perturbative backreaction

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- perturbative backreaction — what works and what doesn't
- relativistic N-body simulations
- Hubble diagram — percent level effects?

Canonical Cosmology

- compute everything as power series in small parameter ε

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \epsilon\delta^{(1)}g_{\mu\nu} + \epsilon^2\delta^{(2)}g_{\mu\nu} + \cdots$$

'real' spacetime

'background' spacetime
FLRW

first-order
perturbation

second-order
correction

fit to 'background' observables - SNIa etc

Canonical Cosmology

- compute everything as power series in small parameter ε

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?

how do these
fit in?

Perturbation theory

metric to second-order

$$ds^2 = -\left[1 + 2\Phi + \Phi^{(2)}\right] dt^2 - aV_i dx^i dt + a^2 \left[(1 - 2\Phi - \Psi^{(2)})\gamma_{ij} + h_{ij}\right] dx^i dx^j$$

Bardeen equation $\ddot{\Phi} + 4H\dot{\Phi} + \Lambda\Phi = 0$

$$v_i^{(1)} = -\frac{2}{3aH^2\Omega_m}\partial_i\left(\dot{\Phi} + H\Phi\right) \qquad \delta = \frac{\delta\rho}{\rho} = \frac{2}{3H^2\Omega_m}\left[a^{-2}\partial^2\Phi - 3H\left(\dot{\Phi} + H\Phi\right)\right]$$

no real backreaction from first-order perturbations

- average of perturbations vanish by assumption

Perturbation theory

metric to second-order

$$ds^2 = -\left[1 + 2\Phi + \Phi^{(2)}\right] dt^2 - aV_i dx^i dt + a^2 \left[(1 - 2\Phi - \Psi^{(2)})\gamma_{ij} + h_{ij}\right] dx^i dx^j$$

second-order potentials *induced* by first-order scalars

$$\Phi^{(2)} \simeq \Psi^{(2)} \sim (\partial\Phi)^2 \qquad V_i \sim \Phi\partial_i\Phi \qquad h_{ij} \sim \Phi\partial_i\partial_j\Phi$$

backreaction as correction to the background

- second-order modes give non-trivial ‘backreaction’

- Hubble rate depends on

$$\mathrm{H}^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)}$$

  - What is it?

backreaction as correction to the background

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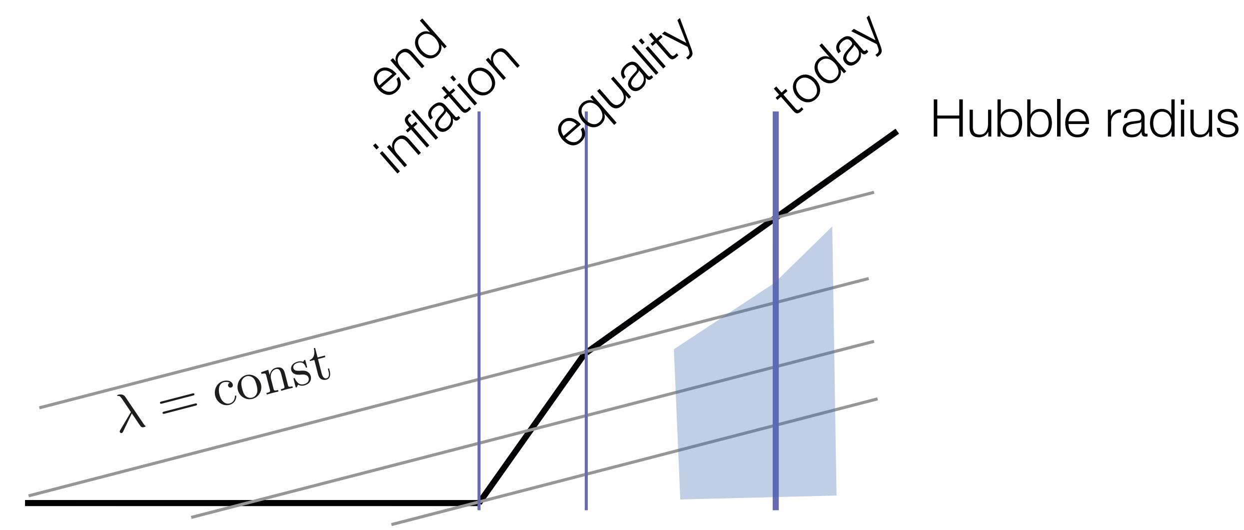
$$\mathrm{H}^{(2)} \sim [\cdots]\Phi^2 + \boxed{[\cdots](\partial\Phi)^2} + [\cdots]\Phi^{(2)}$$

  - What is it?

