# Relativistic Perturbation Theory in the Presence of Non-Linear Structures

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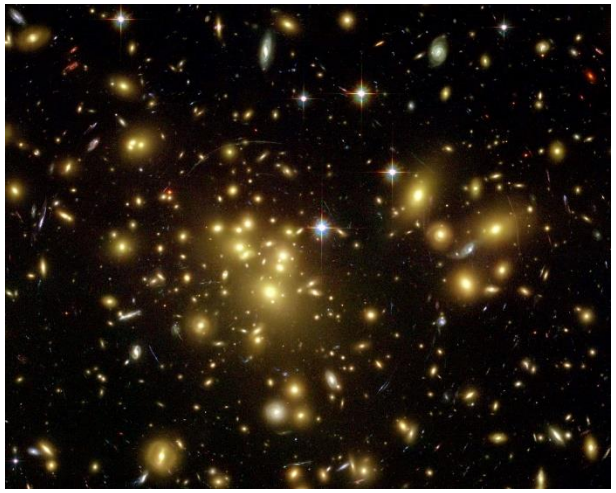
# Modelling Structure in Cosmology

On small scales:

- Newtonian gravitational fields, on an FLRW background
- Non-linear Eulerian equations of hydrodynamics



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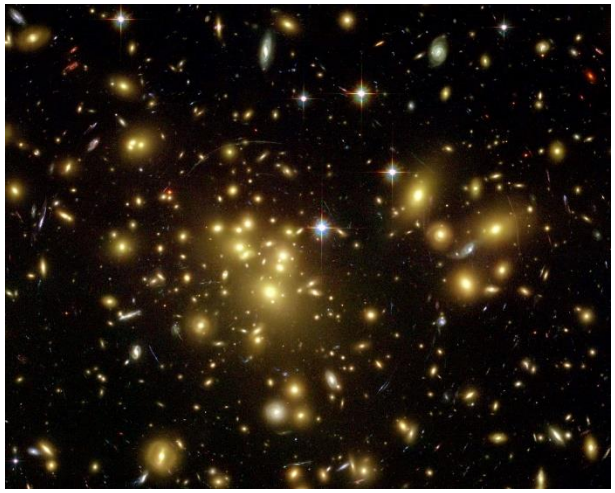

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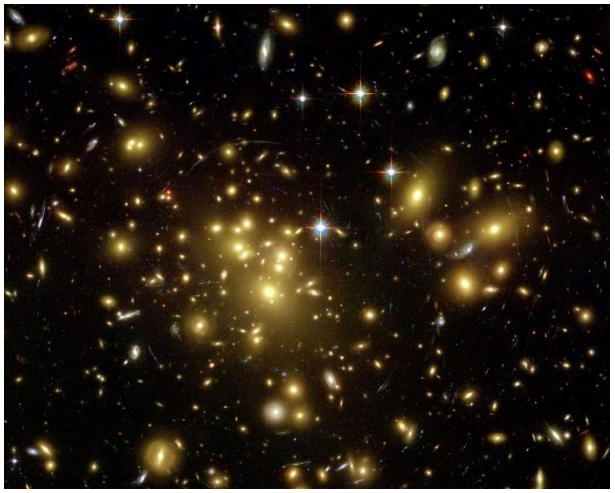

