Cosmic backreaction and Gauss's law

Cosmoback workshop
Marseille, 28th-31st May 2018

Pierre Fleury

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FACULTÉ DES SCIENCES

Presentation based on Phys. Rev. D 95, 124009 (2017), and ongoing work with C. Clarkson, T. Clifton, L. Reverberi

A note on terminology

Paraphrasing *Kolb, Marra, Matarrese (2009)*

« strong backreaction » = inhomogeneities affect expansion law

« weak backreaction » = inhomogeneities affect observations

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(usually)

distance - redshift relation

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« fitting problem » **[Ellis, 1987]**

(usually)

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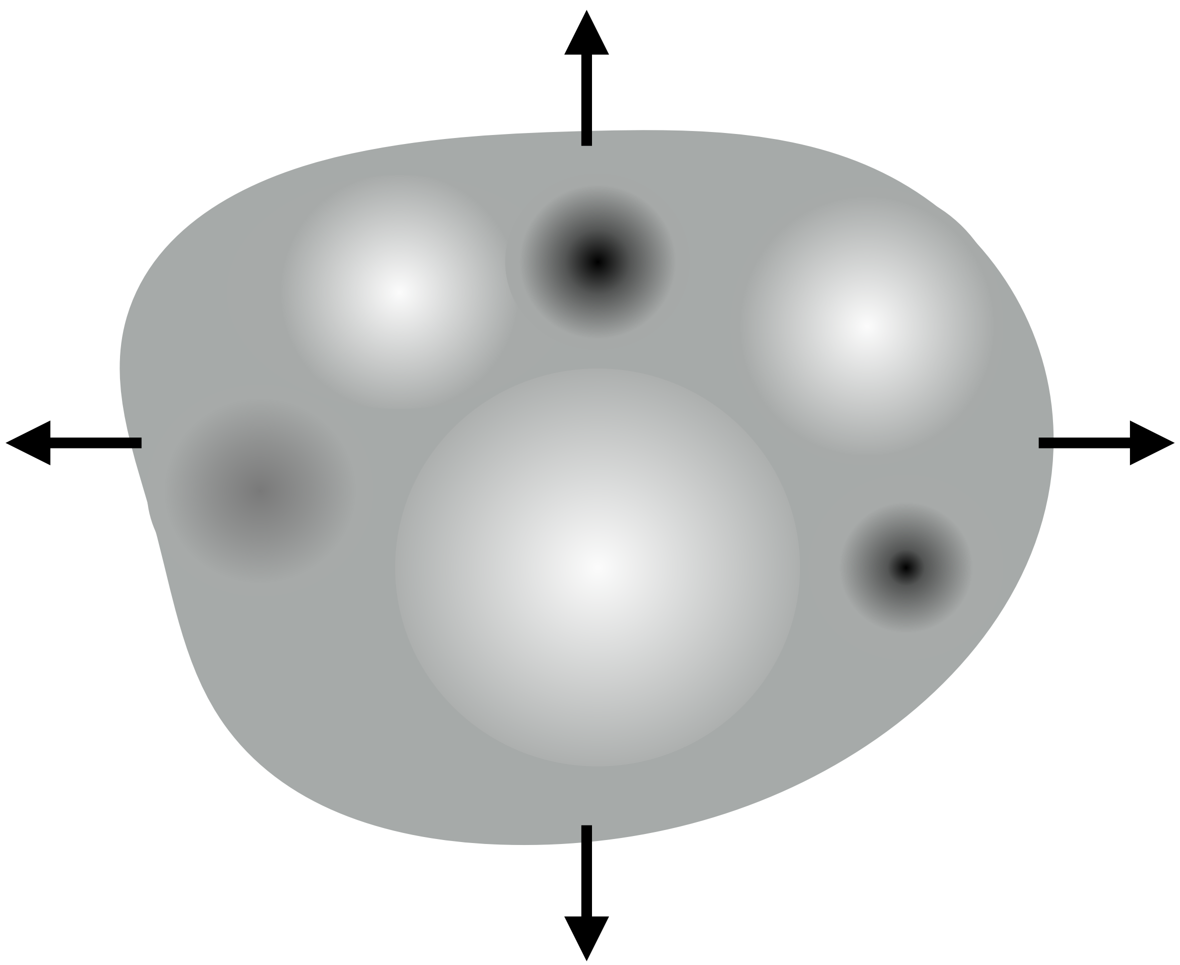
lensing

Backreaction: two questions

inhomogeneity



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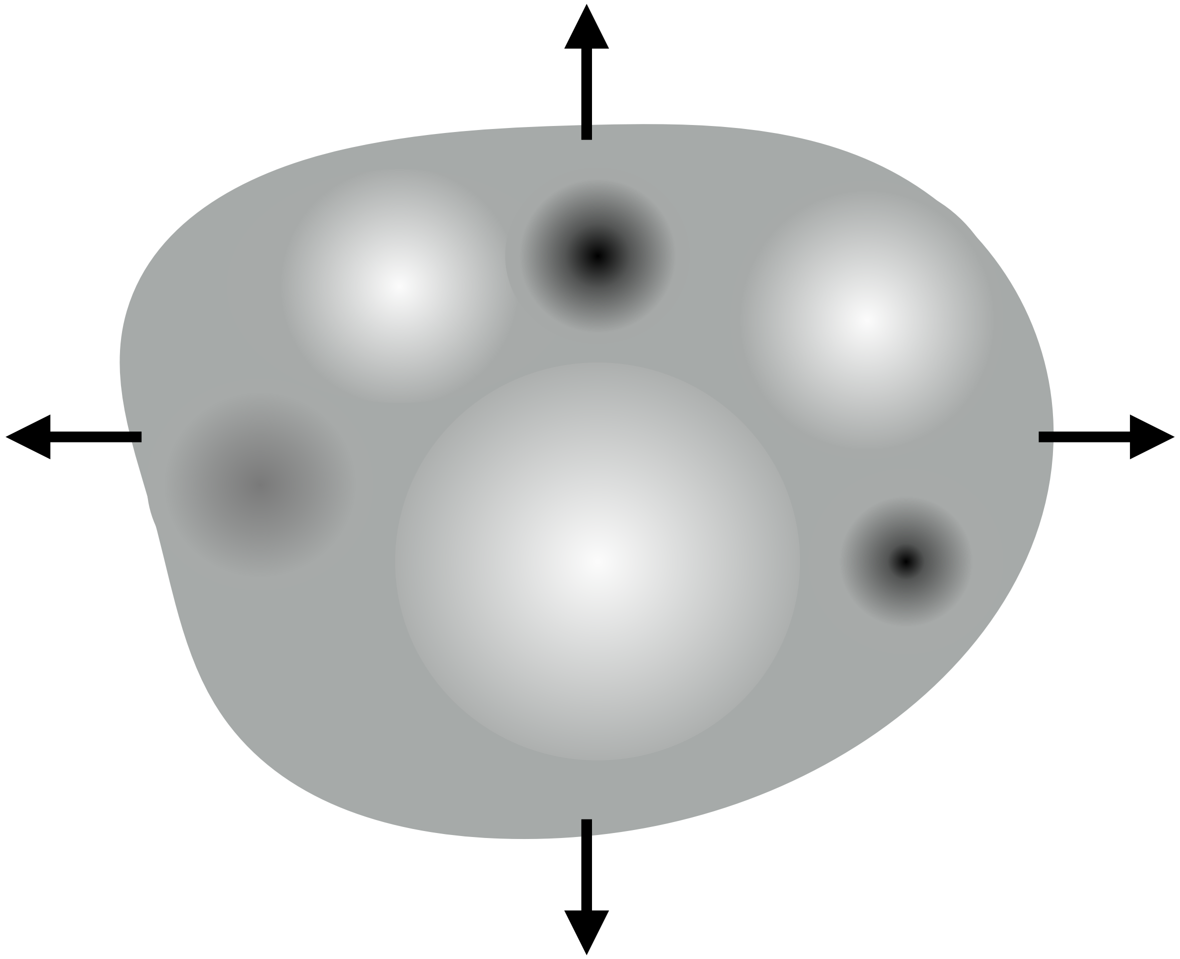
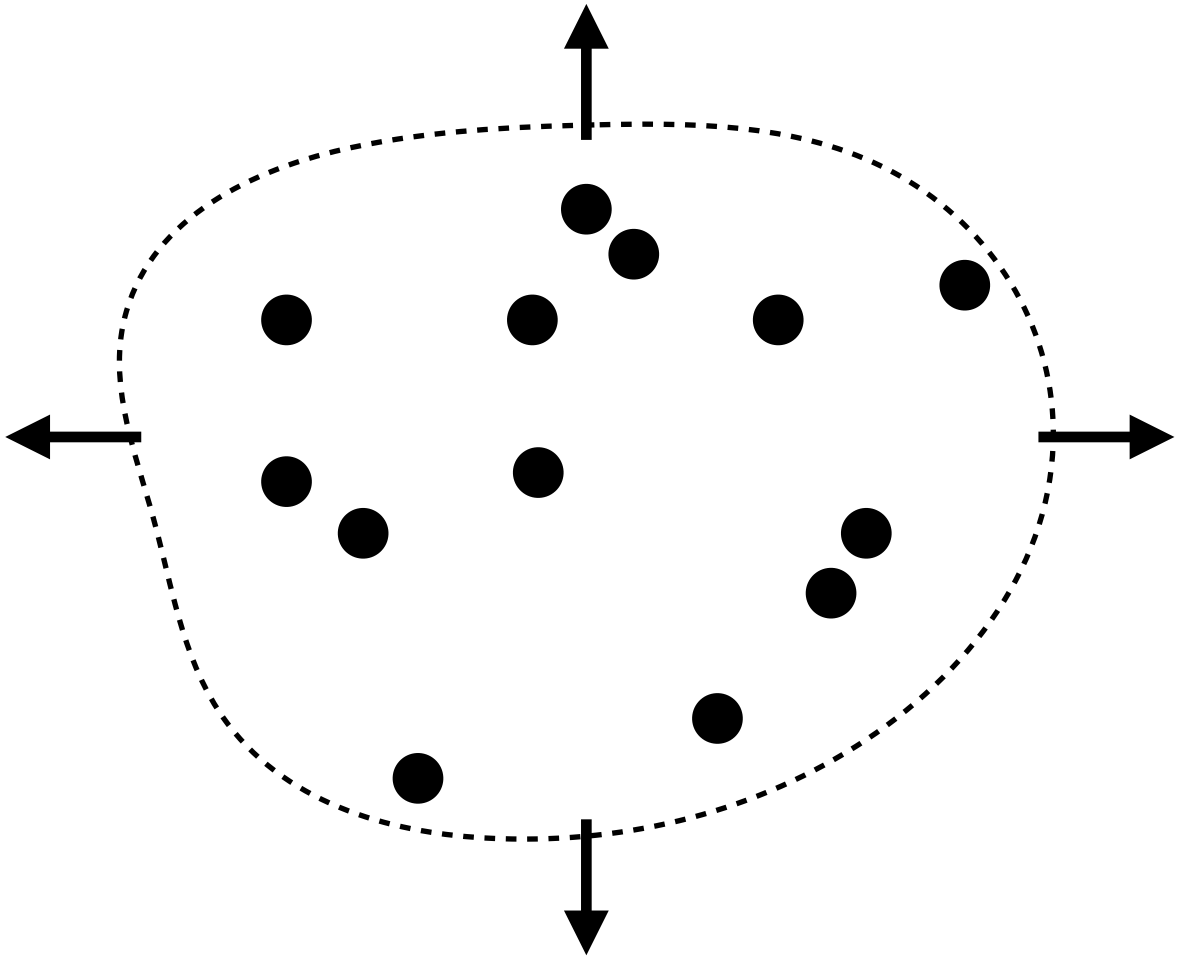