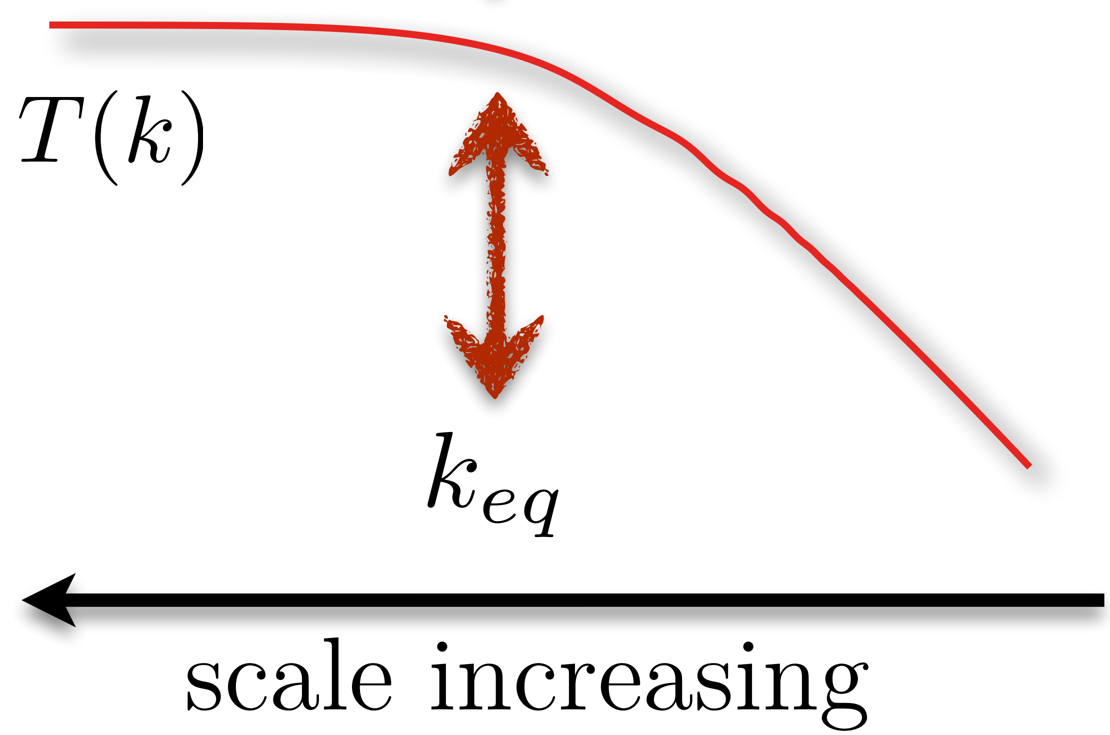
determines amplitude
of backreaction

scaling behaviour
at first-order



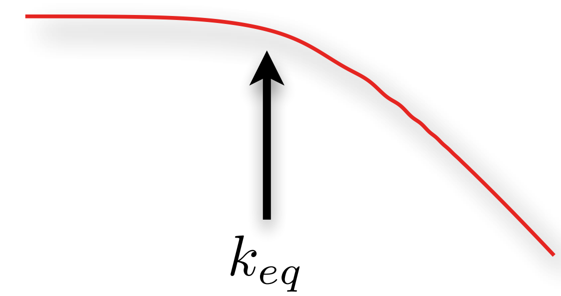
$$\mathcal{P}_\Phi \sim \Delta^2_{\mathcal{R}} T(k)^2$$

$$2.4 \times 10^{-9}$$



amplitude of second-order contributions

$$\overline{\partial^m \Phi \, \partial^n \Phi} \sim \int_0^\infty \frac{\mathrm{d}k}{k} \, k^{m+n} \mathcal{P}_\Phi(k)$$



$$\sim \Delta^2_{\mathcal{R}} \left(\frac{k_{eq}}{k_H} \right)^{m+n} \underbrace{\int_0^\infty \mathrm{d}\kappa \, \kappa^{m+n-1} T(\kappa)^2}$$

amplitude of second-order contributions

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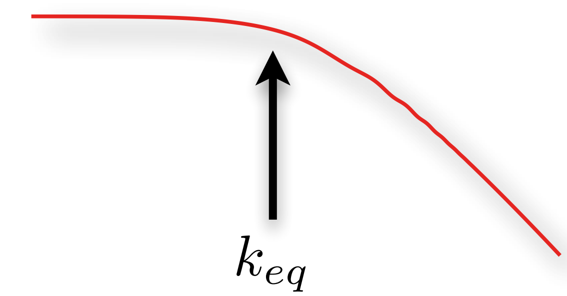
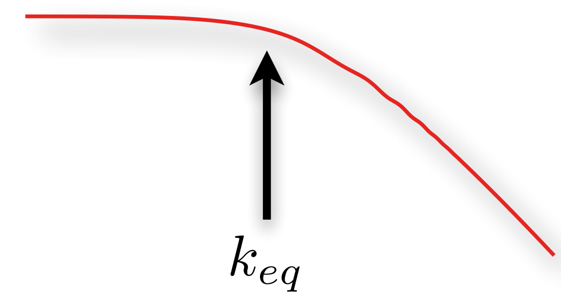


$$\sim \Delta^2_{\mathcal{R}} \left(\frac{k_{eq}}{k_H}\right)^{m+n} \underbrace{\int_0^\infty \mathrm{d}\kappa\,\kappa^{m+n-1}T(\kappa)^2}$$

$$\approx \begin{array}{lll} -\ln(\kappa_{\mathrm{IR}}) & \text{for} & m+n=0 \\ 3.9 & \text{for} & m+n=2 \\ 53\ln^3(\kappa_{\mathrm{UV}}) & \text{for} & m+n=4 \end{array}$$

amplitude of second-order contributions



$$\overline{\partial^m \Phi \, \partial^n \Phi} \sim \int_0^\infty \frac{\mathrm{d}k}{k} \, k^{m+n} \mathcal{P}_\Phi(k)$$

$$\sim \Delta^2_{\mathcal{R}} \left(\frac{k_{eq}}{k_H} \right)^{m+n} \underbrace{\int_0^\infty \mathrm{d}\kappa \, \kappa^{m+n-1} T(\kappa)^2}$$

$$\approx \begin{array}{ccc} -\ln(\kappa_{\mathrm{IR}}) & \text{for} & m+n=0 \\ 3.9 & \text{for} & m+n=2 \\ 53 \ln^3(\kappa_{\mathrm{UV}}) & \text{for} & m+n=4 \end{array}$$

$$\boxed{\Delta_{\mathcal{R}} \left(\frac{k_{eq}}{k_H} \right)^2 \approx 2.4 \Omega_m^2 h^2}$$

large equality scale suppresses backreaction - but overcomes factor(s) of Delta

backreaction - expansion rate

- second-order modes give non-trivial backreaction

- Hubble rate depends on

$$\mathrm{H}^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)}$$

  - UV divergent terms don't contribute on average [Newtonain]

  - well defined and well behaved backreaction

- ***but***, this is only well behaved because of the long radiation era

  - what would we do if the equality scale were smaller?

does everything converge?

