D}}}{16\pi G} - \frac{\tilde{\mathcal{W}}_{\mathcal{D}}}{16\pi G} + \frac{\tilde{\mathcal{L}}_{\mathcal{D}}}{8\pi G}\;;$$

$$p_{\mathrm{eff}} \equiv \langle\tilde{p}\rangle_{\mathcal{D}} - \frac{\tilde{\mathcal{Q}}_{\mathcal{D}}}{16\pi G} + \frac{\tilde{\mathcal{W}}_{\mathcal{D}}}{48\pi G} - \frac{\tilde{\mathcal{L}}_{\mathcal{D}}}{8\pi G} - \frac{\tilde{\mathcal{P}}_{\mathcal{D}}}{12\pi G}$$

$$\mathrm{d}\tau \equiv N/\gamma\,\mathrm{d}t.$$

→ Only dependent on threading lapse

$N/\gamma = 1$

Synchronous Foliation

Buchert, Mourier, Roy
arXiv: 1805.10455

Lagrangian representation :

$$u^\mu = (1,0,0,0),\; N^2 - N^\mu N_\mu = 1$$

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\tau} = \frac{\gamma}{N}\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t}$$

$$3\left(\frac{\dot{a}_\mathcal{D}}{a_\mathcal{D}}\right)^2 = 8\pi G\,\epsilon_{\rm eff} - 3\frac{k_\mathcal{D}}{(a_\mathcal{D})^2} + \Lambda\,;$$
$$3\frac{\ddot{a}_\mathcal{D}}{a_\mathcal{D}} = -4\pi G(\epsilon_{\rm eff} + 3\,p_{\rm eff}) + \Lambda\,;$$
$$\dot{\epsilon}_{\rm eff} + 3\,H_\mathcal{D}\,(\epsilon_{\rm eff} + p_{\rm eff}) = 0\,.$$

$$\epsilon_{\rm eff} = \langle\epsilon\rangle_\mathcal{D} - \frac{\mathcal{Q}_\mathcal{D}}{16\pi G} - \frac{\mathcal{W}_\mathcal{D}}{16\pi G}\,;$$
$$p_{\rm eff} = \langle p\rangle_\mathcal{D} - \frac{\mathcal{Q}_\mathcal{D}}{16\pi G} + \frac{\mathcal{W}_\mathcal{D}}{48\pi G} - \frac{\mathcal{P}_\mathcal{D}}{12\pi G}$$

$$\mathcal{W}_\mathcal{D} = \langle\mathscr{R}\rangle_\mathcal{D} - 6k_\mathcal{D}/(a_\mathcal{D})^2$$

$$\mathcal{Q}^b_\mathcal{D} = \frac{2}{3}\left\langle\left(\Theta - \langle\Theta\rangle^b_\mathcal{D}\right)^2\right\rangle^b_\mathcal{D} - 2\left\langle\sigma^2\right\rangle^b_\mathcal{D} + 2\left\langle\omega^2\right\rangle^b_\mathcal{D}$$

$$\mathcal{P}^b_\mathcal{D} = \left\langle\nabla_\mu a^\mu\right\rangle^b_\mathcal{D}$$

Weak dependence on foliation on cosmological scales

Coarse-grain on a cosmological scale

$N \approx 1$

Nonrelativistic motion of coarse-grained elements

$\gamma \approx 1$

Averaged Equations close to comoving / Lagrangian

$N / \gamma \approx 1$

Perturbations in Poisson / longitudinal gauge

$N / \gamma \approx 1$

Take home summary IV

General Averaged Equations depend on the foliation through the threading lapse – No gauge issues here !

Backreaction depends only weakly on the foliation on cosmological scales

Backreaction has a covariant meaning and assumes it's simplest form in a synchronous foliation

Backreaction depends only functionally on a metric, and can be applied to a statistical ensemble of metrics