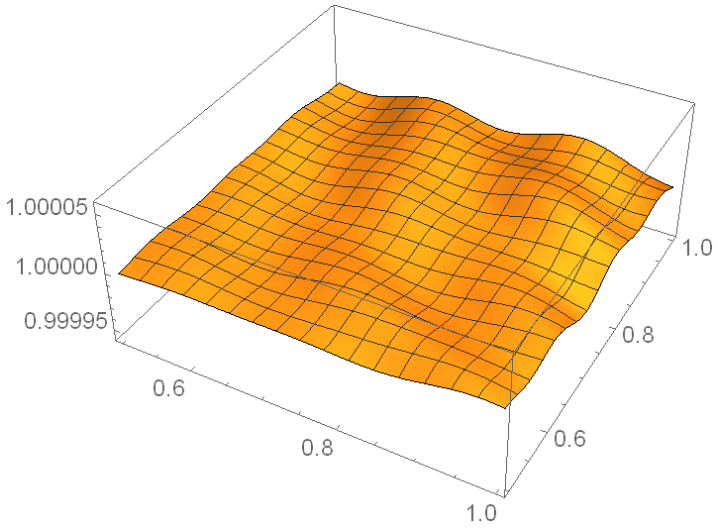


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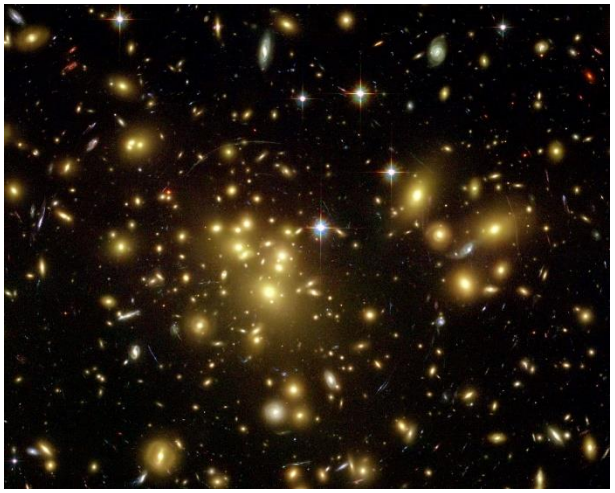

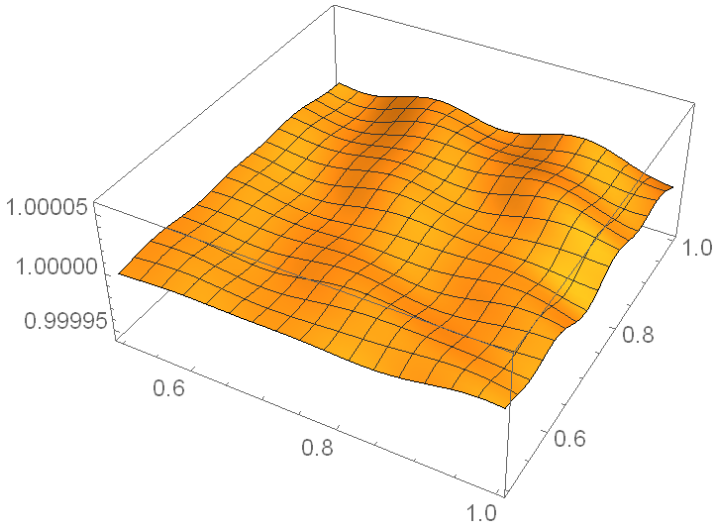


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- Linear cosmological perturbations theory, on an FLRW background
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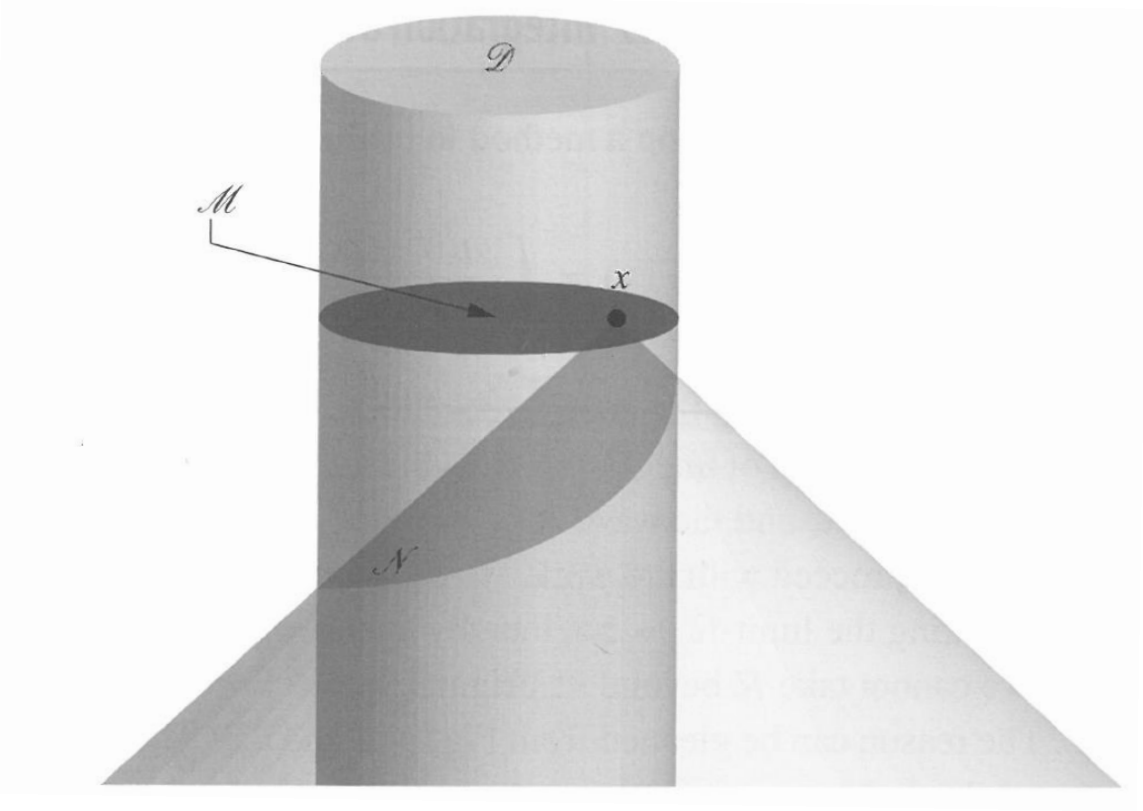
Not a valid assumption in the presence of non-linear structures