- other quantities are much stranger
- time derivative of the Hubble rate represented in the deceleration parameter

$$q^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)} + \cdots + [\cdots](\partial^2\Phi)^2$$

  - UV divergent terms do not cancel out

arXiv:1105.1886 [pdf, other]
Is backreaction really small within concordance cosmology?
Chris Clarkson, Obinna Umeh (Cape Town)

does everything converge?

- other quantities are much stranger
- time derivative of the Hubble rate represented in the deceleration parameter

$$q^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)} + \cdots + \boxed{[\cdots](\partial^2\Phi)^2}$$

  - UV divergent terms do not cancel out

dominates backreaction

arXiv:1105.1886 [pdf, other]

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higher-order ?

- corrections appear like

$$\langle \nabla_i \Phi \nabla^i \Phi (\nabla^2 \Phi)^2 \rangle \sim \langle v^2 \delta^2 \rangle$$

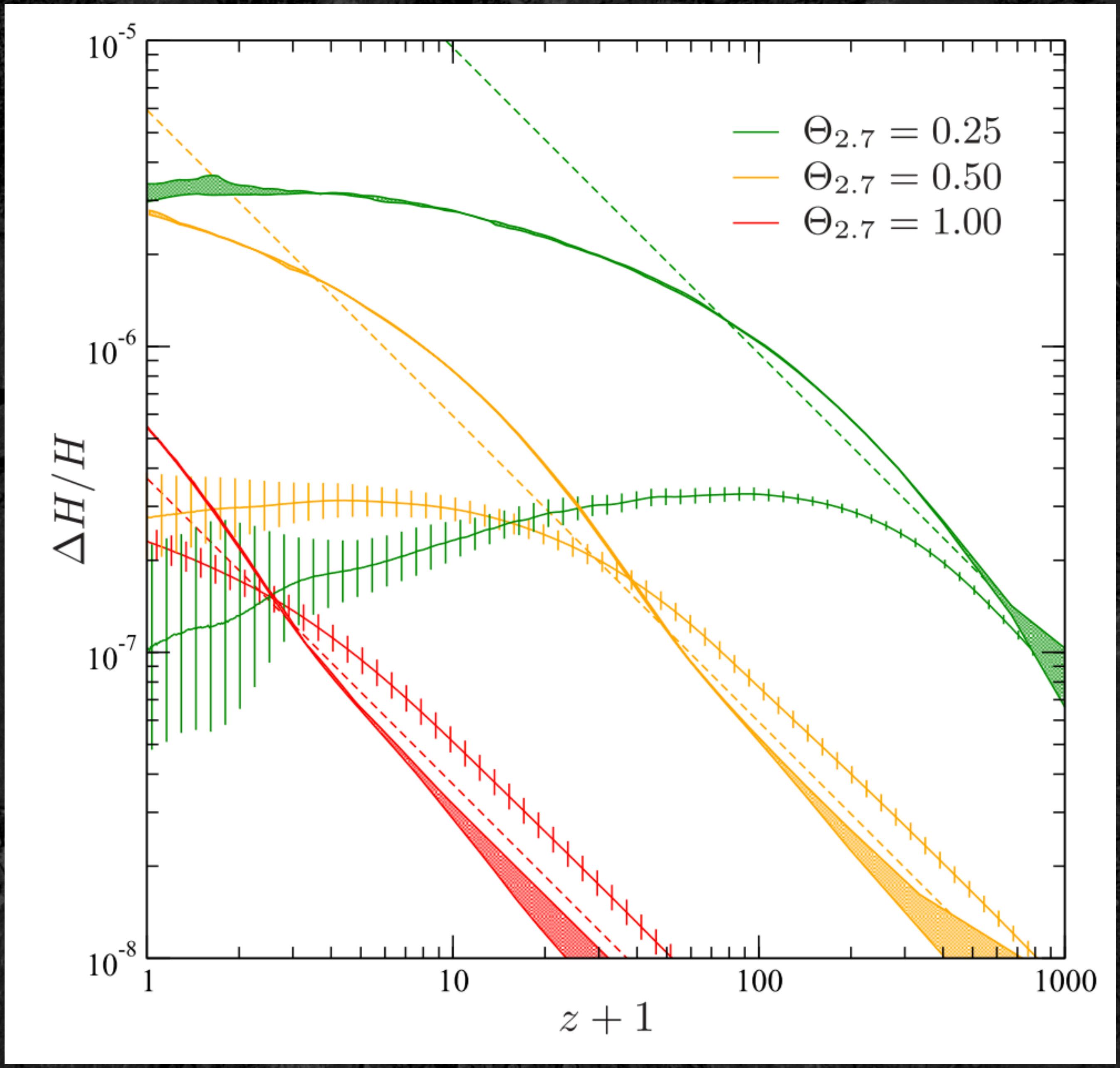
- giving corrections

$$\frac{\Delta H}{H} \sim \left(\frac{k_{\rm eq}}{H_0} \right)^2 \Delta^2_{\mathcal{R}} \langle \delta^2 \rangle \sim \Omega^2_m h^2 \Delta_{\mathcal{R}} \langle \delta^2 \rangle$$

- very sensitive to UV cutoff…

relativistic N-body: does the UV matter?

- studied in gevolution
- perturbative result correct by chance! [long radiation era]
- backreaction generically stabilises — virialised regions don't contribute
- answer: it's small



Does Small Scale Structure Significantly Affect Cosmological Dynamics?