No Backreaction in Newtonian Cosmology and quasi-Newtonian simulations

Properties of Backreaction in Euclidean Space

2.5 Remark: Divergence property of principal scalar invariants

We note the following properties of the *principal scalar invariants* (here written for the invariants of the mean velocity gradient):

$$I = \nabla \cdot \mathbf{v} \quad ; \quad II = \nabla \cdot \Upsilon_{II} \quad ; \quad III = \nabla \cdot \Upsilon_{III} \quad , \text{ with}$$
$$\Upsilon_{II} := \tfrac{1}{2}\left(\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v}\right) \quad ;$$
$$\Upsilon_{III} := \tfrac{1}{3}\left(\tfrac{1}{2}\nabla \cdot (\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v})\mathbf{v} - (\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v}) \cdot \nabla \mathbf{v}\right) \quad . \qquad (36)$$

$$\mathcal{Q}_{\mathcal{D}_t} := 2\langle II \rangle_{\mathcal{D}_t} - \frac{2}{3}\langle I \rangle^2_{\mathcal{D}_t}$$

$$Q_{\mathcal{D}} = \frac{1}{V}\int_{\mathcal{D}} \nabla \cdot \vec{\Psi}\, d^3x + \frac{2}{3V^2}\left(\int_{\mathcal{D}} \nabla \cdot \vec{v}\, d^3x\right)^2$$

Properties of Backreaction in Euclidean Space

2.5 Remark: Divergence property of principal scalar invariants

We note the following properties of the *principal scalar invariants* (here written for the invariants of the mean velocity gradient):

$$I = \nabla \cdot \mathbf{v} \quad ; \quad II = \nabla \cdot \boldsymbol{\Upsilon}_{II} \quad ; \quad III = \nabla \cdot \boldsymbol{\Upsilon}_{III} \quad , \quad \text{with}$$

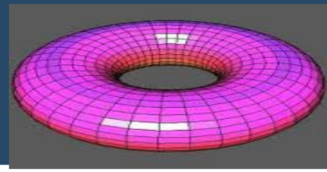
$$\boldsymbol{\Upsilon}_{II} := \tfrac{1}{2}\left(\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v}\right) \quad ;$$

$$\boldsymbol{\Upsilon}_{III} := \tfrac{1}{3}\left(\tfrac{1}{2}\nabla \cdot (\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v})\mathbf{v} - (\mathbf{v}\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v}) \cdot \nabla \mathbf{v}\right) \quad . \qquad (36)$$

$$\mathcal{Q}_{\mathcal{D}_t} := 2\langle II\rangle_{\mathcal{D}_t} - \frac{2}{3}\langle I\rangle^2_{\mathcal{D}_t}$$

$$Q_{\mathcal{D}} = \frac{1}{V}\int_{\partial\mathcal{D}} \vec{\Psi} \cdot \, d\vec{S} \; + \; \frac{2}{3V^2}\left(\int_{\partial\mathcal{D}} \vec{v} \cdot \, d\vec{S}\right)^2$$

No Backreaction Theorem

Theorem: Q = 0 on 3-torus 

Buchert and Ehlers 1997

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G\frac{M}{a_{\mathcal{D}}^3} - \Lambda =: Q_{\mathcal{D}} =$$