# Post-Newtonian Expansions

Perturbations are considered on scales much smaller than the horizon:



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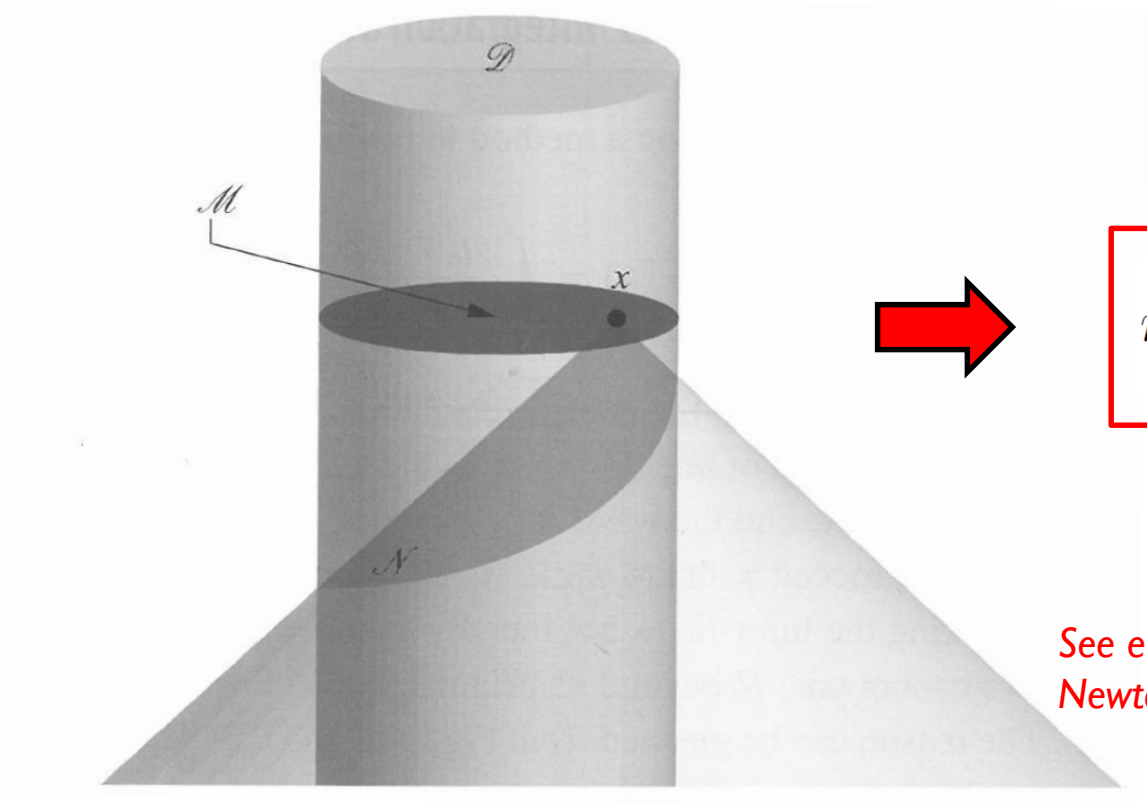


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*See e.g. Will & Poisson, Gravity: Newtonian, Post-Newtonian, Relativistic*

