Relativistic Structure Formation in the Universe

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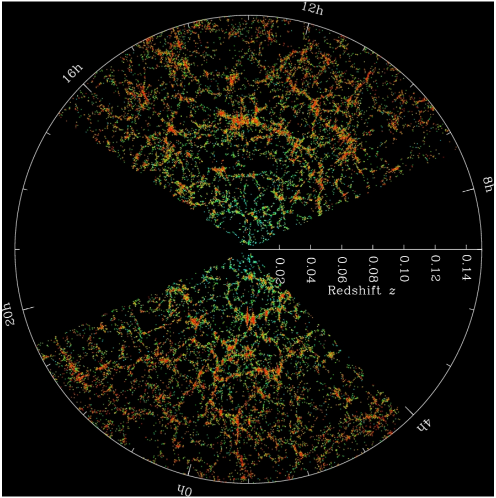
Outline

The observed Universe is not homogeneous and isotropic. It contains galaxies, clusters of galaxies, voids, filaments...



M. Blanton and the Sloan Digital Sky Survey Team.

Introduction

- We assume that on large scales there is a well defined mean density and on intermediate scales, the density differs little from it.
- This is a highly non-trivial assumption, which is best justified by the isotropy of the cosmic microwave background, the CMB.
- Under the hypothesis that cosmic structure grew out of small initial fluctuations we can study their evolution using perturbation theory.
- At late times and sufficiently small scales fluctuations of the cosmic density are not small. The density inside a galaxy is about 10^5 times higher than the mean density of the Universe.
- Perturbation theory is then not adequate and we need N-body simulations to study structure formation on galaxy - cluster scales of a few Mpc and less.
- Since this is mainly relevant on scales much smaller than the Hubble scale ($3000h^{-1}$Mpc), it has been studied in the past mainly with non-relativistic simulations. Since a couple years several groups have started to develop also relativistic simulations.

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Linear cosmological perturbation theory

Linear perturbation theory: Gauge transformation

Splitting the true metric into a background and a perturbation,

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= a^2(t)\left[-(1+2A)dt^2 - 2B_i dt dx^i + (1+2H_L)\gamma_{ij}dx^i dx^j + 2H_{ij}dx^i dx^j\right] \\ &= a^2(t)\left[\bar\eta_{\mu\nu} + \epsilon\delta\eta_{\mu\nu}\right]dx^\mu dx^\nu = \left[\bar g_{\mu\nu} + \epsilon\delta g_{\mu\nu}\right]dx^\mu dx^\nu \end{aligned}$$

is not unique. Under a linearized coordinate transformation (gauge transformation), $x^\mu \to x^\mu + \epsilon X^\mu$, the metric (and any other tensor field) changes like

$$g_{\mu\nu} \mapsto g_{\mu\nu} + \epsilon L_X g_{\mu\nu}\ , \qquad \delta g_{\mu\nu} \mapsto \delta g_{\mu\nu} + L_X \bar g_{\mu\nu}\ .$$

The same is true for the energy momentum tensor,

$$T_{\mu\nu} = \bar T_{\mu\nu} + \epsilon\delta T_{\mu\nu}\ , \qquad \delta T_{\mu\nu} \mapsto \delta T_{\mu\nu} + L_X \bar T_{\mu\nu}\ .$$

A variable (tensor field) V is called gauge invariant if it does not change under gauge transformations. As we see from the above equations, this happens if $L_X \bar V = 0\ \forall\ X$, hence $\bar V =$ constant. (Stewart Walker lemma).

Furthermore, even if the total metric and energy momentum tensor satisfy Einstein's equation, this is in general not true for arbitrary splits into a background and a perturbation. We call a split 'admissible' if it is true. For an admissible split also the perturbations satisfy Einstein's equations.

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We typically split spatial tensor fields into helicity zero (scalar) helicity one (vector) and helicity two (tensor) parts. These do not mix within linear perturbation theory due to rotational symmetry of the background. The same is true for each Fourier component (translational symmetry)

$$B_i = \nabla_i B + B_i^{(V)}\,, \qquad H_{ij} = \left(\nabla_i\nabla_j - \frac{1}{3}\Delta\gamma_{ij}\right) H^{(S)} + \left(\nabla_i H_j^{(V)} + \nabla_j H_i^{(V)}\right) + H_{ij}^{(T)}$$

$$\nabla^i B_i^{(V)} = \nabla^i H_i^{(V)} = \nabla^i H_{ij}^{(T)} = H_i^{i\,(T)} = 0\,.$$

I shall not discuss vector perturbations any further. They seem not to be relevant in cosmology.