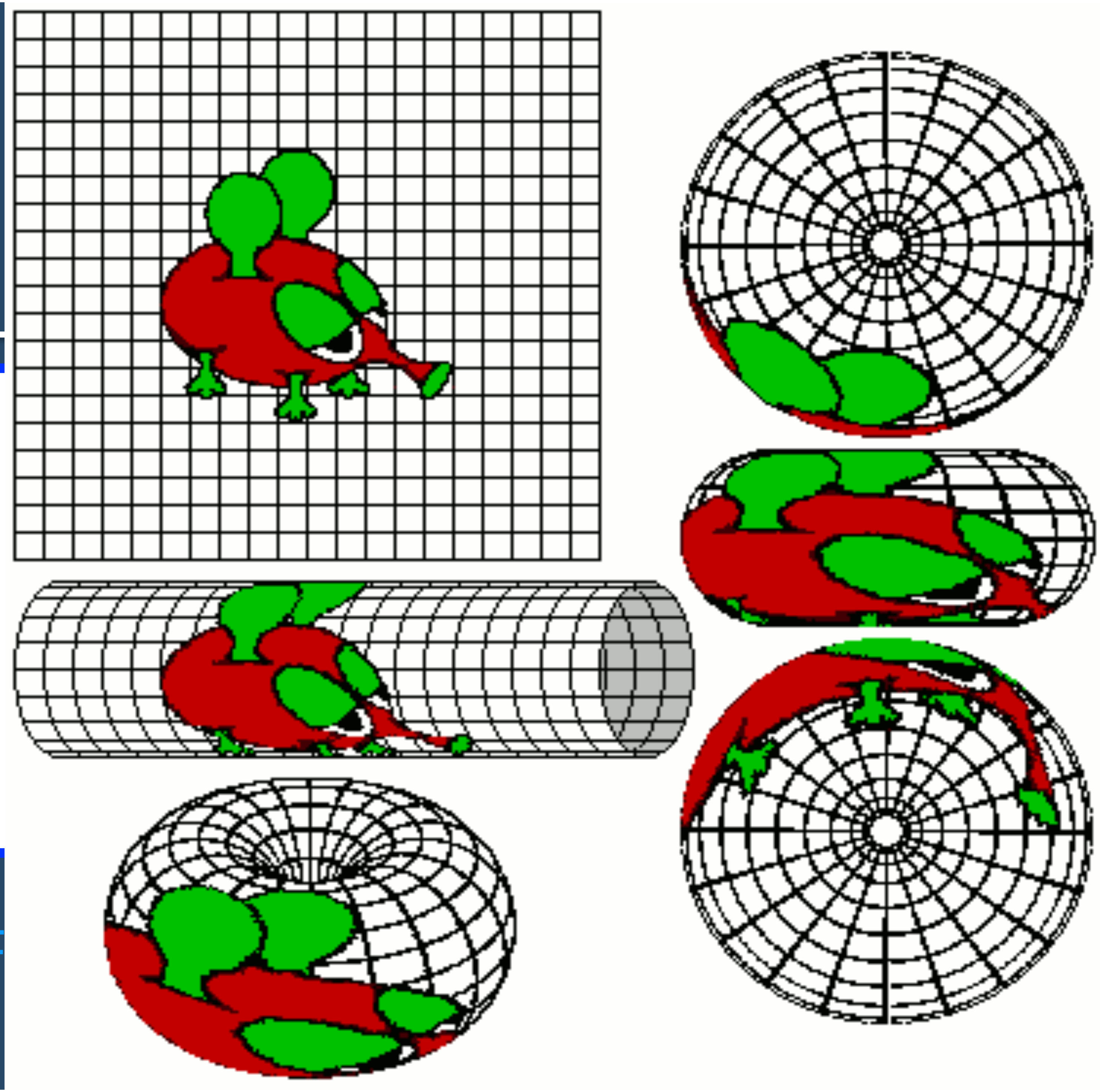
$$\langle \boldsymbol{\nabla} \cdot [\mathbf{u}(\boldsymbol{\nabla} \cdot \mathbf{u}) - (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u}]\rangle_{\mathcal{D}} - \frac{2}{3}\langle \boldsymbol{\nabla} \cdot \mathbf{u}\rangle_{\mathcal{D}}^2$$

$$+2\left(\Omega^2 - \Sigma^2\right) + 2\left(\Omega_{ij}\langle\hat{\omega}_{ij}\rangle_{\mathcal{D}} - \Sigma_{ij}\langle\hat{\sigma}_{ij}\rangle_{\mathcal{D}}\right),$$

Flat Space – Periodicity – Conserved average curvature
Fourier transformation – Background-dependence …

Th

997



F

ure

...

Theorem unchanged by including Shell-crossing

Euler-Jeans Equation with velocity dispersion :

$$\varrho \frac{d}{dt}\overline{v}_i = \varrho\, g_i - \frac{\partial}{\partial x_j}\Pi_{ij} \ .$$

$$\Pi_{ij} = \varrho\, \overline{(v_i - \overline{v}_i)(v_j - \overline{v}_j)}$$

$$\psi_i = \frac{1}{\varrho}\Pi_{ik,k} \ .$$

Raychaudhuri Equation :

$$\frac{d}{dt}\overline{v}_{i,j} + \overline{v}_{k,j}\,\overline{v}_{i,k} = g_{i,j} - \psi_{i,j} \quad \Leftrightarrow \quad \frac{d}{dt}\theta = \Lambda - 4\pi\varrho G + 2\,II - \theta^2 - \psi_{i,i} \ .$$

$$Q_{\mathcal{D}} := \frac{2}{3}\left[\langle(\theta - \langle\theta\rangle_{\mathcal{D}})^2\rangle_{\mathcal{D}}\right] + 2\,\langle\omega^2 - \sigma^2\rangle_{\mathcal{D}} - \langle\psi_{i,i}\rangle_{\mathcal{D}} = 2\langle II(v_{i,j})\rangle_{\mathcal{D}} - \frac{2}{3}\langle I(v_{i,j})\rangle_{\mathcal{D}}^2 - \langle\psi_{i,i}\rangle_{\mathcal{D}} \ .$$