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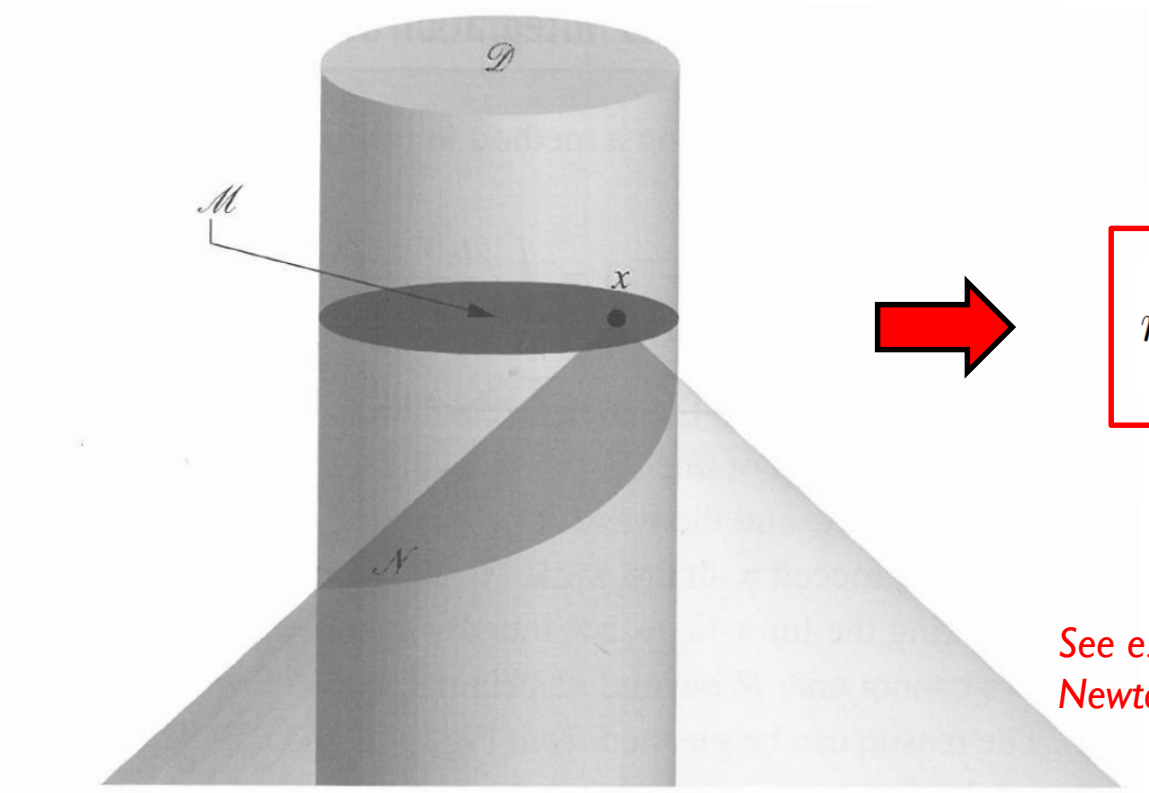