lumpiness/discreteness



Is there backreaction due to discreteness?

Current status

Results towards « discrete = continuous »

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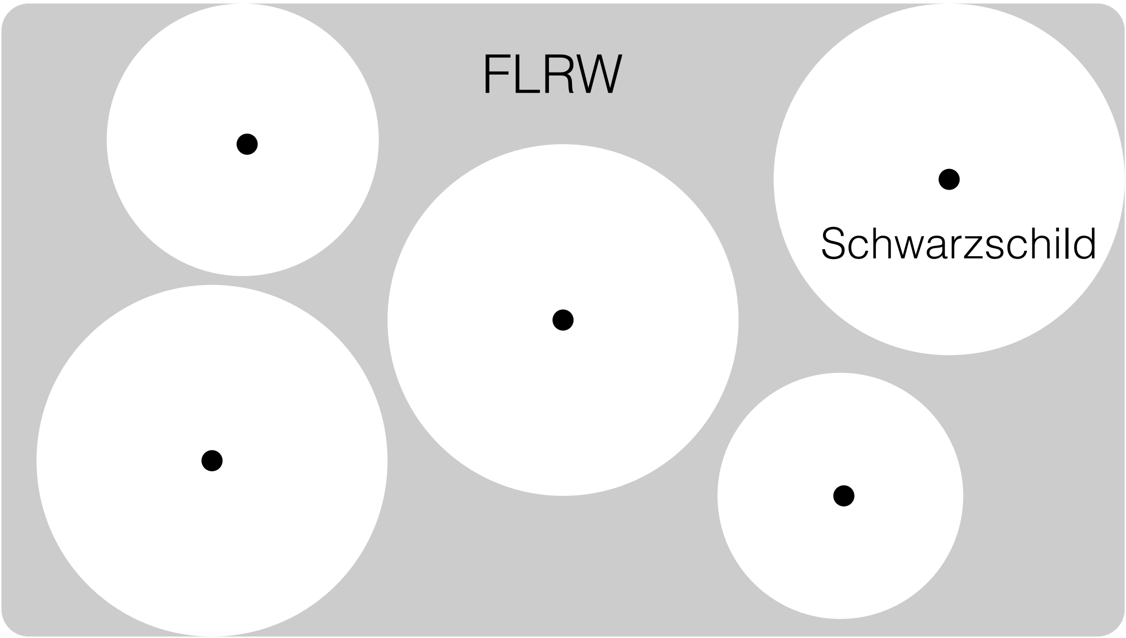
- Swiss-cheese models
  [Einstein & Straus 1945]



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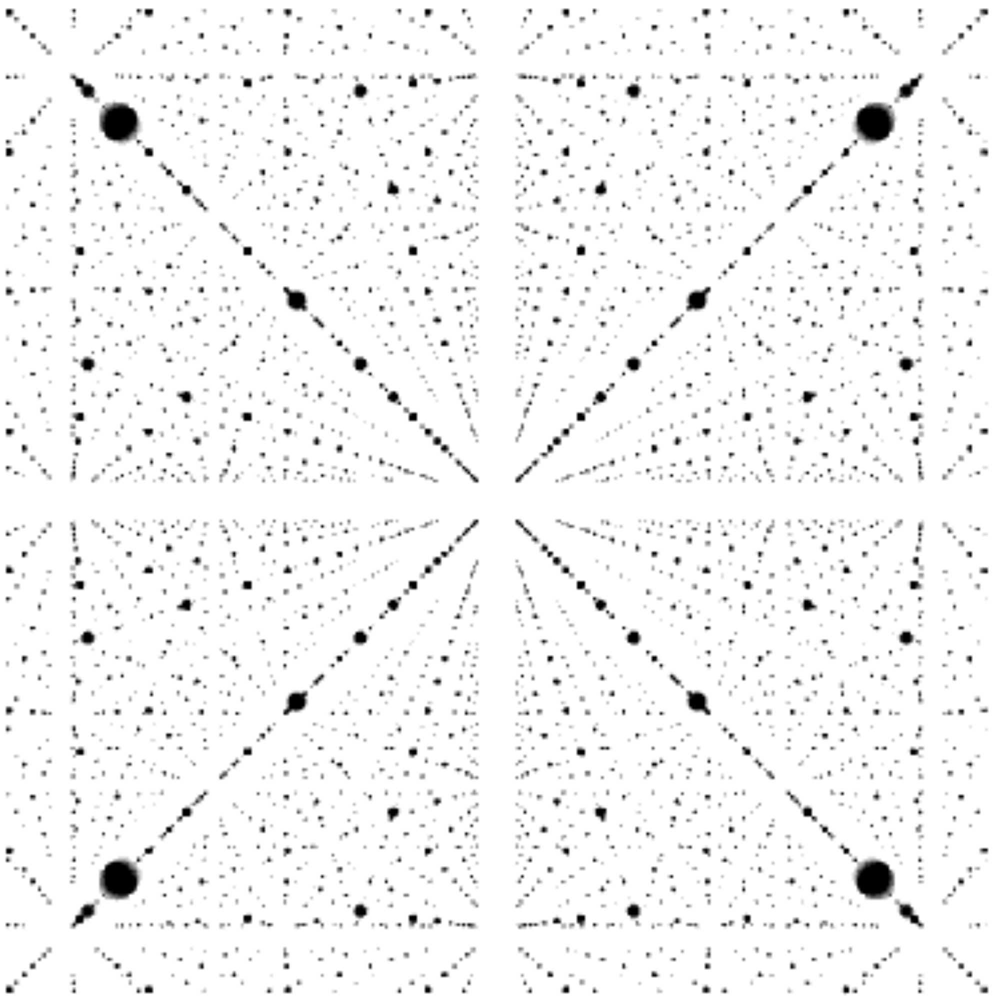