Take home summary V

Newtonian Backreaction is Cosmic Variance
on an assumed background

Newtonian Backreaction vanishes globally,
but it is non-zero in the interior of the simulation

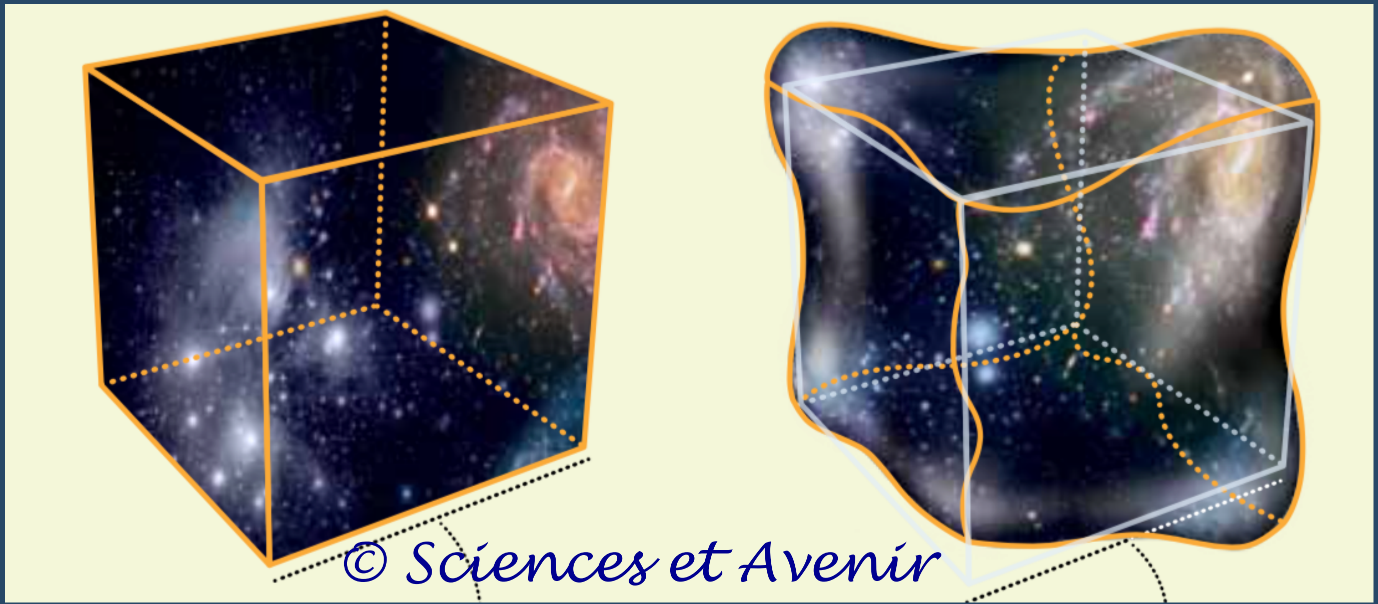
Assuming a background and periodic boundary conditions on
the deviation fields cannot give Backreaction

Any approximate assumption on some domain
is known globally and may lead to spurious Backreaction

Take home message :

Backreaction reduces to Cosmic Variance in fixed-background / periodic / flat-space simulations