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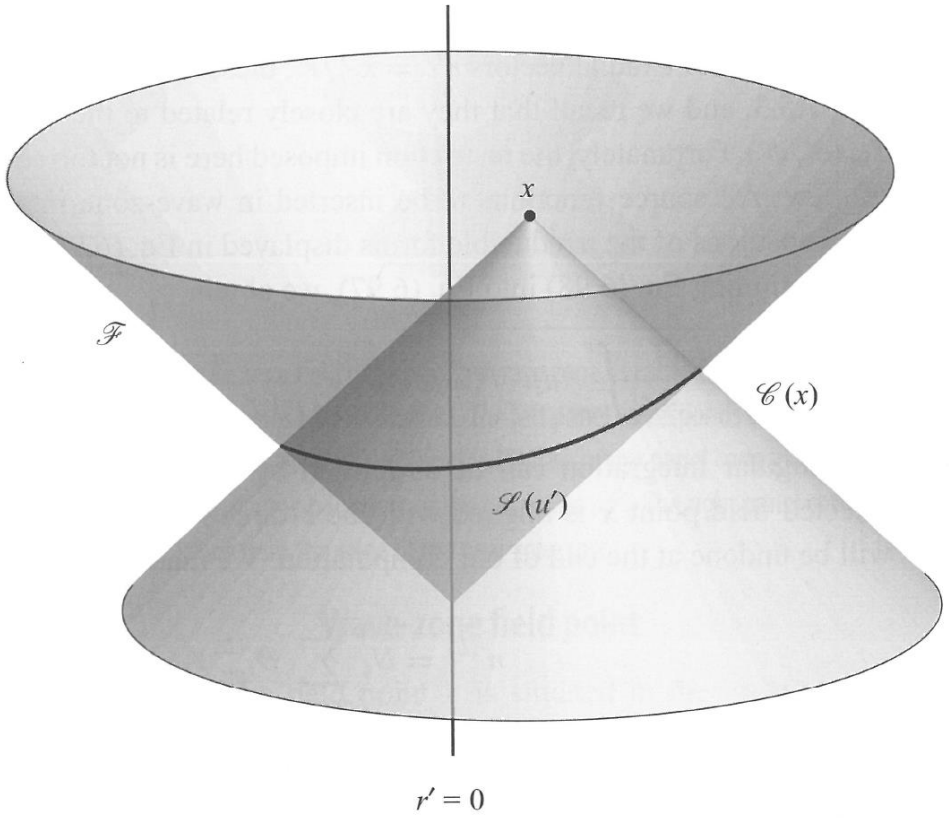
$$g_{00} = g_{00}^{(0)}(t) + g_{00}^{(2)}(t, \mathbf{x}) + \tfrac{1}{2} g_{00}^{(4)}(t, \mathbf{x}) \ldots$$

# Cosmological Perturbation Theory

Perturbations considered on large scales, comparable to the horizon:



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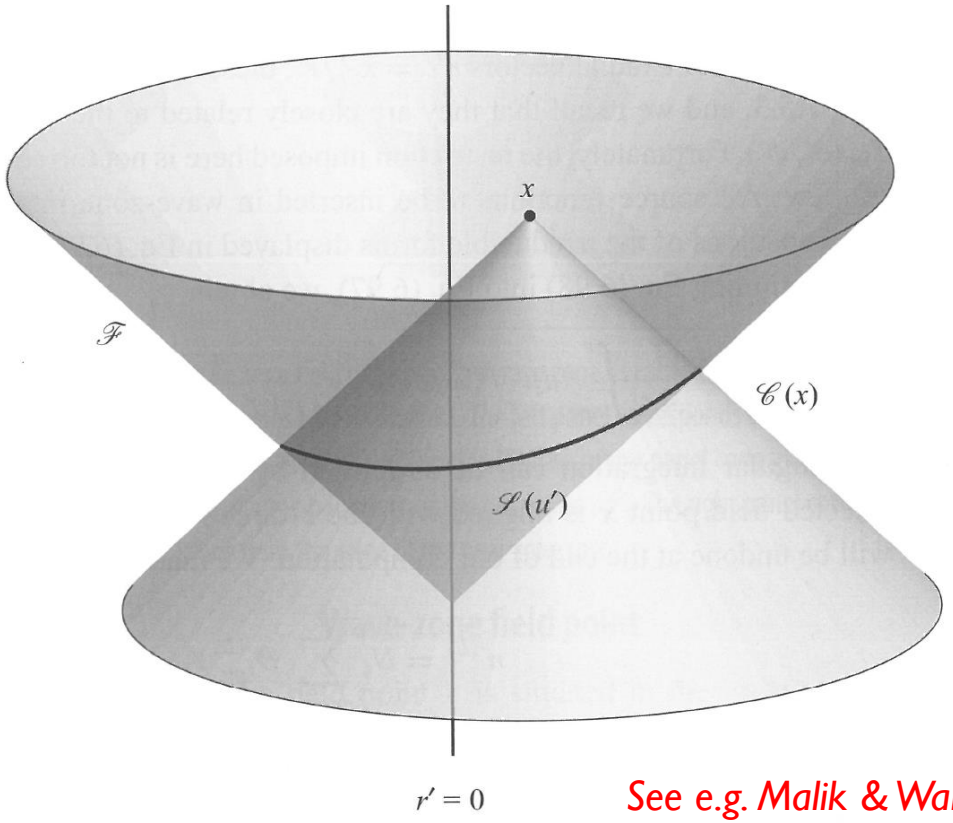


