Simulations Of The Universe In General Relativity

from Newtonian physics to weak field relativity

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Based on:
1505.04756, 1606.05588, 1610.04236, 1702.03221, 1703.08585 and 1708.07769

Newtonian Vs GR Simulations

The Metric in General Relativity

$$
\begin{aligned}
\mathrm{d}s^2 &= -a^2\left(1+2A\right)\mathrm{d}\eta^2 - 2a^2\hat{\boldsymbol{\nabla}}_i B\mathrm{d}\eta\mathrm{d}x^i \\
&\quad + a^2\left[\delta_{ij}\left(1+2H_{\mathrm{L}}\right) + 2\left(\hat{\boldsymbol{\nabla}}_i\hat{\boldsymbol{\nabla}}_j + \frac{\delta_{ij}}{3}\right)H_{\mathrm{T}}\right]\mathrm{d}x^i\mathrm{d}x^j
\end{aligned}
$$

Newtonian simulations have been used to analyse non-linear structure formation

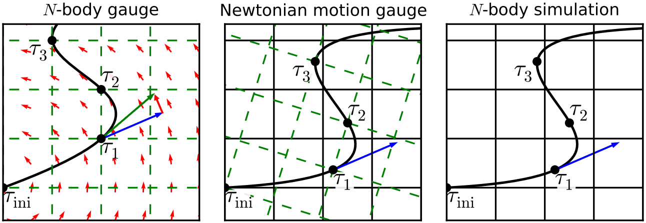
- How can relativistic corrections be implemented into a code?
- Can a Newtonian code be embedded into a fully relativistic point-of-view?
- Chisari & Zaldarriaga find a cancelation of corrections in a specific limit

The Newtonian Motion Gauge Idea

Gauge Freedom of General Relativity

- The gauge defines the coordinates
- The gauge specifies the dynamical equations

Can we find a gauge that has an entirely Newtonian dynamics?



The Newtonian Motion Gauge

Gauge Condition

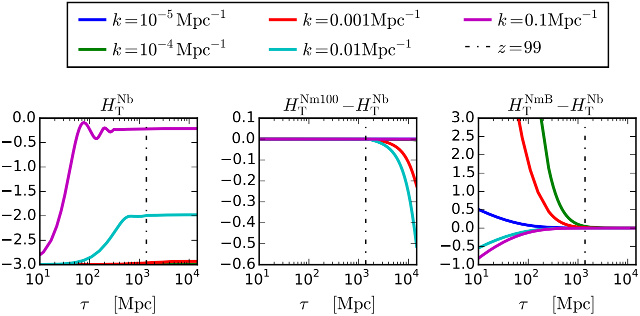
- We want Newtonian trajectories: $v_{\mathrm{cdm}} = v_{\mathrm{N}}$
  - ➔ $F^{\mathrm{N}} = -\boldsymbol{\nabla}\Phi^{\mathrm{N}}$
  - ➔ $F^{\mathrm{GR}} = \boldsymbol{\nabla}A + \boldsymbol{\nabla}\left(\partial_\tau + \mathcal{H}\right)\mathfrak{K}^{-2}\dot{H}_{\mathrm{T}}$
- The potentials are sourced by the matter content
  - ➔ $\boldsymbol{\nabla}^2\Phi^{\mathrm{N}} = 4\pi G a^2 \delta\rho_N$
  - ➔ $\boldsymbol{\nabla}^2 A = -4\pi G a^2\left(\delta\rho + 3\mathcal{H}(\rho + p)\mathfrak{K}^{-1}(v - \mathfrak{K}^{-1}\dot{H}_{\mathrm{T}})\right) - 8\pi G a^2\Sigma$
- Volume distortions affect the relativistic density
  - ➔ $\rho_{\mathrm{cdm}} = (1 - 3H_{\mathrm{L}})\rho_{\mathrm{N}}$
- Combined the gauge condition becomes $(\partial_\tau + \mathcal{H})\dot{H}_{\mathrm{T}} = 4\pi G a^2(\delta\rho_\gamma + 3\mathcal{H}(\rho_\gamma + p_\gamma)\mathfrak{K}^{-1}(v - \mathfrak{K}^{-1}\dot{H}_{\mathrm{T}}) - \rho_{\mathrm{cdm}}(3\zeta - H_{\mathrm{T}})) + 8\pi G a^2\Sigma$

H_{T} remains small and decouples from non-linear matter perturbations (weak field limit of GR)

The Newtonian Motion Gauges

The Newtonian motion gauge decouples the full relativistic evolution

- Into the non-linear but Newtonian collapse of matter
  - ➔ Can be simulated by existing N-body codes
- And the relativistic but linear analysis of the underlying space-time
  - ➔ Can be implemented in existing linear Boltzmann codes



GR Dictionary

The Newtonian simulation plus H_{T} provides the full information to construct all relativistic perturbations