In the background Friedmann-Lemaître universe the Weyl tensor $C^{\mu}{}_{\nu\alpha\beta}$ and the anisotropic stress tensor $\Pi_{\mu\nu}$ vanish $\Rightarrow$ their perturbations are gauge-invariant,

$$\Pi_{\mu\nu} = T_{\mu\nu} - (\rho + P)u_\mu u_\nu - P g_{\mu\nu}\,.$$

Here ρ and u are defined by $T^\mu_\nu u^\nu = -\rho u^\mu$, $u^\mu u_\mu = -1$ and $T^\mu_\mu + \rho = 3P$.

Linear perturbation theory: Gauge invariant variables

For scalar perturbations we can define the so called Bardeen potentials. In Fourier space (k = wavenumber) they are given by

$$\begin{aligned}
\Psi &= A - \mathcal{H}k^{-1}\sigma - k^{-1}\dot{\sigma}\,, \\
\Phi &= -H_L - \frac{1}{3}H^{(S)} + \mathcal{H}k^{-1}\sigma = -\mathcal{R} + \mathcal{H}k^{-1}\sigma \\
\sigma &= k^{-1}\dot{H}^{(S)} - B \qquad \mathcal{R} = H_L + \frac{1}{3}H^{(S)}\,.
\end{aligned}$$

The non-vanishing components of $C^{\mu}_{\nu\alpha\beta}$ and $\Pi_{\mu\nu}$ are then given by

$$\begin{aligned}
E_{ij} &\equiv C^{\mu}_{i\nu j}u_\mu u^\nu = -C^{0}_{i0j} = -\frac{1}{2}\left[(\Psi+\Phi)_{|ij} - \frac{1}{3}\Delta(\Psi+\Phi)\gamma_{ij}\right] \\
8\pi G a^2 \Pi_{ij} &= (\Psi-\Phi)_{|ij} - \frac{1}{3}\Delta(\Psi-\Phi)\gamma_{ij}\,.
\end{aligned}$$

For the last eqn. we used Einstein's equation.
In longitudinal (Newtonian) gauge, $B = H^{(S)} = 0$, the perturbed metric is given by

$$\delta g_{\mu\nu} = -2a^2\left[\Psi dt^2 + \Phi\gamma_{ij}dx^i dx^j\right]\,.$$

Tensor perturbations are gauge invariant (there are no tensor-type gauge-transformations).

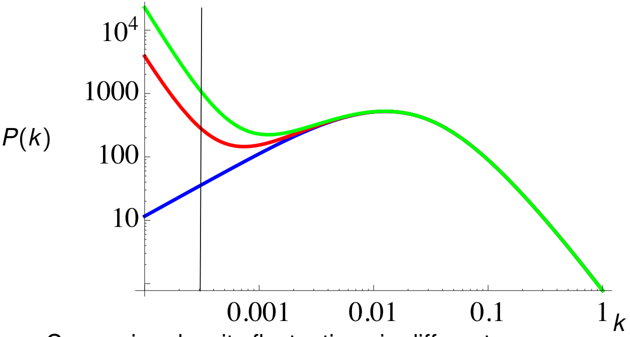
Apart from the anisotropic stress, the entropy perturbation

$$\Gamma = (\delta P + \frac{c_s^2}{w}\delta\rho)/\bar\rho$$

is gauge invariant. To obtain gauge invariant perturbations of the velocity and density perturbations, $\delta = (\rho - \bar\rho)/\bar\rho$ one has to combine them with metric perturbations. The most common combinations are

$$\begin{aligned}
V &\equiv v - \frac{1}{k}\dot H_T = v^{\rm long} \\
D_s &\equiv \delta + 3(1+w)\mathcal{H}(k^{-2}\dot H_T - k^{-1}B) \equiv \delta^{\rm long}\ , \\
D &\equiv \delta^{\rm long} + 3(1+w)\frac{\mathcal{H}}{k}V = \delta + 3(1+w)\frac{\mathcal{H}}{k}(v - B) \\
&= D_s + 3(1+w)\frac{\mathcal{H}}{k}V\ , \\
D_g &\equiv \delta + 3(1+w)\left(H_L + \frac{1}{3}H^{(S)}\right) = \delta^{\rm long} - 3(1+w)\Phi \\
&= D_s - 3(1+w)\Phi\,.
\end{aligned}$$

The resulting density perturbation spectrum depends on the choice of the perturbation variable only on very large scales, $k \lesssim H_0^{-1}$.