Julian Adamek, Chris Clarkson, Ruth Durrer, and Martin Kunz
Phys. Rev. Lett. **114**, 051302 – Published 3 February 2015

conclusion 1: spatial averaging small and irrelevant…

- N-body => perturbation theory gets predictions right by chance
  - virialised regions don’t contribute
- some things can’t be computed in perturbation theory
  - eg - the expectation value of the acceleration rate…
- but who cares? [maybe the wrong conference to say this…]

backreaction at the percent level

- computation of average observables is critical …

Non-linear effects in light propagation

a single line of sight gives $D(z)$

gives magnification relative to background

$$
\left\{
\begin{aligned}
\frac{\mathrm{d}^2 D_A}{\mathrm{d}\lambda^2} &= -\left[\frac{1}{2}R_{ab}k^a k^b + \Sigma_{ab}\Sigma^{ab}\right] D_A \\
\frac{\mathrm{d}z}{\mathrm{d}\lambda} &= -(1+z)^2\left[\frac{1}{3}\Theta - A_a n^a + \sigma_{ab}n^a n^b\right]
\end{aligned}
\right.
$$

Hubble rate along the line of sight

distance modifications

linear convergence

$$\Delta(z_s) = \frac{D_A(z_s) - \bar{D}_A(z_s)}{\bar{D}_A(z_s)}$$

$$\Delta = \Delta_{\text{loc}} + \Delta_{\text{loc-int}} + \Delta_{\text{int}} \,.$$

Umeh, CC, Maartens, arXiv:1207.2109, 1402.1933
Ben-Dyan, et al., arXiv:arXiv:1209.4326

distance modifications

linear convergence

$\downarrow$

2nd order

$$\Delta(z_s) = \frac{D_A(z_s) - \bar{D}_A(z_s)}{\bar{D}_A(z_s)}$$

$$\Delta = \Delta_{\rm loc} + \Delta_{\rm loc\text{-}int} + \Delta_{\rm int} \, .$$

Umeh, CC, Maartens, arXiv:1207.2109, 1402.1933

Ben-Dyan, et al., arXiv:arXiv:1209.4326

$$\Delta = \Delta_{\mathrm{loc}} + \Delta_{\mathrm{loc\text{-}int}} + \Delta_{\mathrm{int}} \,.$$

$$\Delta = \Delta_{\text{loc}} + \Delta_{\text{loc-int}} + \Delta_{\text{int}} \,.$$

$$\Delta_{\text{loc}} = \Delta_{\text{loc}}^{\Phi} + \Delta_{\text{loc}}^{v} + \Delta_{\text{loc}}^{\Phi\times v} \,.$$

$$\Delta_{\text{int}} = \Delta_{\text{int}}^{\text{direct}} + \Delta_{\text{int}}^{\text{multiple}} \,.$$

$$\Delta_{\text{loc-int}} = \Delta_{\text{loc-int}}^{\Phi} + \Delta_{\text{loc-int}}^{v} \,,$$

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$$\Delta_{\text{int}}^{\text{multiple}} = \Delta_{\text{int}}^{\Phi-\Phi} + \Delta_{\text{int}}^{\Phi-\kappa} + \Delta_{\text{int}}^{\alpha-\alpha} + \Delta_{\text{int}}^{\kappa-\kappa} + \Delta_{\text{int}}^{\alpha-\nabla\kappa} + \Delta_{\text{int}}^{\gamma-\gamma}$$

$$\Delta = \Delta_{\text{loc}} + \Delta_{\text{loc-int}} + \Delta_{\text{int}}\,.$$

$$\Delta_{\text{loc}} = \Delta^{\Phi}_{\text{loc}} + \Delta^{v}_{\text{loc}} + \Delta^{\Phi\times v}_{\text{loc}}\,.$$

$$\Delta_{\text{loc-int}} = \Delta^{\Phi}_{\text{loc-int}} + \Delta^{v}_{\text{loc-int}},$$

$$\Delta_{\text{int}} = \Delta^{\text{direct}}_{\text{int}} + \Delta^{\text{multiple}}_{\text{int}}\,.$$

$$\Delta^{\text{multiple}}_{\text{int}} = \Delta^{\Phi-\Phi}_{\text{int}} + \Delta^{\Phi-\kappa}_{\text{int}} + \Delta^{\alpha-\alpha}_{\text{int}} + \Delta^{\kappa-\kappa}_{\text{int}} + \Delta^{\alpha-\nabla\kappa}_{\text{int}} + \Delta^{\gamma-\gamma}_{\text{int}}$$

$$\Delta^{\kappa-\kappa}_{\text{int}} = -\int_0^{\chi_s} \mathrm{d}\chi\, \frac{(\chi-\chi_s)}{\chi_s} \nabla^2_\perp \Phi \int_0^{\chi} \mathrm{d}\tilde\chi\, (\tilde\chi-\chi)\tilde\chi \nabla^2_\perp \Phi(\tilde\chi)\,.$$

$$\begin{aligned}\Delta^{\alpha-\nabla\kappa}_{\text{int}} = &+4 \int_0^{\chi_s} \mathrm{d}\chi\, \frac{(\chi-\chi_s)}{\chi_s} \int_0^{\chi} \mathrm{d}\tilde\chi\, \nabla_{\perp i}\Phi(\tilde\chi) \int_0^{\chi} \mathrm{d}\tilde\chi\, \frac{\tilde\chi^2}{\chi} \nabla^i_\perp \nabla^2_\perp \Phi(\tilde\chi) \\ &-2 \int_0^{\chi_s} \mathrm{d}\chi\, \frac{(\chi-\chi_s)}{\chi_s} \nabla_{\perp i}\Phi \int_0^{\chi} \mathrm{d}\tilde\chi\, \frac{(\tilde\chi-\chi)\tilde\chi^2}{\chi} \nabla^i_\perp \nabla^2_\perp \Phi(\tilde\chi)\,.\end{aligned}$$