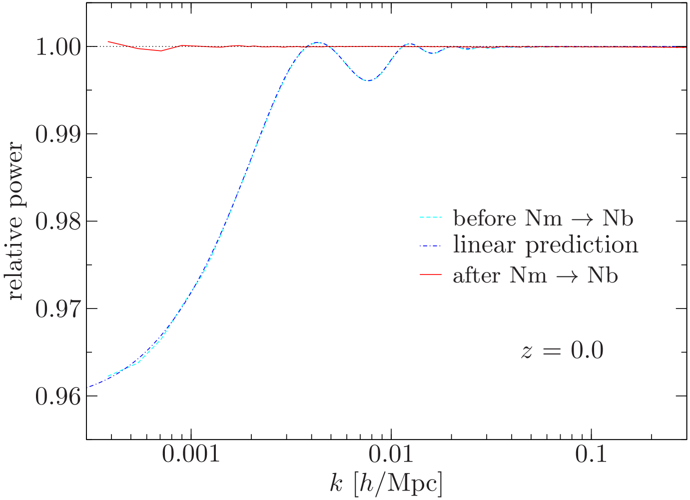
- $A = -\Phi^{\mathrm{N}} - (\partial_\tau + \mathcal{H})\, \mathfrak{K}^{-2}\, \dot{H}_{\mathrm{T}}$
- $B = \mathfrak{K}^{-1}\, \dot{H}_{\mathrm{T}}$
- $H_{\mathrm{L}} = \Phi^{\mathrm{N}} - \frac{1}{3} H_{\mathrm{T}} - \gamma$
- $H_{\mathrm{T}} = H_{\mathrm{T}}$
- $v = v^{\mathrm{N}}$
- $\delta = \delta^{\mathrm{N}} - 3H_{\mathrm{L}} = \delta^{\mathrm{N}} - 3\Phi^{\mathrm{N}} + H_{\mathrm{T}} + 3\gamma$

With $\gamma = -\,(\partial_\tau + \mathcal{H})\, \mathfrak{K}^{-2}\, \dot{H}_{\mathrm{T}} + 8\pi G a^2 \mathfrak{K}^{-2} \Sigma$

Poisson gauge is simple!

$\Phi = \Phi^{N} - \gamma$, $\Psi = A$, $\delta^{\mathrm{P}} = \delta$, $v^{\mathrm{P}} = v^{\mathrm{N}} - \mathfrak{K}^{-1}\, \dot{H}_{\mathrm{T}}$

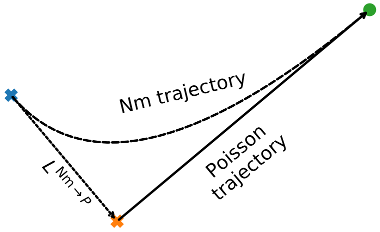
Comparison To GEVOLUTION



The ICS Effect

Light Transport on a Non-Trivial Metric

- The simulation potential Φ^N bends light rays: lensing
- Corrections from H_{T} introduce a rotation in the photon direction
  - ➔ The effect is integrated along a trajectory comparable to the ISW
  - ➔ ICS = Integrated coordinate shift



Dark Matter - Baryon Fluid

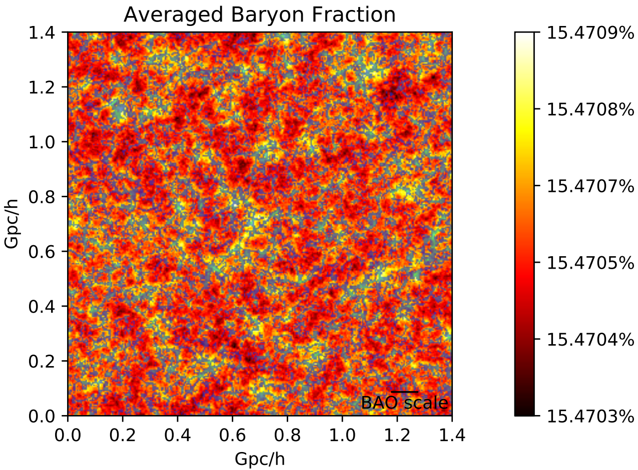
Impact of Baryons

- Same dynamics
  - ➔ Baryons evolve as a cold species
- Baryons are not dark
  - ➔ Interactions with the microwave background after reionisation
- Different initial distribution
  - ➔ Baryons were in contact to photons and carry frozen baryon acoustic oscillations

How to define a gauge for two fluids?

- Employ centre-of-mass coordinates
  - ➔ Metric only depends on centre-of-mass system
  - ➔ The particles in the simulation no longer represent physical particles
  - ➔ Additional information is required to disentangle the fluids

Dark Matter - Baryon Fluid



Conclusions

- Newtonian motion gauges allow a consistent embedding of Newtonian simulations in general relativity, from the large to the small scales
- Generalisation of C&Z (non-linearities, initial conditions, general cosmology)
- Numerically very efficient and simple to use
- Caution is needed in the interpretation of the data, a Newtonian simulation lives on a NM gauge
- Many future applications of this method (Baryons, Neutrinos, ...)

`Thank You For Your Attention`