Comparing density fluctuations in different gauges: comoving gauge, D (blue), longitudinal gauge, D_s (red) and spatially flat gauge, D_g (green)

Linear perturbation theory: Perturbation equations

Einstein's equations ($\mathcal{H} = \dot{a}/a = Ha$)

$$\begin{aligned}
4\pi G a^2 \rho D &= -(k^2 - 3K)\Phi && (00)\\
4\pi G a^2 (\rho + P) V &= k\left(\mathcal{H}\Psi + \dot{\Phi}\right) && (0i)\\
8\pi G a^2 P \Pi^{(S)} &= k^2(\Phi - \Psi) && (i \neq j)\\
4\pi G a^2 \rho \left[\tfrac{1}{3} D + c_s^2 D_s + w\Gamma\right] &= \ddot{\Phi} + 2\mathcal{H}\dot{\Phi} + \mathcal{H}\dot{\Psi} + \left[2\dot{\mathcal{H}} + \mathcal{H}^2 - \tfrac{k^2}{3}\right]\Psi && (ii)\\
8\pi G a^2 P \Pi^{(T)} &= \ddot{H}^{(T)} + 2\mathcal{H}\dot{H}^{(T)} + \left(2K + k^2\right) H^{(T)} && (\text{tensor})
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Conservation equations for scalar perturbations ($w = P/\rho$, $c_s^2 = \dot{P}/\dot{\rho}$)

$$\begin{aligned}
\dot{D} - 3w\mathcal{H}D &= -\left(1 - \frac{3K}{k^2}\right)\left[(1+w)kV + 2\mathcal{H}w\Pi\right],\\
\dot{V} + \mathcal{H}V &= k\left[\Psi + \frac{c_s^2}{1+w}D + \frac{w}{1+w}\Gamma - \frac{2}{3}\left(1 - \frac{3K}{k^2}\right)\frac{w}{1+w}\Pi\right].
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Bardeen equation for scalar perturbations

$$\begin{aligned}
&\ddot{\Phi} + 3\mathcal{H}(1 + c_s^2)\dot{\Phi} + \left[3(c_s^2 - w)\mathcal{H}^2 - (2 + 3w + 3c_s^2)K + c_s^2 k^2\right]\Phi\\
&= \frac{8\pi G a^2 P}{k^2}\left[\mathcal{H}\dot{\Pi} + [2\dot{\mathcal{H}} + 3\mathcal{H}^2(1 - c_s^2/w)]\Pi - \frac{1}{3}k^2\Pi + \frac{k^2}{2}\Gamma\right].
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\end{aligned}
$$

We consider adiabatic fluctuations of a perfect fluid ($\Gamma = \Pi = 0$) with $K = 0$ and $w = c_s^2$ =constant. In this case the Bardeen eqn. simplifies to

$$\ddot{\Phi} + 3\mathcal{H}(1+w)\dot{\Phi} + 3wk^2\Phi = 0\,, \qquad \mathcal{H} = \frac{2}{1+3w}t^{-1} = \frac{q}{t}$$

with solution

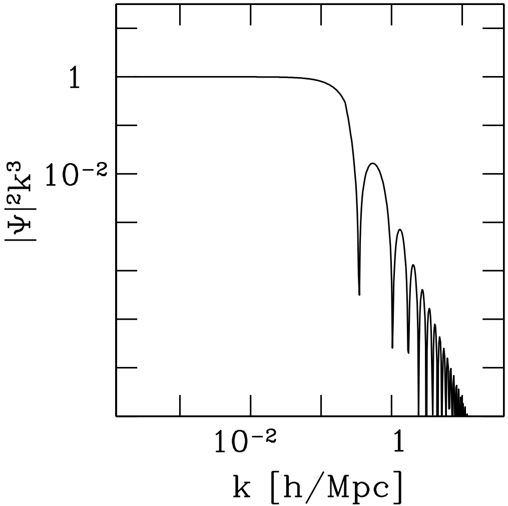
$$\Phi = \frac{1}{a}\left[Aj_q(c_s kt) + By_q(c_s kt)\right]$$

On large scales, $c_s kt < 1$ the growing mode $\propto A$ is constant, on small scales, $c_s kt > 1$ it oscillates and decays.
For radiation, $w = c_s^2 = 1/3$, $q = 1$.
The case of dust, $c_s^2 = w = 0$ has been treated apart. In this case the solution is

$$\Phi = A + \frac{B}{(kt)^5}\,.$$

Again, the growing mode is constant.