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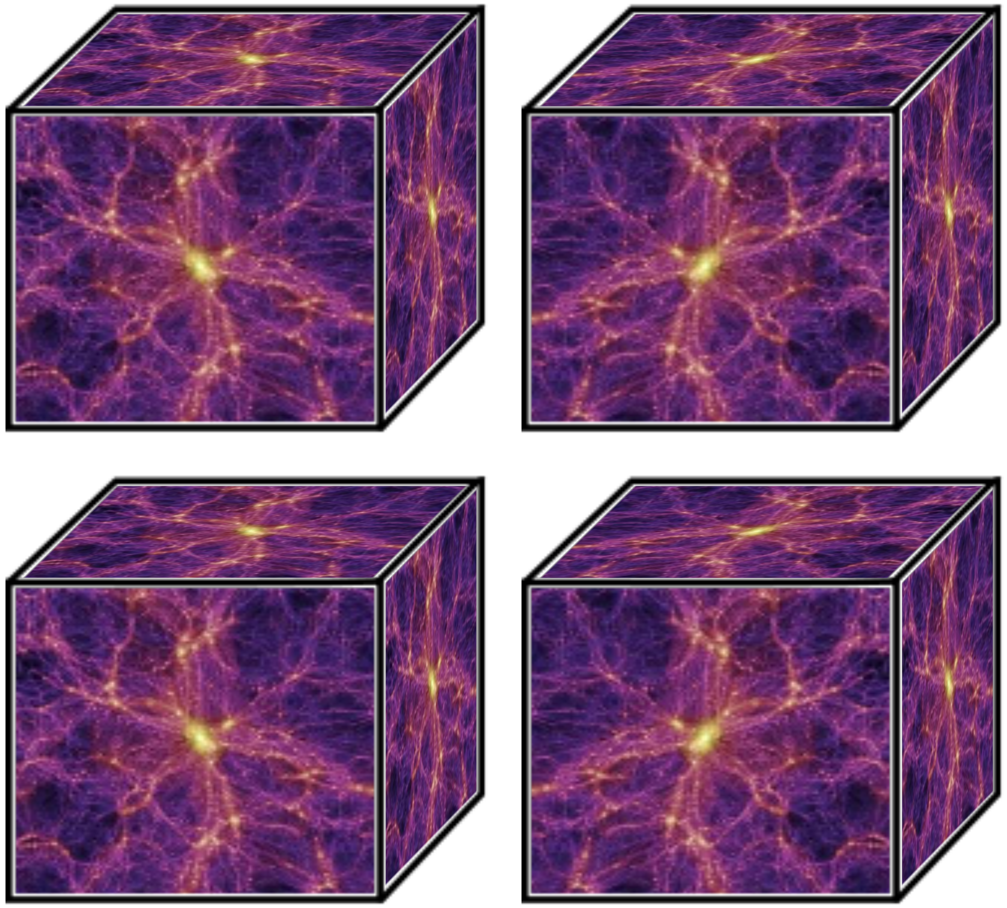
  [Sanghai & Clifton 2015]



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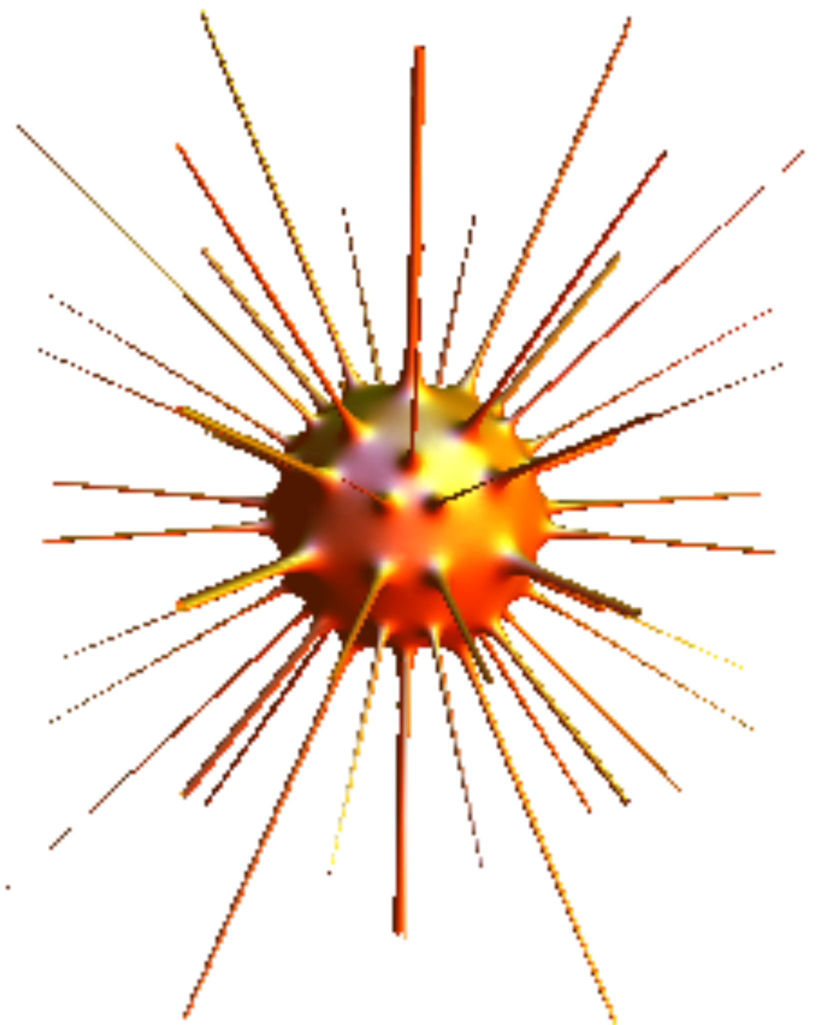

…but some backreaction found in *finite* lattice universes

[Clifton et al. 2012]
[Korzynski 2014]

A Newtonian approach

homogeneous self-gravitating system

A Newtonian approach

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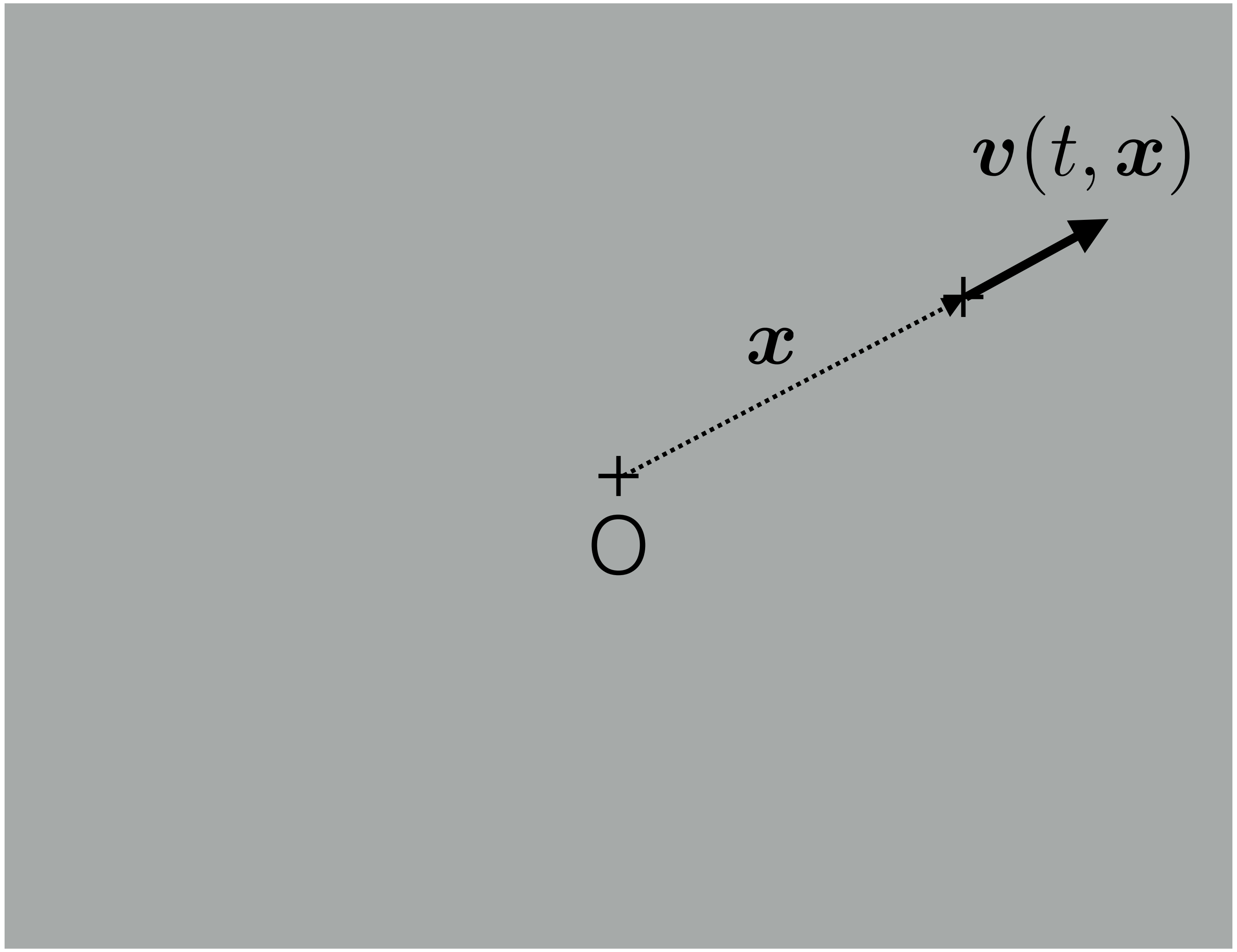
A Newtonian approach