$$\Delta^{\gamma-\gamma}_{\text{int}} = -2 \int_0^{\chi_s} \mathrm{d}\chi\, \frac{(\chi-\chi_s)\chi}{\chi_s} \int_0^{\chi} \mathrm{d}\tilde\chi\, \nabla_{\perp\langle i}\nabla_{\perp j\rangle}\Phi(\tilde\chi) \int_0^{\chi} \mathrm{d}\tilde\chi\, \nabla^{\langle i}_\perp \nabla^{j\rangle}_\perp \Phi(\tilde\chi)$$

$$\Delta = \Delta_{\text{loc}} + \Delta_{\text{loc-int}} + \Delta_{\text{int}} \,.$$

$$\Delta_{\text{loc}} = \Delta_{\text{loc}}^{\Phi} + \Delta_{\text{loc}}^{v} + \Delta_{\text{loc}}^{\Phi\times v} \,.$$

$$\Delta_{\text{int}} = \Delta_{\text{int}}^{\text{direct}} + \Delta_{\text{int}}^{\text{multiple}} \,.$$

$$\Delta_{\text{loc-int}} = \Delta_{\text{loc-int}}^{\Phi} + \Delta_{\text{loc-int}}^{v} ,$$

$$\Delta_{\text{int}}^{\text{multiple}} = \Delta_{\text{int}}^{\Phi-\Phi} + \Delta_{\text{int}}^{\Phi-\kappa} + \Delta_{\text{int}}^{\alpha-\alpha} + \Delta_{\text{int}}^{\kappa-\kappa} + \Delta_{\text{int}}^{\alpha-\nabla\kappa} + \Delta_{\text{int}}^{\gamma-\gamma}$$

$$\Delta_{\text{int}}^{\kappa-\kappa} = -\int_0^{\chi_s} \mathrm{d}\chi \, \frac{(\chi-\chi_s)}{\chi_s} \nabla_\perp^2\Phi \int_0^{\chi} \mathrm{d}\tilde{\chi}\, (\tilde{\chi}-\chi)\tilde{\chi}\nabla_\perp^2\Phi(\tilde{\chi}) \,.$$

$$\begin{aligned}\Delta_{\text{int}}^{\alpha-\nabla\kappa} = {} & +4\int_0^{\chi_s} \mathrm{d}\chi \, \frac{(\chi-\chi_s)}{\chi_s} \int_0^{\chi} \mathrm{d}\tilde{\chi}\, \nabla_{\perp i}\Phi(\tilde{\chi}) \int_0^{\chi} \mathrm{d}\tilde{\chi}\, \frac{\tilde{\chi}^2}{\chi} \nabla_\perp^i \nabla_\perp^2 \Phi(\tilde{\chi}) \\ & -2\int_0^{\chi_s} \mathrm{d}\chi \, \frac{(\chi-\chi_s)}{\chi_s} \nabla_{\perp i}\Phi \int_0^{\chi} \mathrm{d}\tilde{\chi}\, \frac{(\tilde{\chi}-\chi)\tilde{\chi}^2}{\chi} \nabla_\perp^i \nabla_\perp^2 \Phi(\tilde{\chi}) \,.\end{aligned}$$

$$\Delta_{\text{int}}^{\gamma-\gamma} = -2\int_0^{\chi_s} \mathrm{d}\chi \, \frac{(\chi-\chi_s)\chi}{\chi_s} \int_0^{\chi} \mathrm{d}\tilde{\chi}\, \nabla_{\perp\langle i}\nabla_{\perp j\rangle}\Phi(\tilde{\chi}) \int_0^{\chi} \mathrm{d}\tilde{\chi}\, \nabla_\perp^{\langle i}\nabla_\perp^{j\rangle}\Phi(\tilde{\chi})$$

$$\begin{aligned}\Delta_{\text{loc,s}}^{v} = {} & \left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\| v_s + \frac{1}{2}\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\| v_s^{(2)} + \frac{1}{2}\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right) v_{\| s}^{(2)} \,. \\ & +\chi_s\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\| v_s \left(\nabla_\| v_s' - \nabla_\|^2 v_s\right) + \left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_{\perp i} v_s \nabla_\perp^i v_s\end{aligned}$$