Starting from scale invariant initial conditions, $\langle|\Psi|^2\rangle k^3 = A_S(k/H_0)^{n-1}$, $n = 1$ one finds today a Bardeen potential of roughly the following form:



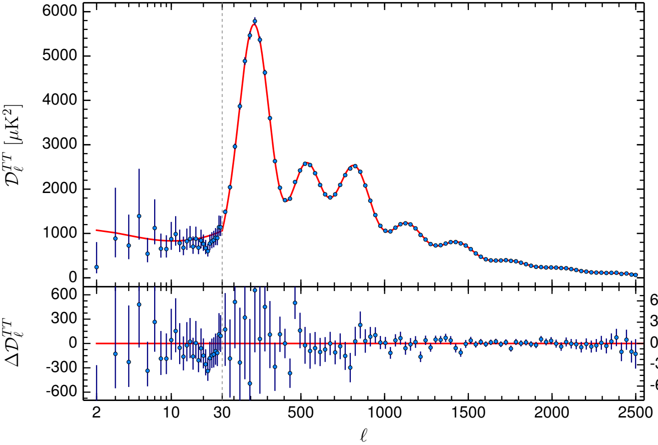
From the Bardeen potential we can now compute the density and velocity perturbations.
For dust we obtain for the growing modes

$$V \propto t \qquad D \propto t^2 \propto a\,.$$

For radiation we find, $x = kt/\sqrt{3}$

$$\begin{aligned} D_g &= 2A\left[\cos(x) - \frac{2}{x}\sin(x)\right]\,, \\ V &= -\frac{\sqrt{3}}{4}D_g' = \frac{\sqrt{3}A}{2}\left[\sin(x) + \frac{2}{x}\cos(x) - \frac{2}{x^2}\sin(x)\right]\,. \end{aligned}$$

These are the acoustic oscillations which we see in the CMB temperature anisotropy spectrum.



The Planck Collaboration, 2015

What do we observe?

Measured quantities must be gauge invariant. They do not depend on the coordinate system in which we compute them.

We have long ago (1990-95) computed what we measure (within linear perturbation theory) when we masure CMB anisotropies. A simplified formula neglecting Silk damping is (RD 1990)

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi + \Phi\right](t_{\rm dec}, \mathbf{x}_{\rm dec}) + \int_{t_{\rm dec}}^{t_0} (\dot\Psi + \dot\Phi)(t, \mathbf{x}(t))\, dt \;.$$

This result is obtained by studying lightlike geodesics.

To go beyond this and take into account polarisation and Silk damping, we have to study the propagation of photons with a Boltzmann equation approach.

For density perturbations (even neglecting bias and non-linearities) it is more complicated. We consider the following measurement: we count the galaxies in a small redshift element dz around redshift z and solid angle $d\Omega_{\mathbf{n}}$ around a direction $\mathbf{n}$. To obtain the fluctuation of this number count we subtract from it the mean number of galaxies inside a redshift bin dz and a solid angle $d\Omega_{\mathbf{n}}$ and we also divide by this mean,

$$\Delta(z,\mathbf{n}) = \frac{N(z,\mathbf{n}) - \bar{N}(z)}{\bar{N}(z)} .$$

Assuming that the number of galaxies is proportional to the matter density times the volume we have

$$dN(z,\mathbf{n}) = \rho(z,\mathbf{n})dV(z,\mathbf{n}) = \rho(z,\mathbf{n})\nu(z,\mathbf{n})dzd\Omega_{\mathbf{n}} \qquad \nu(z,\mathbf{n}) = \frac{dV(z,\mathbf{n})}{dzd\Omega_{\mathbf{n}}} .$$

To continue we have also to take into account that also the redshift and the direction from which we see the galaxy are perturbed. Taking this all into account we find the perturbed expression for the observed number counts,

$$
\begin{aligned}
\Delta_g(\mathbf{n}, z, m_*) \quad = \quad & bD + \frac{1}{\mathcal{H}}\left[\dot\Phi + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] - (3 - b_e)\mathcal{H}V + \\
& \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2-5s}{r_s\mathcal{H}} + 5s - b_e\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s} dr(\dot\Phi + \dot\Psi)\right) \\
& -(2-5s)\Phi + \Psi + \frac{2-5s}{2r_s}\int_0^{r_s} dr\left[2(\Phi+\Psi) - \frac{r_s - r}{r}\Delta_\Omega(\Phi+\Psi)\right]
\end{aligned}
$$