*See e.g. Malik & Wands arXiv:0809.4944*

$$\epsilon \sim v^{(1)} \sim g^{(1)}_{\mu\nu} \sim L_C^2 \rho^{(1)} \sim L_C^2 p^{(1)}$$

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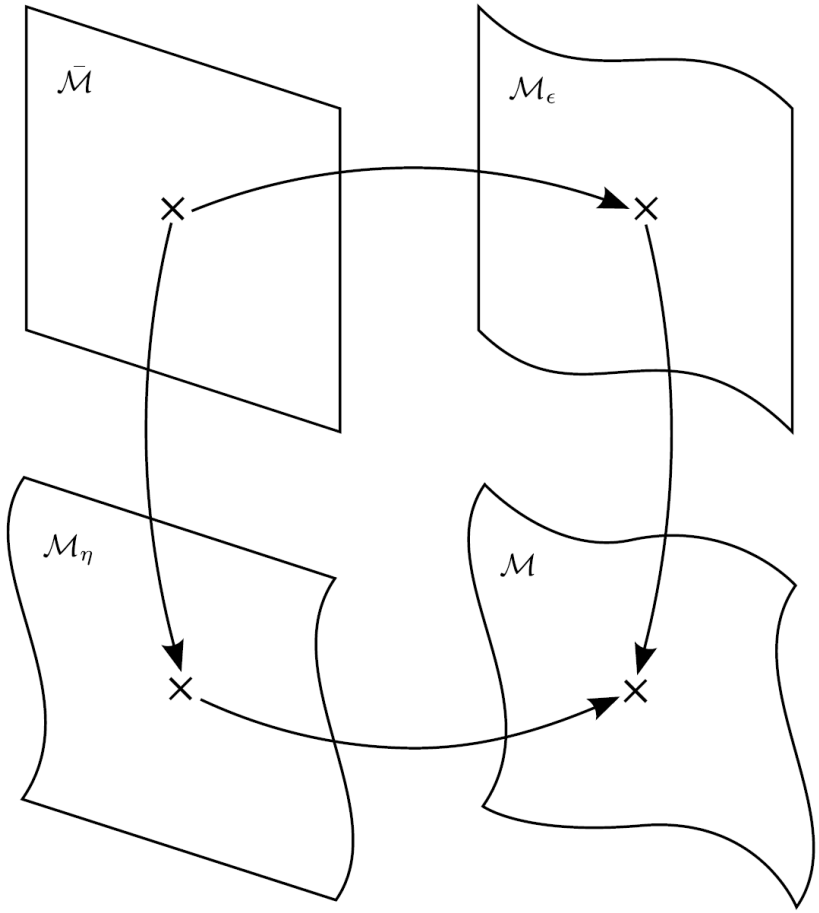
**Solution:** **Perform each of the expansions simultaneously, at every point in space-time. Use post-Newtonian gravity to model small-scale fluctuations, and cosmological perturbation theory to model large-scales.**

# Two-parameter Perturbations

Our proposed solution requires perturbing a background geometry in both post-Newtonian and cosmological perturbation theory formalisms:

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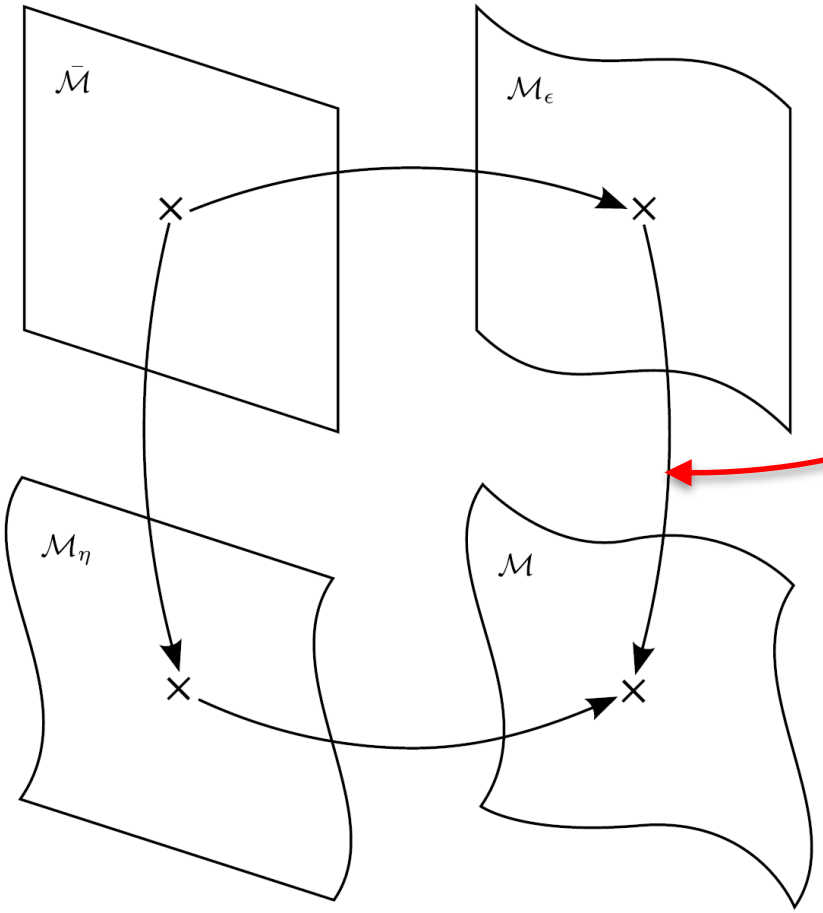