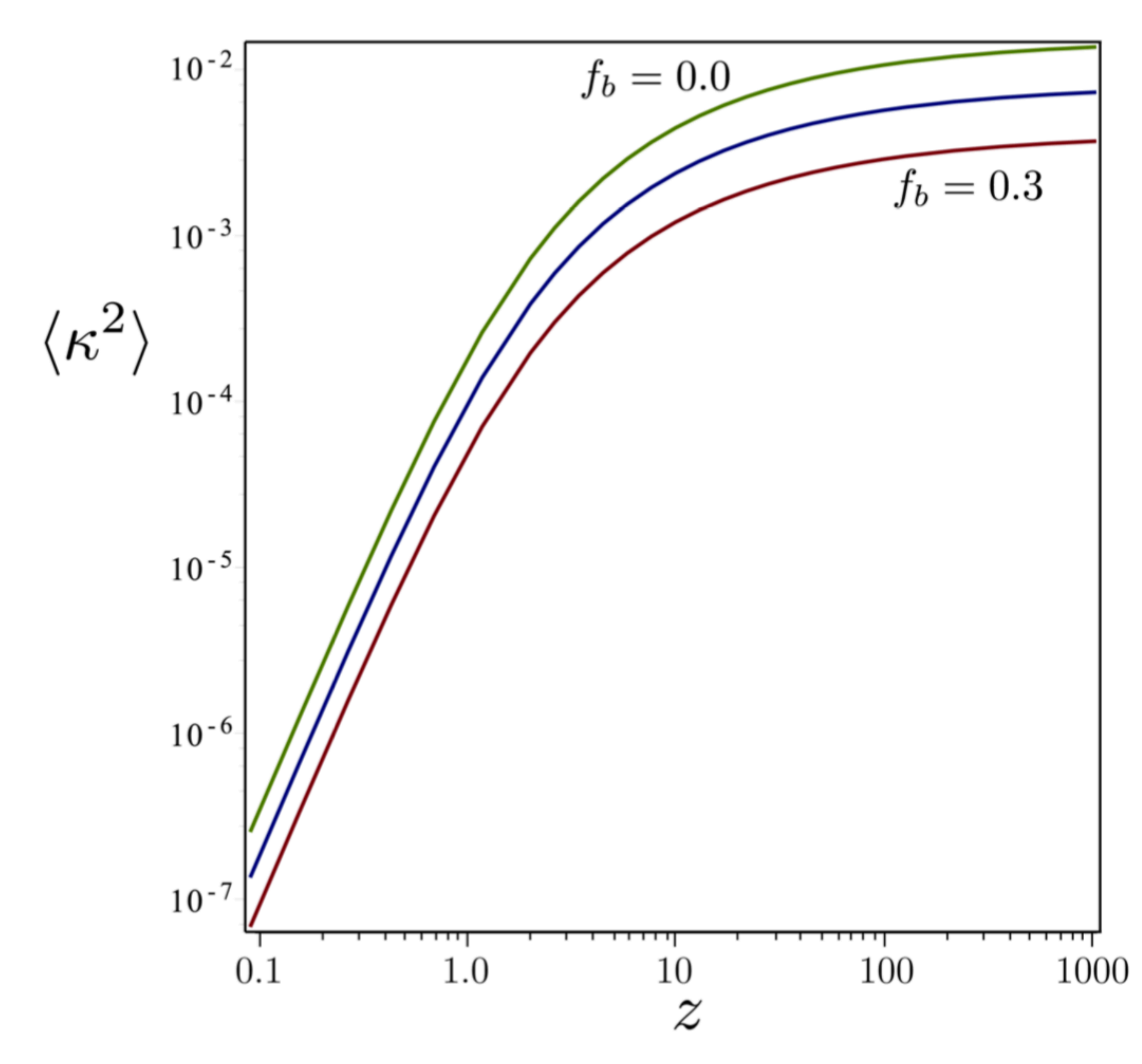
for ‘backreaction’ only 1 term counts

- only linear convergence squared important

$$\langle \kappa_1^2 \rangle \;=\; \left[\int_0^\chi d\chi' \frac{\chi-\chi'}{\chi\chi'} \Delta_\Omega \Psi(\chi)\right]^2$$

- very sensitive to small scale power
- gives %-level changes at CMB distance

$$\frac{3}{2}\left\langle \kappa_1^2 \right\rangle \sim 2\Delta_{\mathcal{R}}^2 (k_{\rm eq}\chi_*)^3 \simeq 0.014 \left(\frac{\Omega_m h^2}{0.14}\right)^3 \left(\frac{\chi_*}{14\,{\rm Gpc}}\right)^3$$



Camille Bonvin, CC, Ruth Durrer, Roy Maartens, Obinna Umeh
arXiv: 1405.7860,1503.07831,1504.01676

distance changes …

- expectation value of distance along a single line of sight *increases*

$$\langle d_A(\boldsymbol{n}_o)\rangle = d_0\left(1+\frac{3}{2}\langle\kappa_1^2\rangle\right)$$

  - structures generate de-focusing with higher probability

- average over observed angles gives *decrease* to the distance

$$\left\langle\,\overline{d_A}\,\right\rangle = \langle d_A(\boldsymbol{n})\rangle = d_0\left(1-\frac{1}{2}\langle\kappa_1^2\rangle\right)$$

  - lensing also changes the distribution of lines of sight in solid angle
  - contribution from under-densities does not exactly cancel contribution from over-densities

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invariants

- expectation value of solid angle

$$\langle \Delta\Omega(\boldsymbol{n}_o)\rangle = \left\langle d_A^{-2}(\boldsymbol{n}_o)\right\rangle \Delta S = \Delta\Omega_0$$

- same for flux

$$\langle \mathcal{F}(\boldsymbol{n}_o)\rangle = \mathcal{F}_0$$

  - but not angle averaged flux

$$\left\langle \overline{\mathcal{F}(z)} \right\rangle = \langle \mathcal{F}(z,\boldsymbol{n})\rangle = \mathcal{F}_0(z)\Big(1 + 4\langle \kappa_1^2\rangle\Big)$$