(Bonvin & RD 2011, Challinor & Lewis 2011)

The first term is simply the biased density contrast in comoving gauge. The second term is the redshift space distortion (the radial volume distortion) the last term is the lensing term (the transversal volume distortion). The other terms (Doppler, Shapiro time delay, integrated Sachs Wolfe term) are relevant mainly on very large scales.

m_* is the magnitude limit of the survey and L_* is the limiting luminosity.

b_e is the evolution bias,

$$\frac{5}{2}s = \left.\frac{\partial \log n_g(z, L)}{\partial \log L}\right|_{L=L_*} \quad \text{is the magnification bias.}$$

The observed number count

The observed number count fluctuation is not a function of position but a function of (observed) direction **n** and redshift z; $(\mathbf{n}, z)$ are observable coordinates on the background lightcone.

We cannot observe distances. To infer a distance from observations in cosmology, we always use a model. Hence the real space correlation function and its Fourier transform, the power spectrum are model dependent.
We can directly observe the angular-redshift correlation function,

$$\xi(\mathbf{n}\cdot\mathbf{n}', z, z') = \langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle$$

or its angular power spectrum, $C_\ell(z, z')$, $\mathbf{n}\cdot\mathbf{n}' = \cos\theta$

$$\xi(\theta, z, z') = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell(z, z')P_\ell(\cos\theta)$$

The distance between two galaxies in directions $\mathbf{n}$ and $\mathbf{n}'$ at redshifts z and z', for $K = 0$

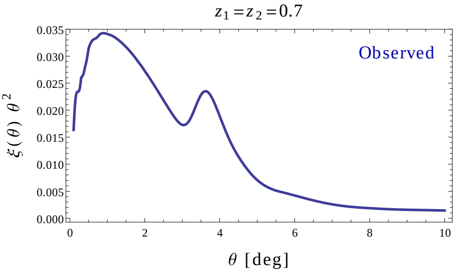
$$d = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z')\cos\theta}$$

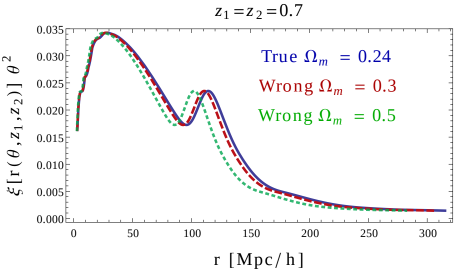
depends on the cosmological model via

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_{\rm de}f(z')}}$$

One therefore has to be very careful when estimating cosmological parameters via the real space correlation function or the power spectrum.

The observed number count





Parameter dependence of the real space correlation function.
(Figure by F. Montanari)

Density and RSD

Most present analyses take into account only the density and rsd terms and consider s small survey in direction $\mathbf{n}$ in the sky.
Using $kV/\mathcal{H} = \mathcal{H}^{-1}\dot{D} = f(z)D$, $f(z) = d\log D/d\log a \sim \Omega_m(z)^{0.57}$ one finds

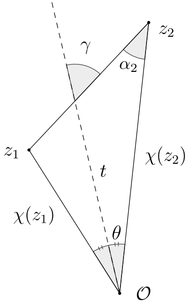
$$\xi_{\rm obs}(\mathbf{d}, z) = \xi(d)\,T_D^2(z)\left[\beta_0(z)P_0(\mu) - \beta_2(z)P_2(\mu) + \beta_4(z)P_4(\mu)\right]\,,$$

$$\mathcal{P}_{\rm obs}(\mathbf{k}, z) = \mathcal{P}(k)\,T_D^2(z)\left[\beta_0(z)P_0(\mu) + \beta_2(z)P_2(\mu) + \beta_4(z)P_4(\mu)\right]\,,$$

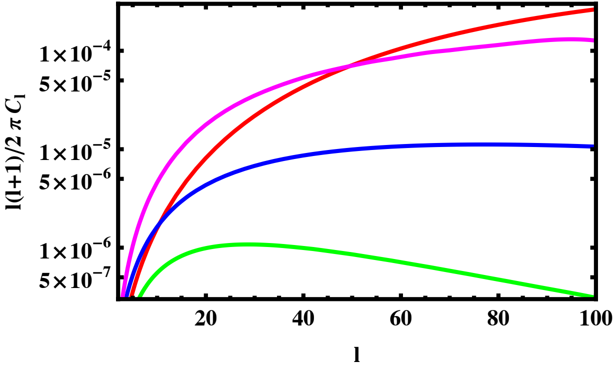
with

$$\beta_0 = b^2 + \frac{2bf}{3} + \frac{f^2}{5}, \quad \beta_2 = \frac{4bf}{3} + \frac{4f^2}{7}, \quad \beta_4 = \frac{8f^2}{35}\,.$$

$\mu = \hat{\mathbf{d}}\cdot\mathbf{n} = \cos\gamma$,
or $\mu = \hat{\mathbf{k}}\cdot\mathbf{n}$.