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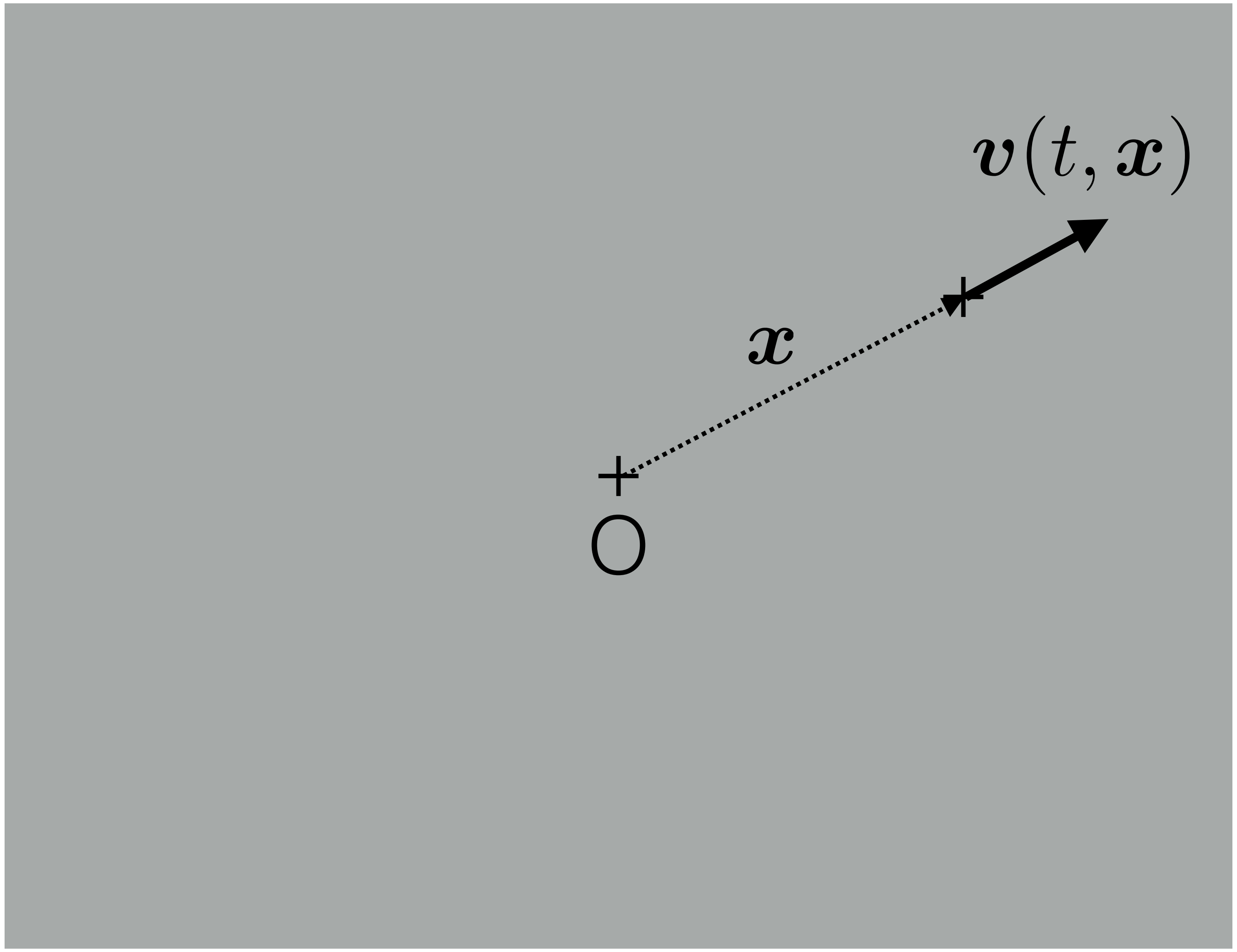
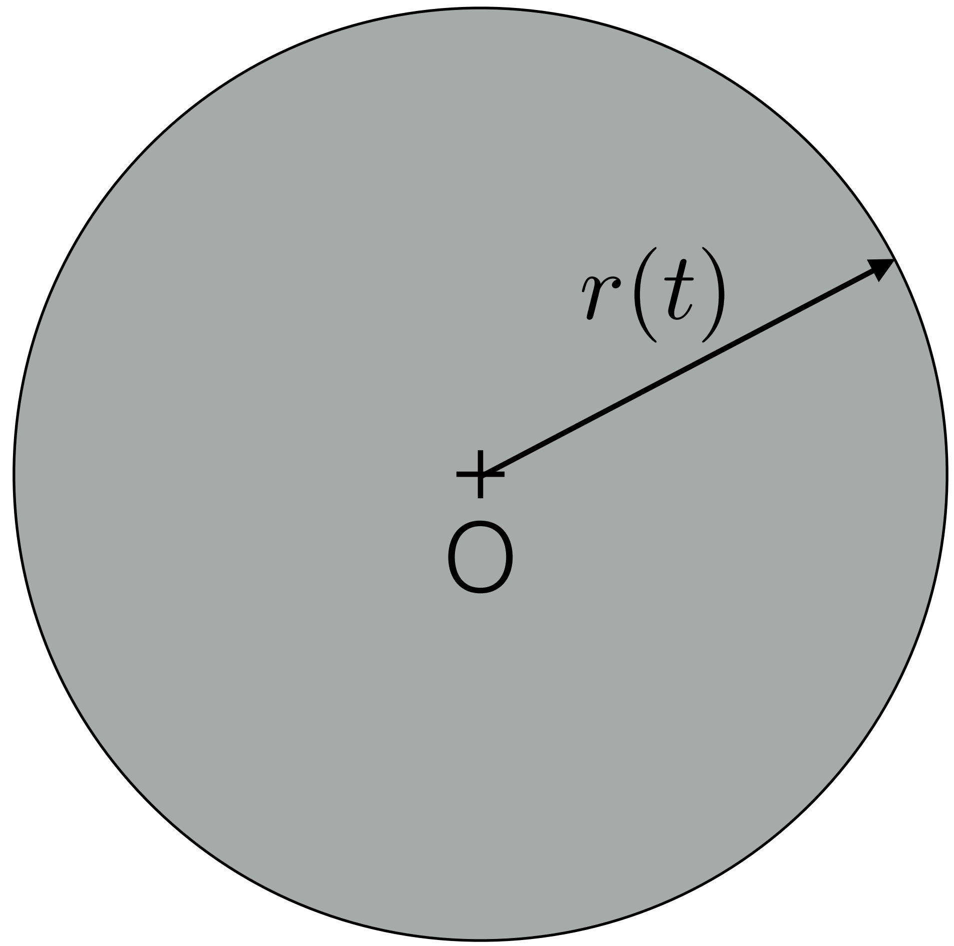