Camille Bonvin, CC, Ruth Durrer, Roy Maartens, Obinna Umeh
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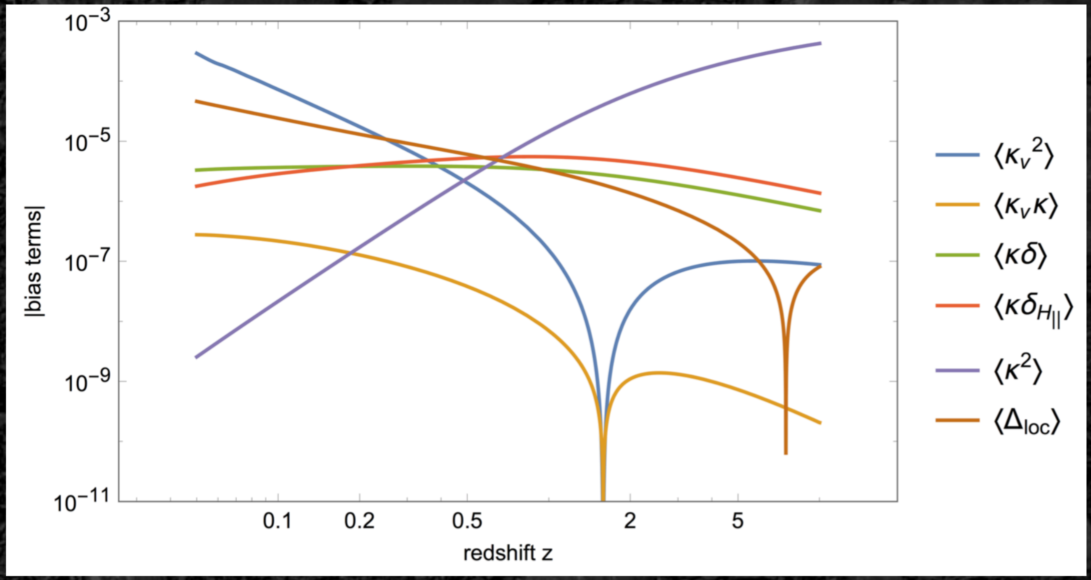
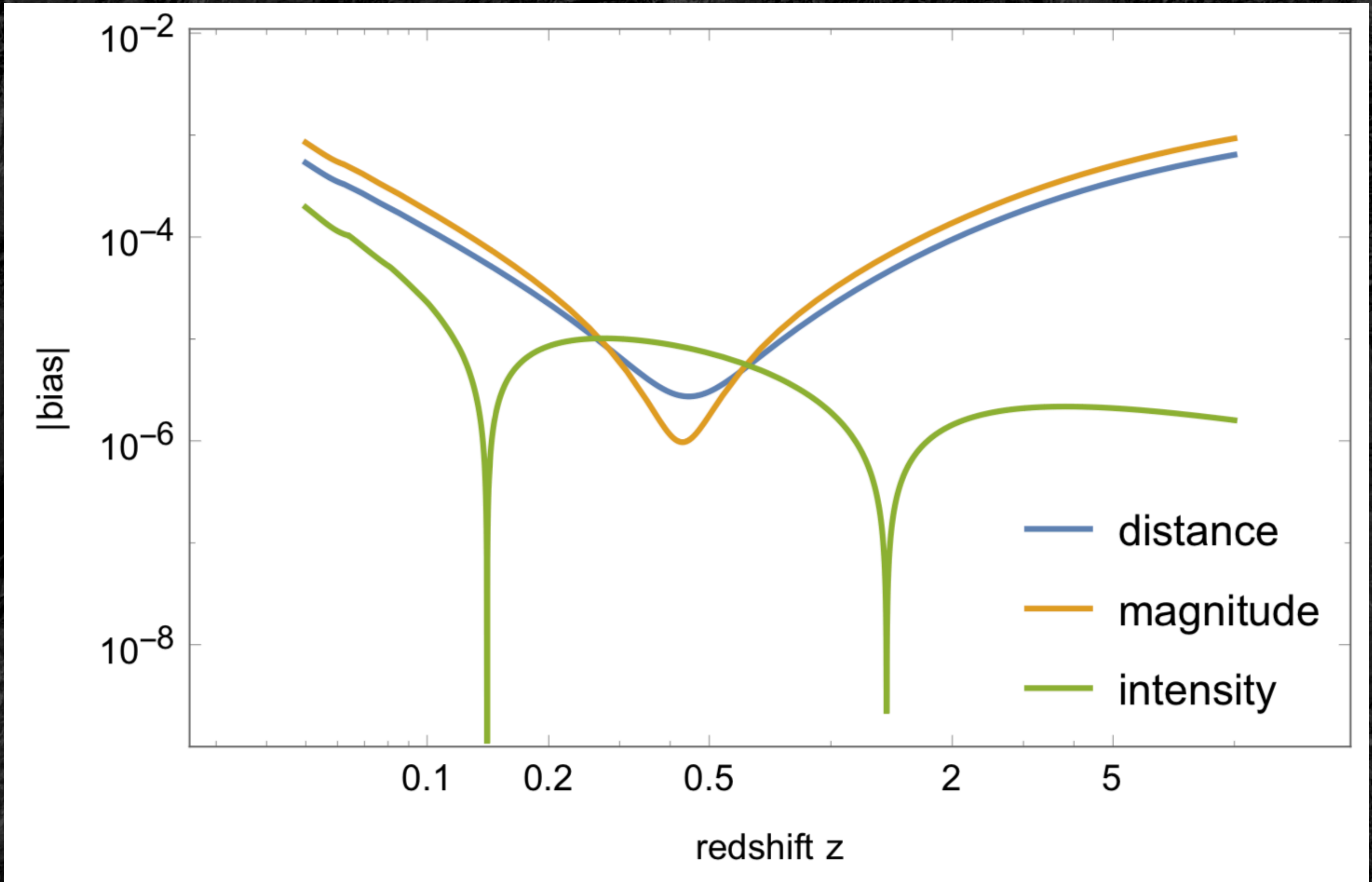
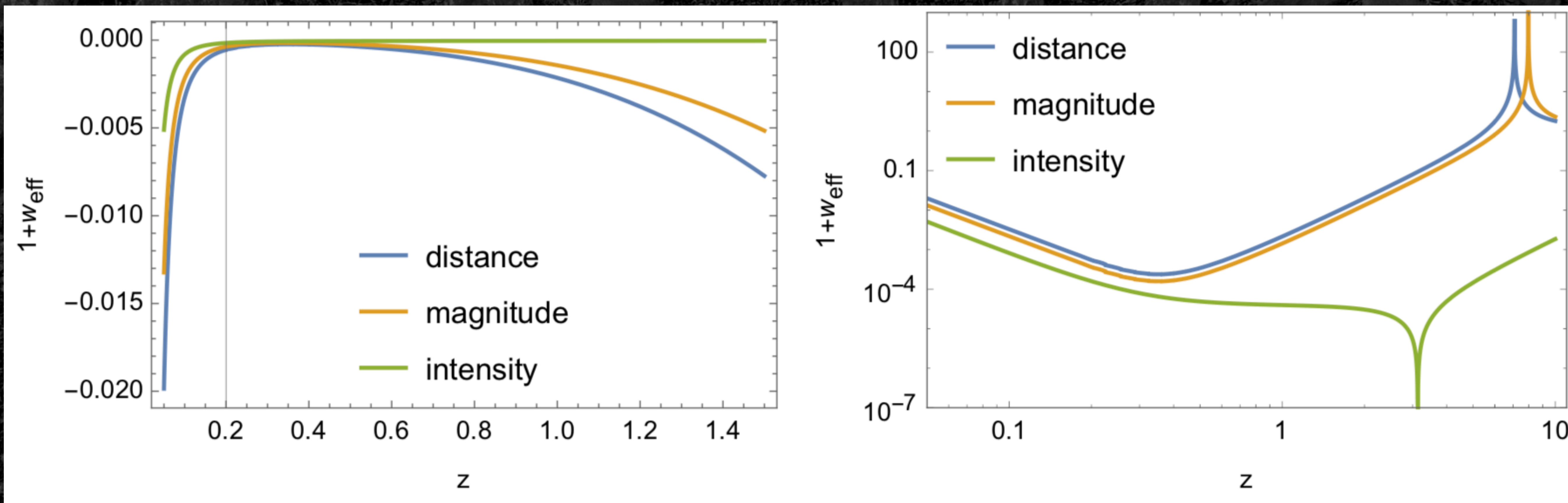
bias in practice