The lensing term can become very important at high redshift and for different redshifts, $z \neq z'$.
Contributions to the power spectrum at redshift $z' = z = 3, \Delta z = 0.3$
(from Bonvin & RD '11)

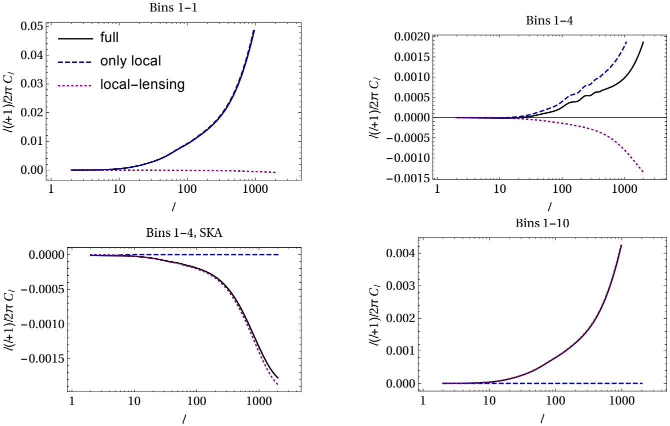


C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{lensing}$ (magenta).

Well separated redshift bins measure mainly the lensing-density correlation:

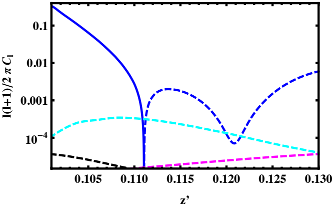
$$\langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle \simeq \langle \Delta^L(\mathbf{n}, z)\delta(\mathbf{n}', z')\rangle \quad z > z'$$

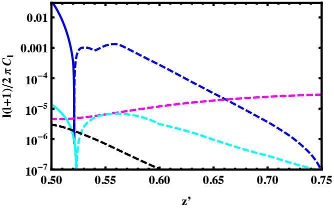
$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$

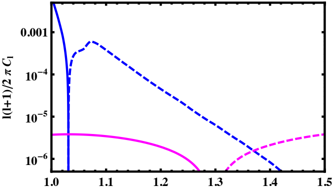


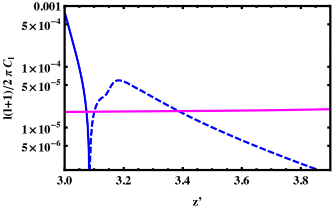
(Montanari & RD) [1506.01369]

The radial power spectrum









The radial power spectrum $C_\ell(z, z')$
for $\ell = 20$
Left, top to bottom: $z = 0.1,\ 0.5,\ 1,$
top right: $z = 3$

Standard terms (blue), $C_\ell^{lensing}$ (magenta),
$C_\ell^{Doppler}$ (cyan), C_ℓ^{grav} (black),
(from Bonvin & RD '11)

Relativistic N-body simulations

Going beyond linear perturbation theory

On intermediate to small scales, $d < 10$Mpc, density perturbations can become large. Inside a galaxy we have $\delta \sim 10^5$ and even inside a galaxy cluster the motion of galaxies is decoupled from the Hubble flow, clusters due not expand. $\Rightarrow$ We have to treat clustering non-linearly .

But the gravitational potential of a galaxy remains small,

$$\Phi \sim R_s/D \sim \frac{2\times 10^{12}\mathrm{km}}{10\mathrm{kpc}} \simeq 10^{-5}$$

Therefore, in Newtonian (longitudinal) gauge, metric perturbations remain small.

In the past, this together with the smallness of peculiar velocities has been used to argue that Newtonian N-body simulations are sufficient. Caveats:

- We start simulations at $z \sim 100$ when the box size is comparable to the horizon and neglecting radiation is not a good approximation (aim: 1% accuracy).
- Neutrino velocities, even for massive neutrinos are not very small at $z = 100$.
- The relativistic gravitational field has 6 degrees of freedom (2 helicity 0, 2 helicity 1 and 2 helicity 2) and Newtonian simulations only consider 1 of them.
- Observations are made on the relativistic, perturbed lightcone. We want to correctly simulate this.