Backreaction requires liberation from strict meter



Some words on the link to observations

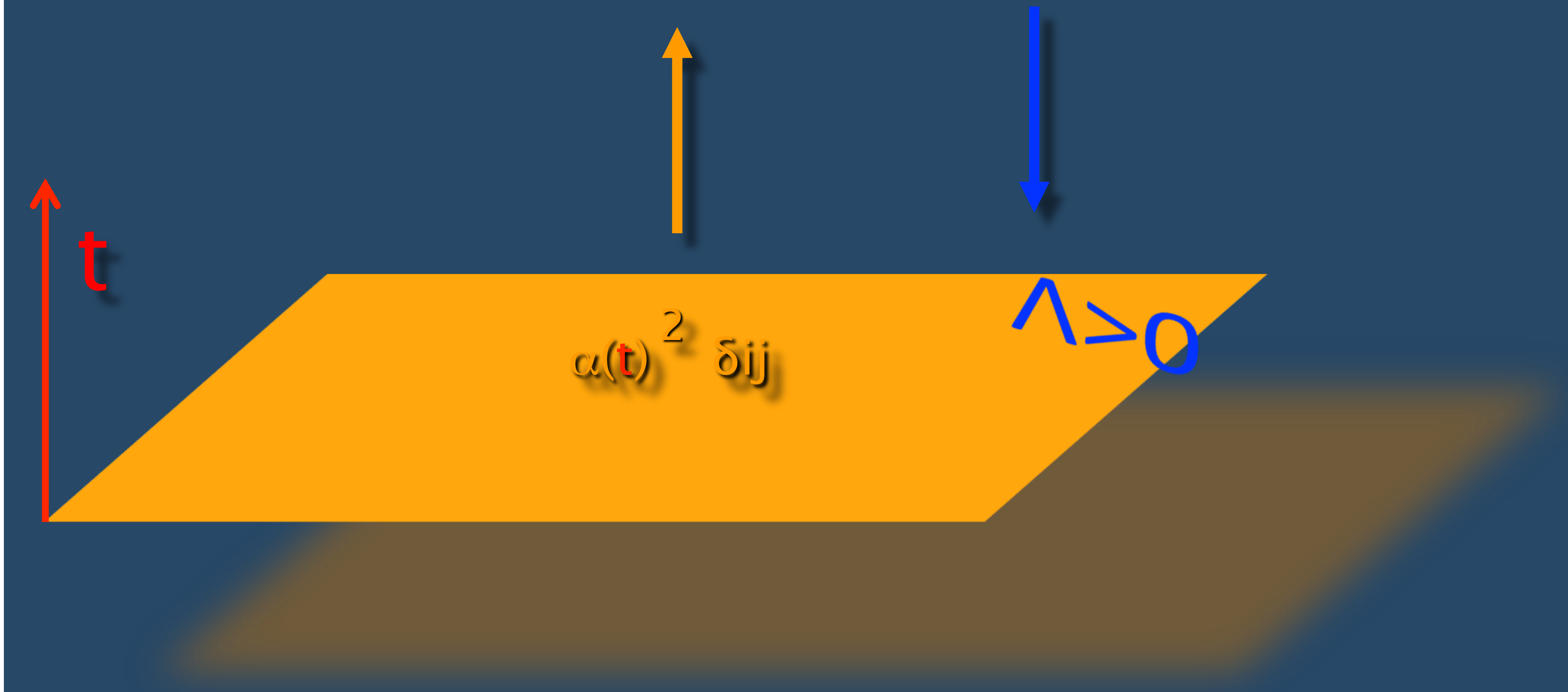
Acceleration in the Standard Model

local acceleration

global acceleration

$$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$

$$\alpha(t) = V(t)^{⅓}$$



Acceleration in the Standard Model

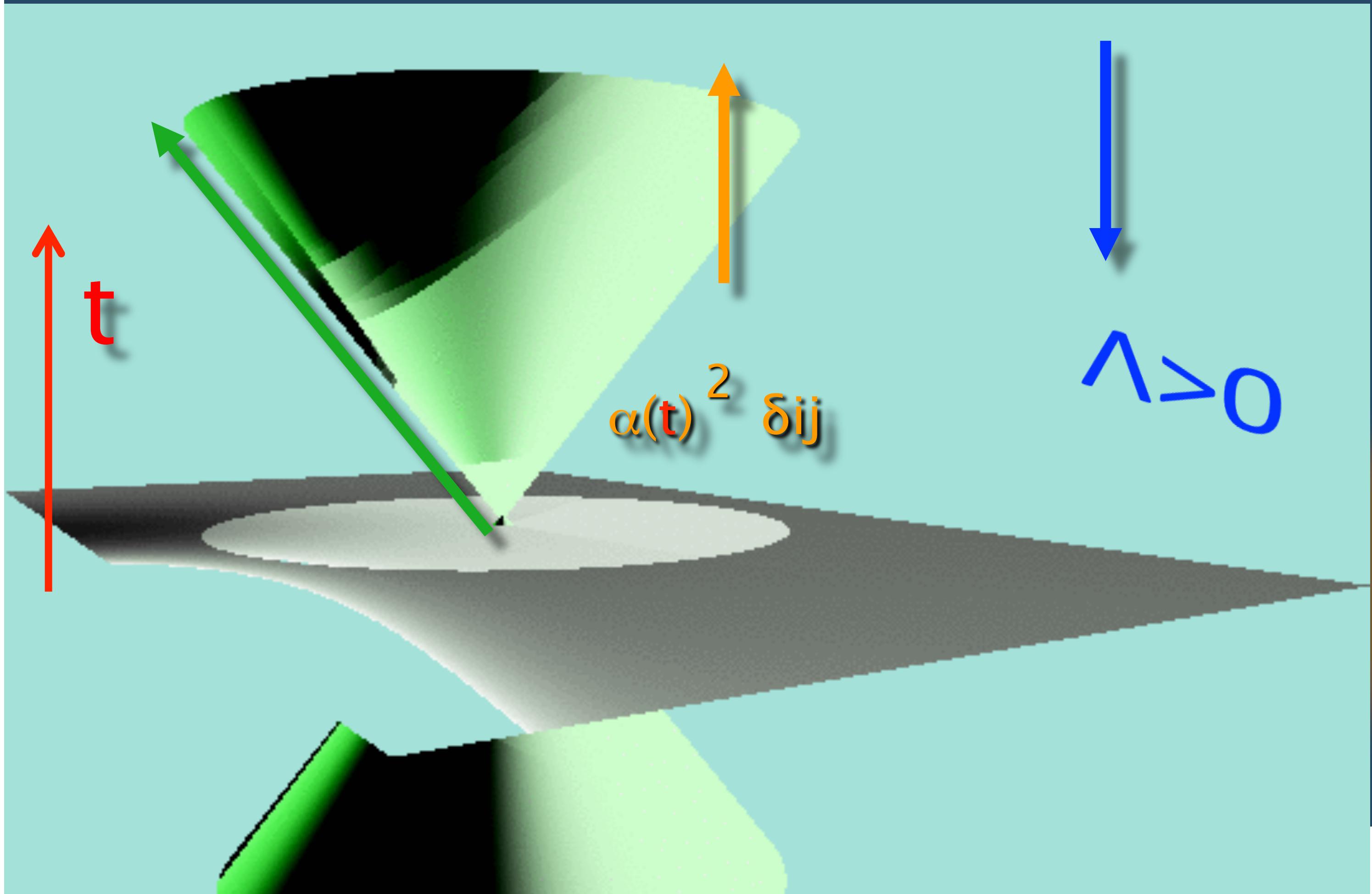
local acceleration

global acceleration