homogeneous self-gravitating system

$$\boldsymbol{v}(t, \boldsymbol{x}) = H(t)\boldsymbol{x}$$

A Newtonian approach



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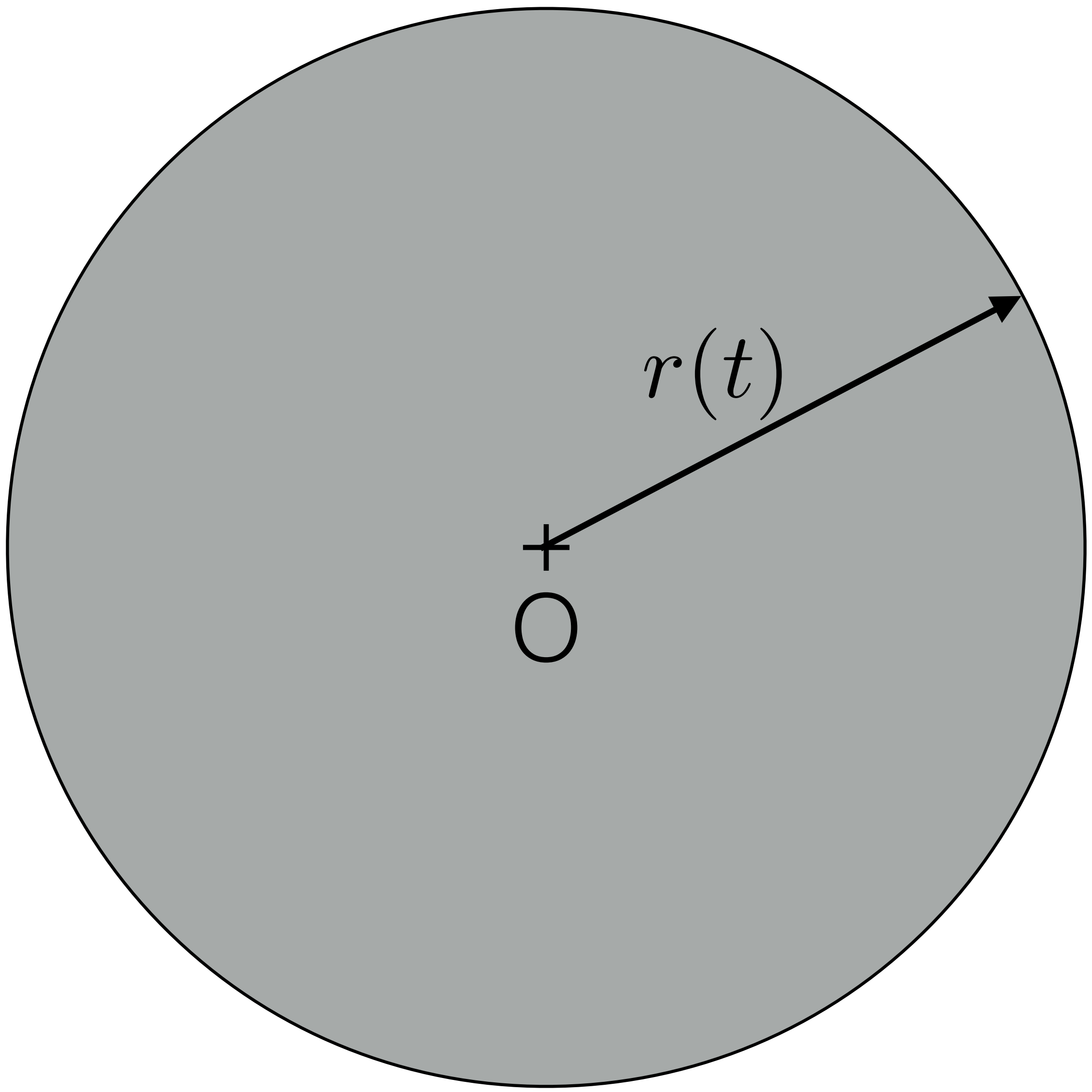

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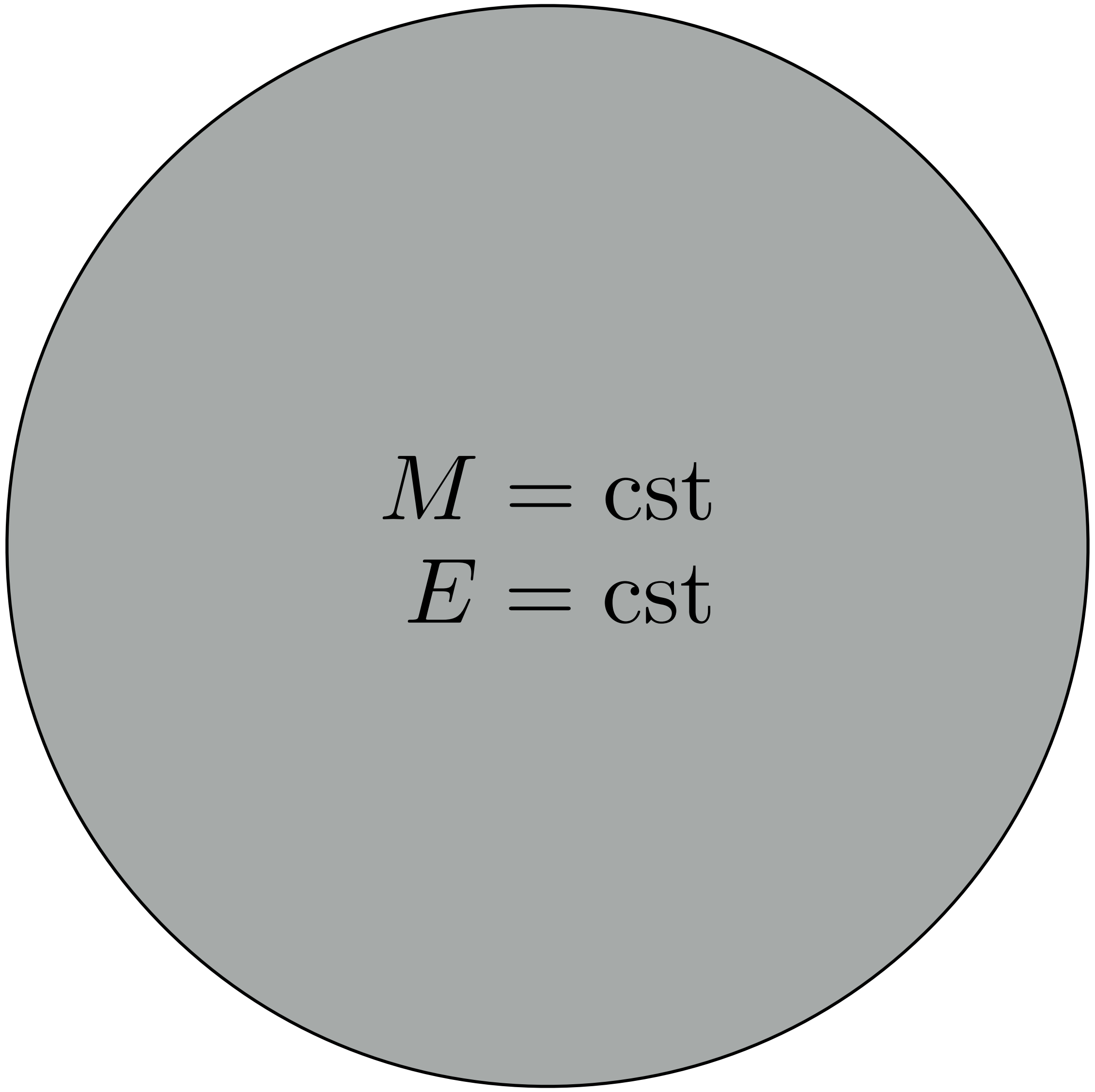


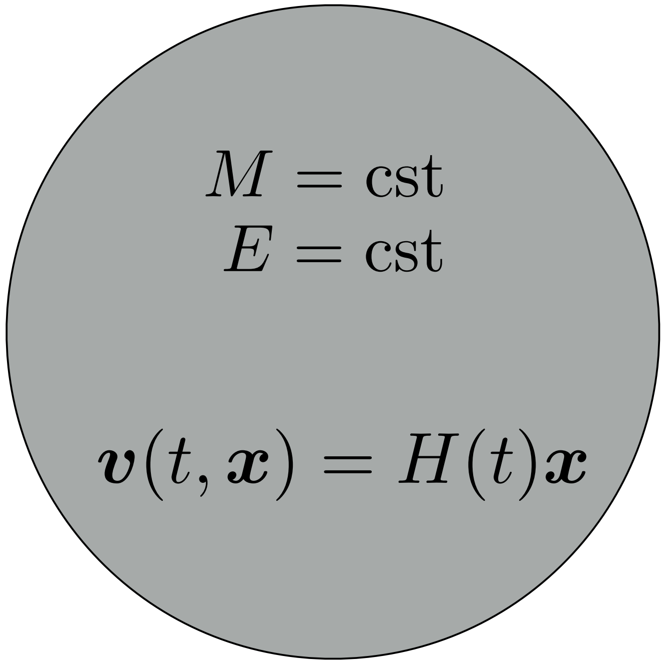
$$M = \mathrm{cst}$$
$$E = \mathrm{cst}$$

homogeneous self-gravitating system

$$\boldsymbol{v}(t, \boldsymbol{x}) = H(t)\boldsymbol{x}$$

A Newtonian approach

$$
\begin{aligned}
E &= E_{\mathrm{kin}} + E_{\mathrm{grav}} \\
&= \int \mathrm{d}^3\boldsymbol{x}\ \frac{\rho v^2}{2} - \int \mathrm{d}^3\boldsymbol{x}\,\mathrm{d}^3\boldsymbol{x}'\ \frac{G\rho^2}{|\boldsymbol{x}-\boldsymbol{x}'|} \\
&= \frac{3}{10} M H^2 r^2 - \frac{3}{5}\frac{GM^2}{r},
\end{aligned}
$$