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Relativistic N-body simulations

A full relativistic N-body simulation might, starting from the initial metric and particle positions at time t_1

1. Move dark matter particles (and baryons) along geodesics of the metric at t_1 to obtain their position at $t_1 + \Delta t$.

$$m\dot{p}^\mu + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$$

2. Compute the energy-momentum tensor with a particle to mesh projection.

$$T^{\mu\nu}(\mathbf{x}, t) = m^{-1} \sum_n p^\mu(t) p^\nu(t) \delta(\mathbf{x} - \mathbf{x}_n(t))$$

3. Solve Einstein's equations to determine the metric at $t_1 + \Delta t$.
4. Return to step 1 replacing t_1 by $t_1 + \Delta t_1$

In practice the time-stepping is replaced by a staggered leap-frog to render his procedure stable.

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At present, we do not solve the full Einstein equations.

We work in longitudinal gauge ($K = 0$),

$$ds^2 = -a^2 e^{2\Psi} dt^2 - 2a^2 B_i dt dx^i + a^2 [e^{-2\Phi}\delta_{ij} + h_{ij}] dx^i dx^j$$

and use the fact that $\Psi \simeq \Phi$ is very small. However, spatial derivatives, $\nabla\Psi/\mathcal{H} \sim V$ and second derivatives, $\Delta\Psi/\mathcal{H}^2 \simeq D$ are not small.

Therefore, when computing the Einstein tensor go only to first order in the gravitational potentials and their time derivatives. We include quadratic terms of first spatial derivatives and all orders for second spatial derivatives. Since the Einstein tensor is linear in its second derivatives of the metric, this simplifies the equations significantly.

We include also vector and tensor perturbations only at linear order. They are induced from scalar perturbations at second order and always remains small.

When solving the geodesic equation we also neglect tensor perturbations as they only enter together with v^2 and are very suppressed.

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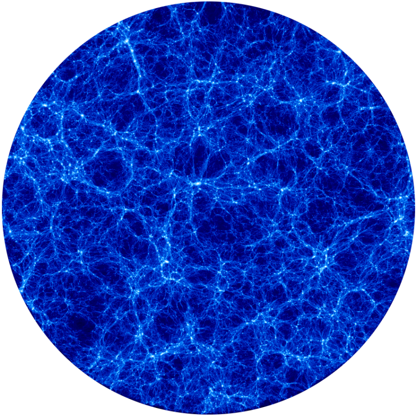
Gevolution

The code doing this is 'gevolution'. It is written by Julian Adamek and extensively uses packages of 'LATfield2' written for gevolution by David Daverio.
The code is publicly available at github.com/gevolution-code/gevolution-1.0



(Figure by Julian Adamek)

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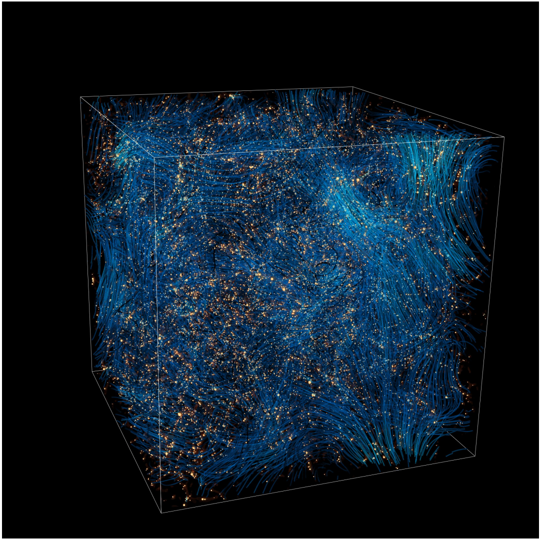
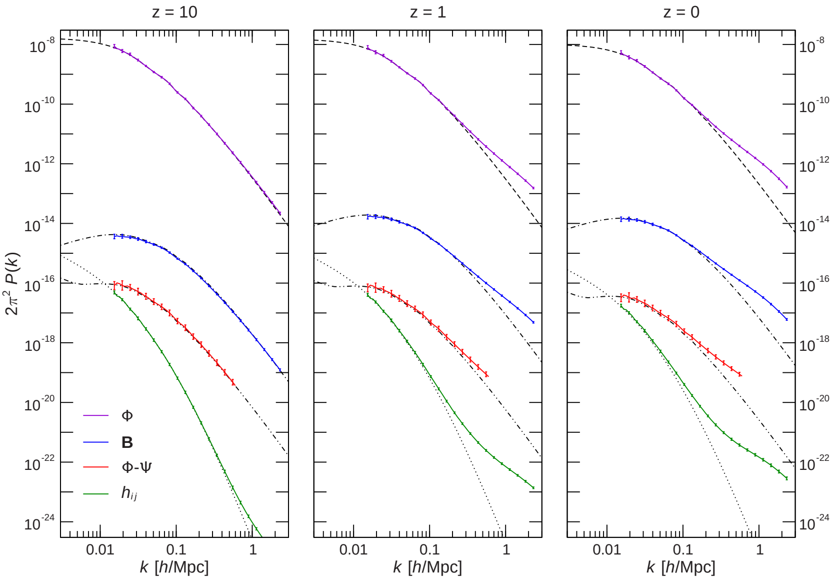