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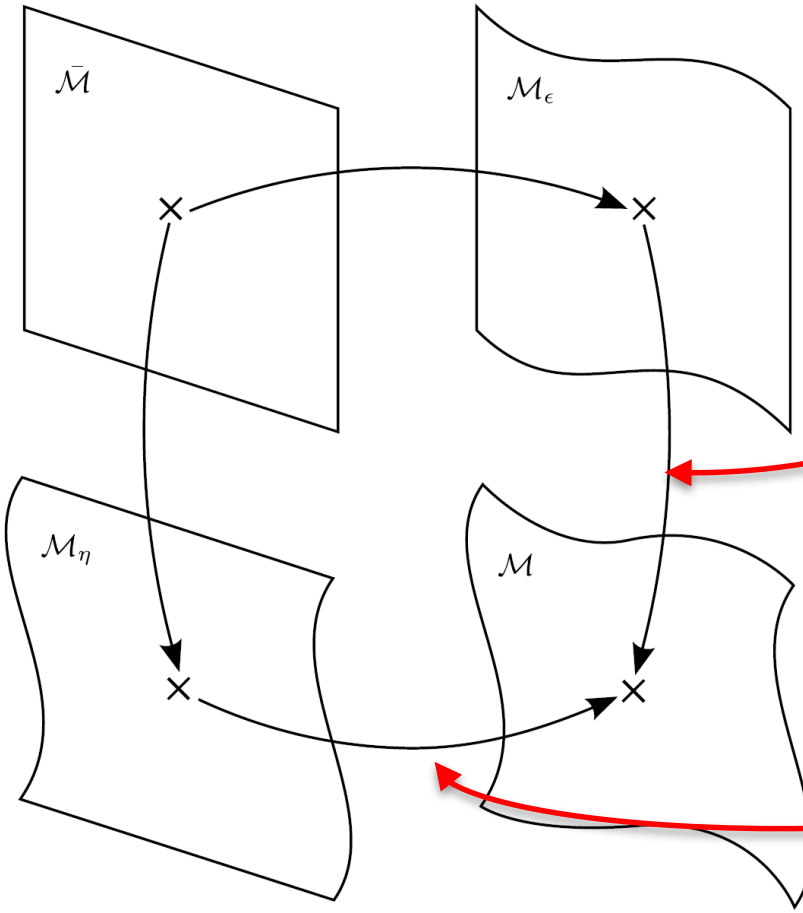


Can be viewed as post-Newtonian gravity operating on a perturbed FLRW background.

Or, equivalently, as cosmological perturbation theory operating on an FLRW background with post-Newtonian perturbations.

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# Results

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- Large-scale gravitational potentials are sourced at leading-order by terms that are quadratic (or higher) in small-scale gravitational fields.
- The large-scale gravitational potentials can no longer be decomposed in non-interacting scalar, vector and tensor parts.

# Applications

- Back-reaction on the large-scale expansion of space is found to occur at the level of $1$ part in $10^5$, with a leading-order contribution that scales like $a^{-4}$ in the effective Friedmann equations. *Sanghai & TC, arXiv:1512.04824*

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*in progress...*

Thanks for listening