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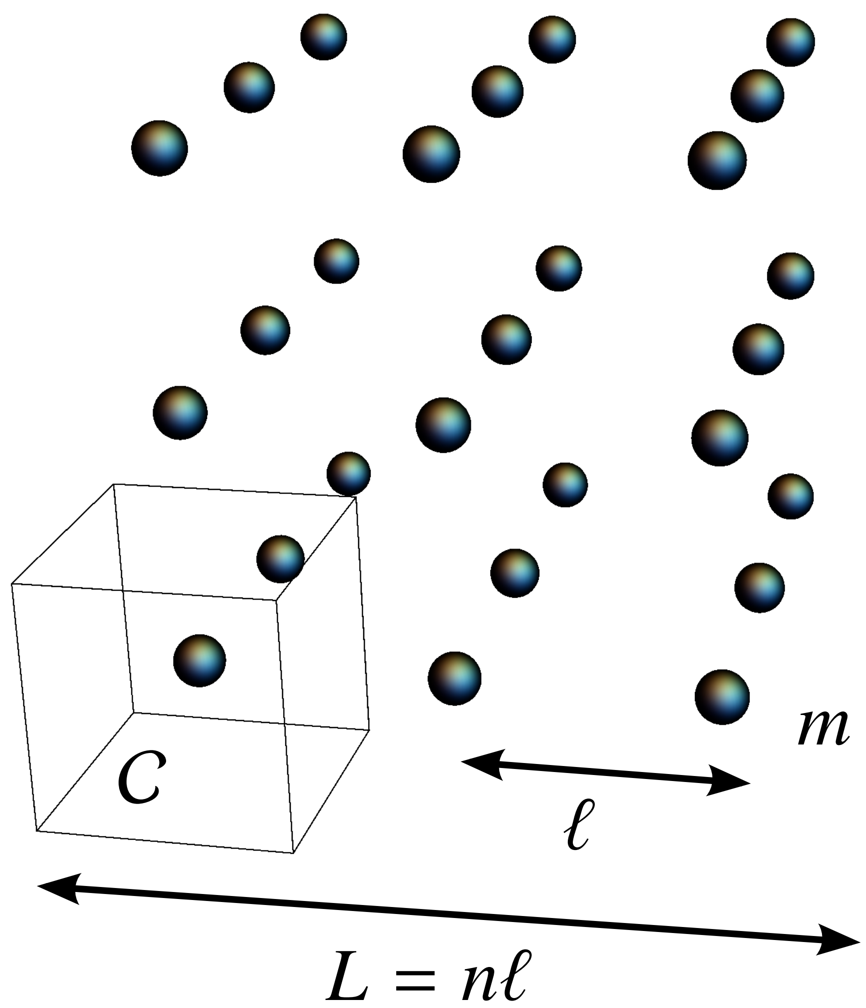
$$\boxed{H^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}}$$

$M = \mathrm{cst}$
$E = \mathrm{cst}$

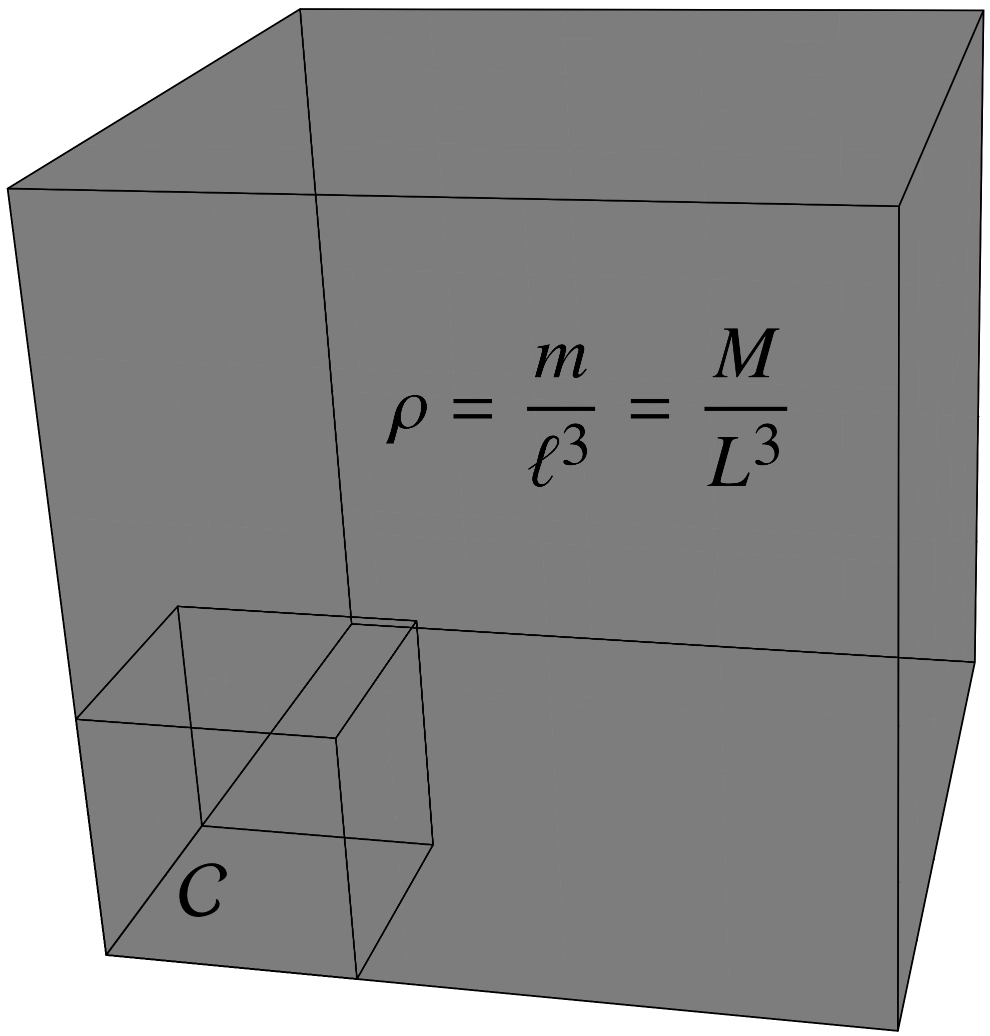
$\boldsymbol{v}(t,\boldsymbol{x}) = H(t)\boldsymbol{x}$

Two models

discrete model (D)

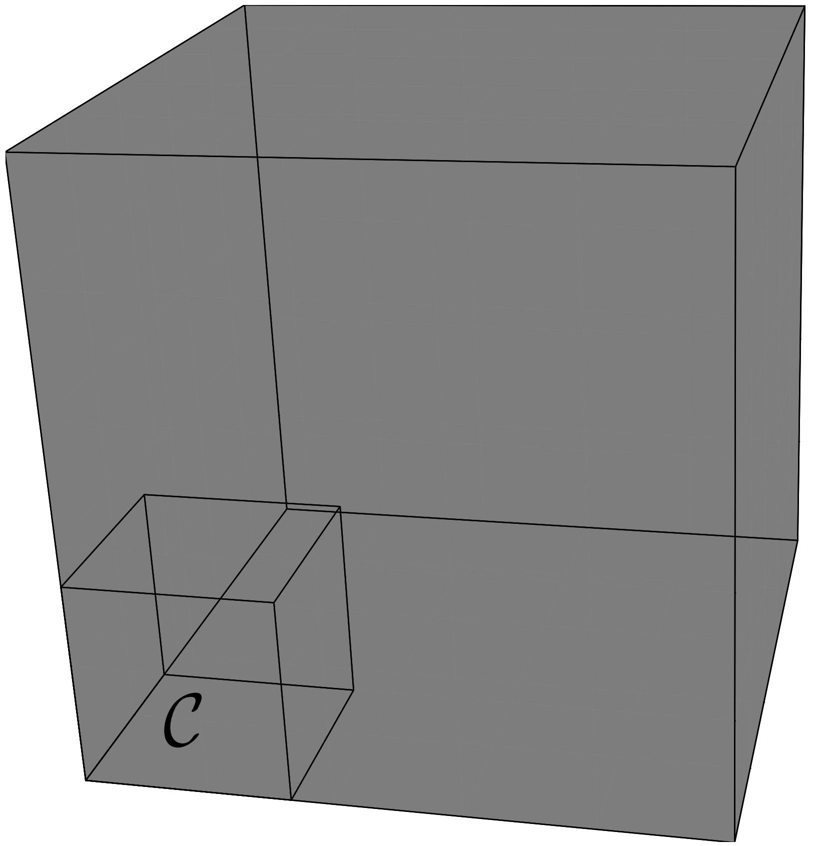


continuous model (C)