(Figure by Julian Adamek)

Spectra from gevolution



(From Adamek et al., 2016)

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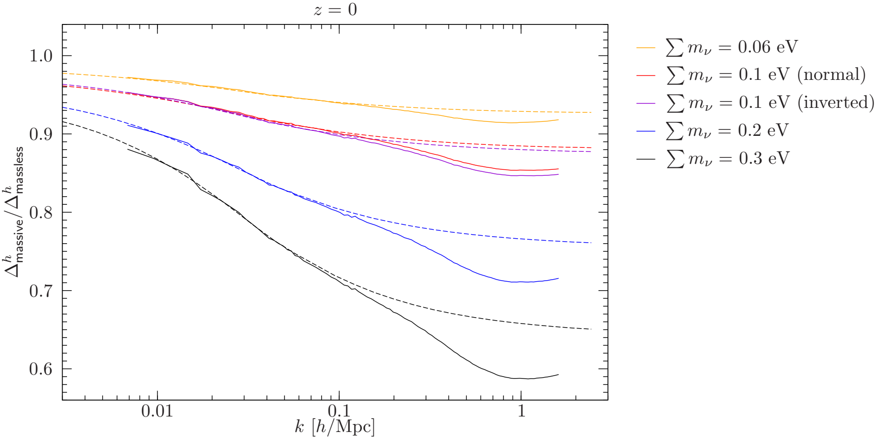
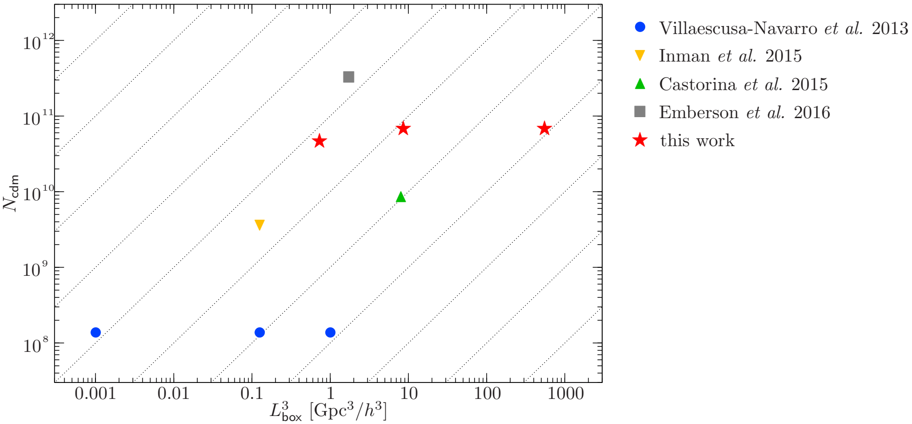


Difference of tensor spectra in presence of massive neutrinos.
(From Adamek et al., 2017)

Performance of gevolution



Gevolution has been run on the Swiss Supercomputer 'Piz Daint'.
These simulations are among the largest worldwide.
(From Adamek et al., 2017)

Conclusions

- At sufficiently large scales and at early times structure formation can be treated within linear cosmological perturbation theory.
- Perturbations variables are usually gauge dependent, but observations are not.
- In cosmology we can observe directions, redshifts, fluxes, $\cdots$, but not distances. Therefore, the real space correlation function and the power spectrum are model dependent.
- Galaxy number counts contain apart from density and rsd an important lensing term which is most relevant for $z \neq z'$ correlations and at high redshift.
- At intermediate scales we need simulations as density fluctuations become large.
- Relativistic weak field simulations are not much more costly than Newtonian simulations and they capture all the relativistic effects relevant in cosmology.

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