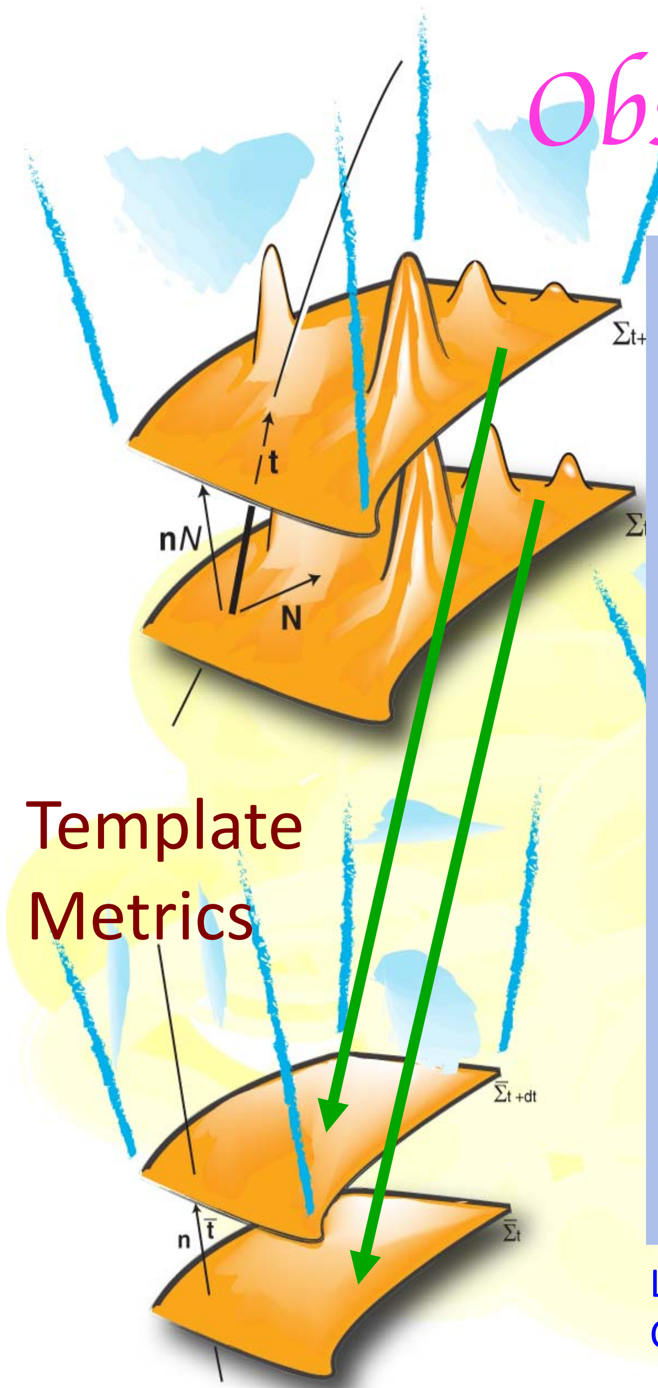
apparent acceleration

$$3\frac{\ddot{a}}{a} + 4\pi G \varrho_H - \Lambda = 0$$

$$\alpha(t) = V(t)^{1/3}$$



Observational Strategies

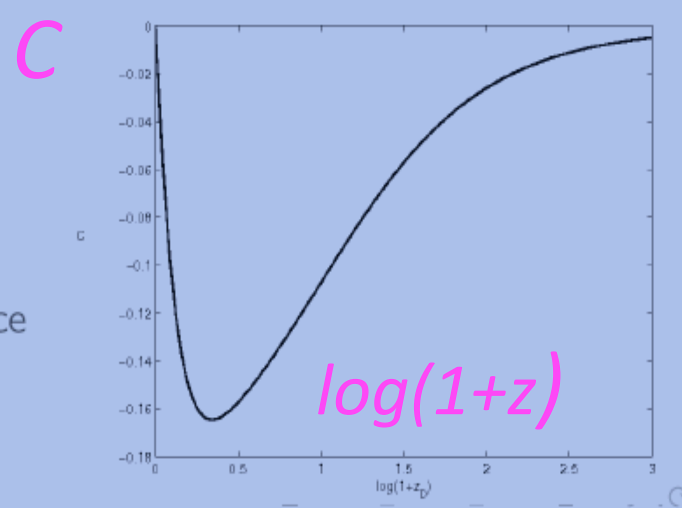


Template Metrics

- $C(z) = 1 + H^2(DD'' - D'^2) + HH'DD'$ with $D \equiv (1+z)d_A$ is identically zero for FRW, different from 0 otherwise [C. Clarkson & al, arXiv:0712.3457]
- In our models:

$$C(z_{\mathcal{D}}) = -\frac{H_{\mathcal{D}}(z_{\mathcal{D}})r(z_{\mathcal{D}})\kappa'_{\mathcal{D}}(z_{\mathcal{D}})}{2H_{\mathcal{D}_0}\sqrt{1-\kappa_{\mathcal{D}}(z_{\mathcal{D}})r^2(z_{\mathcal{D}})}} .$$

- Testable prediction of the model.
- Can allow to make the difference with a quintessence field with the same n.



Larena, Alimi, Buchert, Kunz, Corasaniti arXiv: 0808.1161

Euclid

Further Reading :

arXiv:

- gr-qc/0001056
- 0707.2153
- 1103.2016
- 1112.5335