Gravitational energy

$$E_{\mathrm{grav}}^{(\mathrm{C})} = -\int \mathrm{d}^3\boldsymbol{x}\,\mathrm{d}^3\boldsymbol{x}'\ \frac{G\rho^2}{|\boldsymbol{x}-\boldsymbol{x}'|}$$



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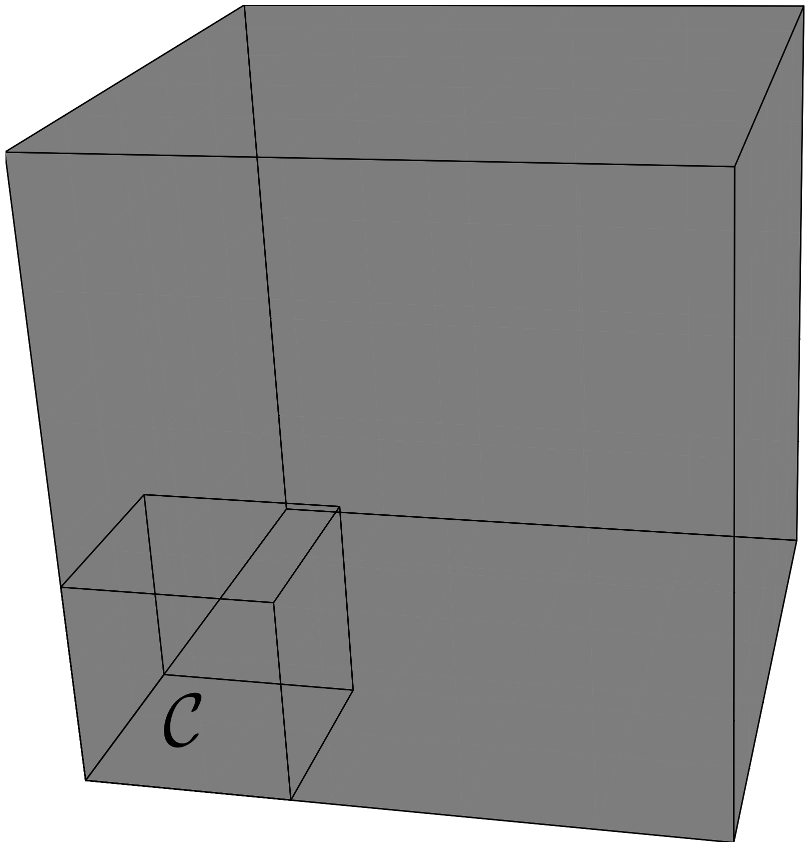
$$- \sum_{\mathcal{C} \neq \mathcal{C}'} \int_{\boldsymbol{x} \in \mathcal{C}, \boldsymbol{x}' \in \mathcal{C}'} \mathrm{d}^3\boldsymbol{x}\, \mathrm{d}^3\boldsymbol{x}'\ \frac{G\rho^2}{|\boldsymbol{x} - \boldsymbol{x}'|}$$



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$$E_{\rm grav}^{\rm (C)} = \overbrace{-\sum_{\mathcal{C}} \int_{\boldsymbol{x},\boldsymbol{x}'\in\mathcal{C}} \mathrm{d}^3\boldsymbol{x}\,\mathrm{d}^3\boldsymbol{x}'\, \frac{G\rho^2}{|\boldsymbol{x}-\boldsymbol{x}'|}}^{N E_{\rm grav,self}}$$

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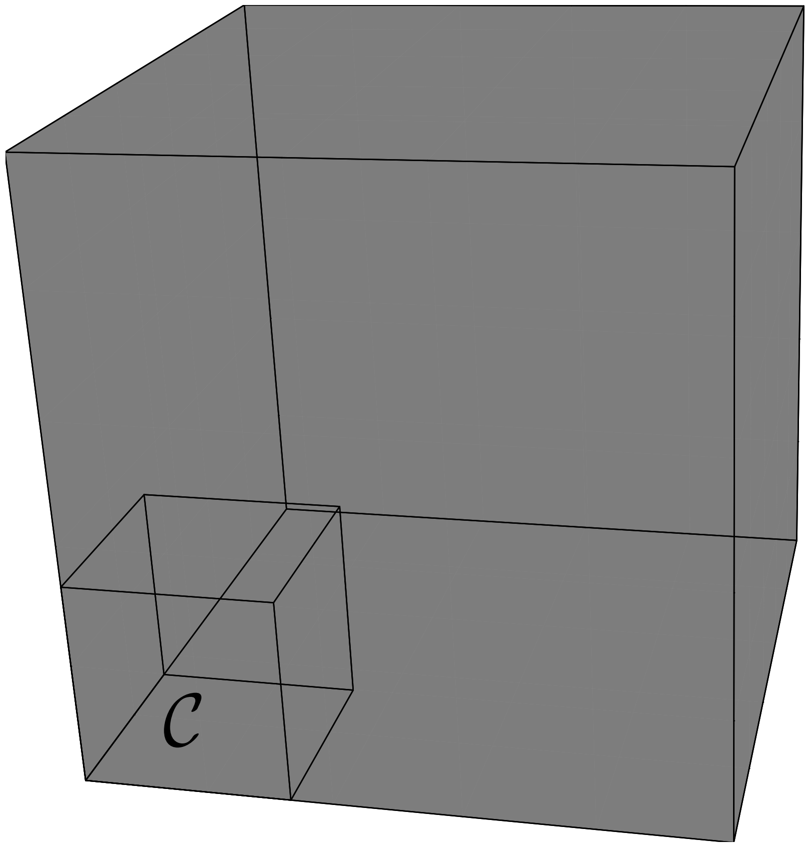

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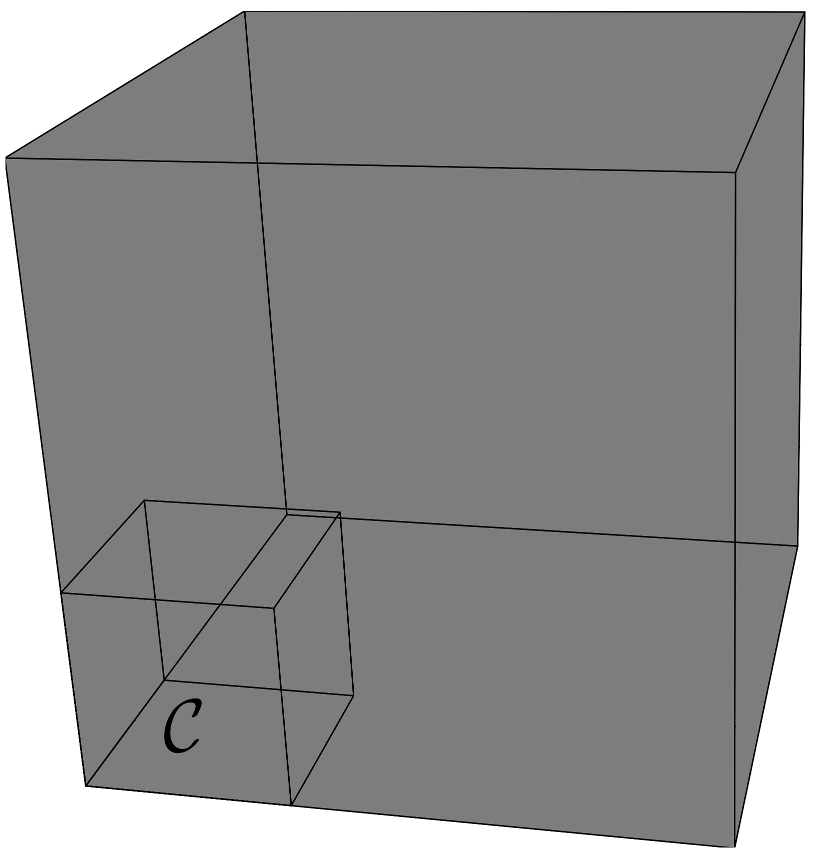