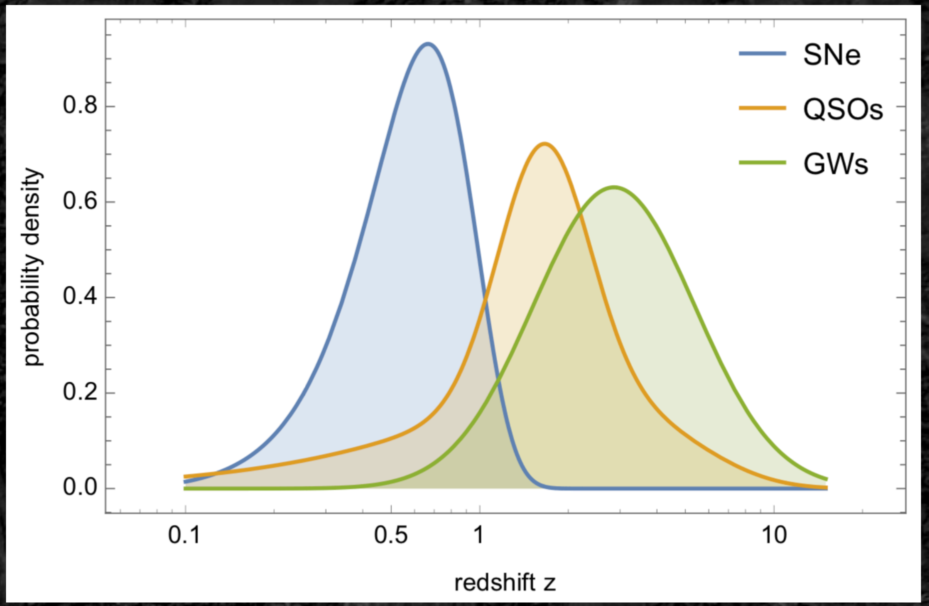


Pierre Fleury, CC, Roy Maartens arXiv: 1612.03726

bias in practice: effective DE



parameter estimation



		$w_0 w_a$CDM			ΛKCDM	
survey	D	$\Omega^*_{\mathrm{m}0} - \bar{\Omega}_{\mathrm{m}0}$	$w^*_0 + 1$	w^*_a	$\Omega^*_{\mathrm{m}0} - \bar{\Omega}_{\mathrm{m}0}$	Ω^*_{K0}
SNIa	d_{L}	-4.5×10^{-4}	4.3×10^{-4}	4.6×10^{-3}	-2.7×10^{-4}	4.7×10^{-4}
	m	-6.1×10^{-4}	5.4×10^{-4}	7.0×10^{-3}	4.9×10^{-5}	-1.2×10^{-4}
	I	-2.0×10^{-3}	2.6×10^{-3}	1.7×10^{-2}	1.0×10^{-3}	-1.7×10^{-3}
QSOs	d_{L}	-9.0×10^{-4}	-9.7×10^{-4}	2.0×10^{-2}	-1.1×10^{-3}	2.4×10^{-3}
	m	-5.2×10^{-4}	4.0×10^{-5}	8.4×10^{-3}	-5.3×10^{-4}	1.0×10^{-3}
	I	-6.0×10^{-3}	8.0×10^{-3}	5.1×10^{-2}	5.9×10^{-3}	-9.3×10^{-3}
GWs	d_{L}	-1.0×10^{-3}	-2.7×10^{-3}	3.0×10^{-2}	-1.4×10^{-3}	3.1×10^{-3}
	m	-6.0×10^{-4}	-4.6×10^{-4}	1.2×10^{-2}	-7.2×10^{-4}	1.5×10^{-3}
	I	-3.3×10^{-6}	3.7×10^{-5}	-1.2×10^{-4}	-2.2×10^{-5}	4.8×10^{-5}

conclusions

- spatial averaging very small, probably irrelevant
  - virialised structures don’t contribute
  - study viral theorem in GR to understand more
- average observables can have significant changes ito dark energy biasing
  - sensitive to small scale power — needs more analysis to quantify and correct for
  - a problem for the future, small effects atm