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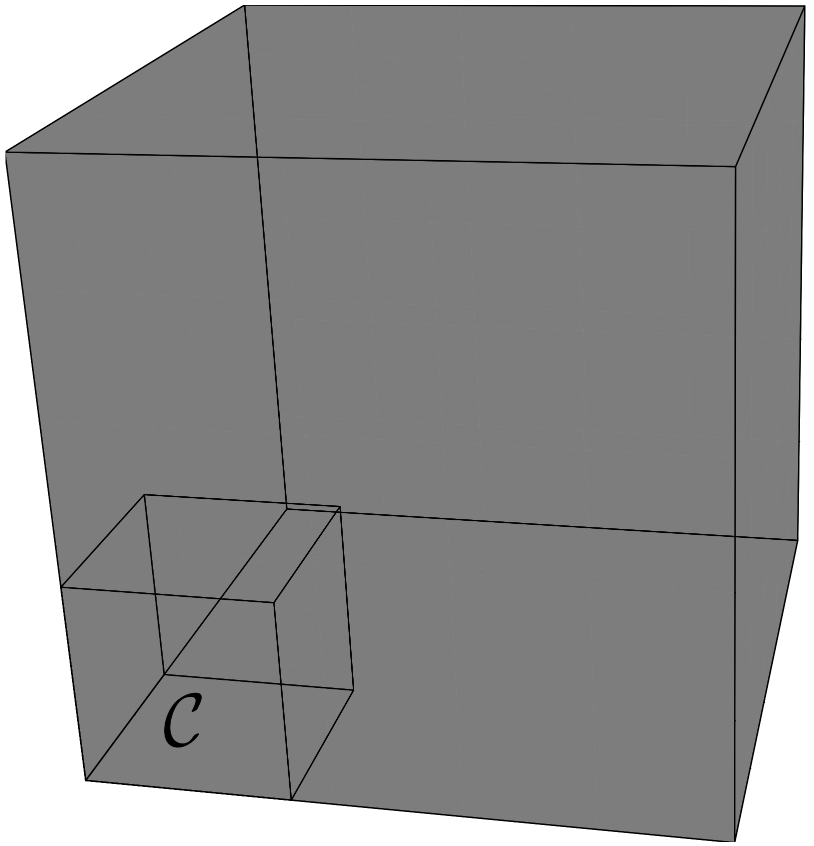

$$E_{\rm grav,self} \propto Gm^2/\ell$$

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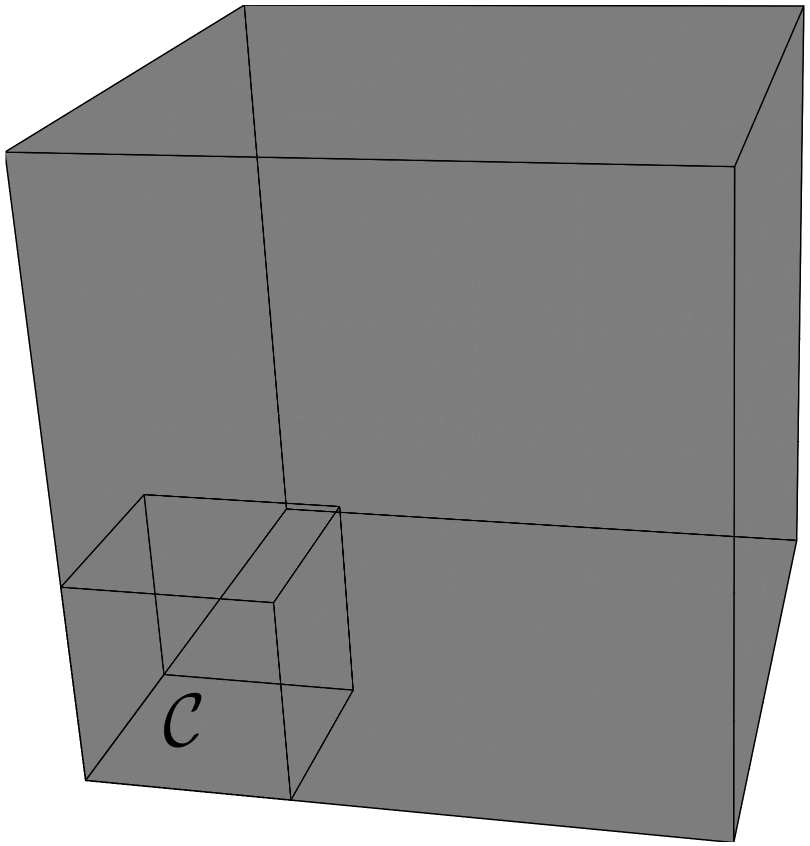

$$E_{\mathrm{grav,self}} \propto Gm^2/\ell \longrightarrow \frac{E^{(\mathrm{C})}_{\mathrm{grav}}}{E_{\mathrm{grav,self}}} = \frac{M^2\ell}{m^2 L} = N^{5/3}$$

Gravitational energy

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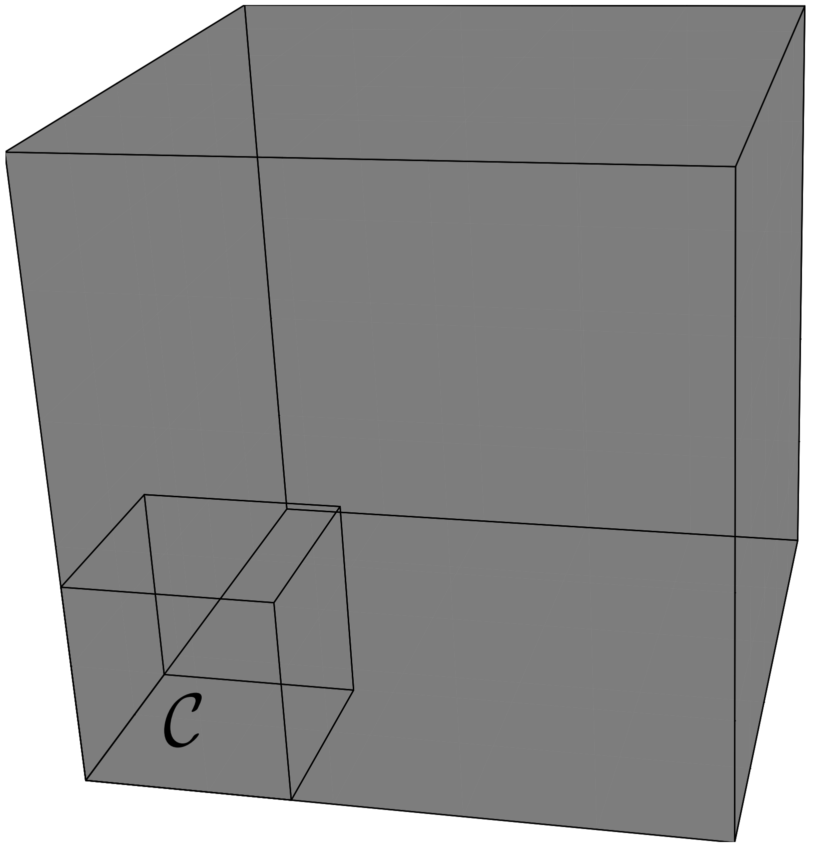

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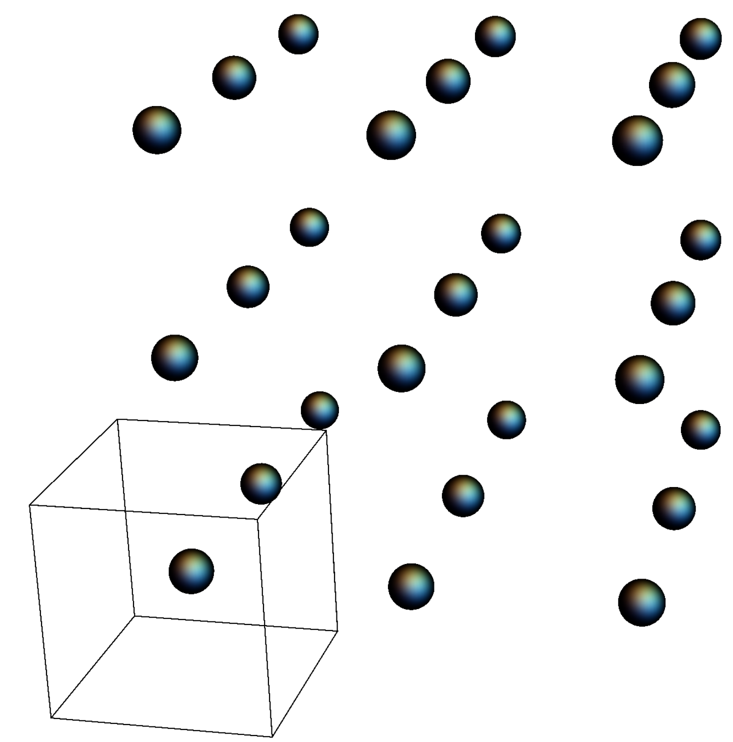

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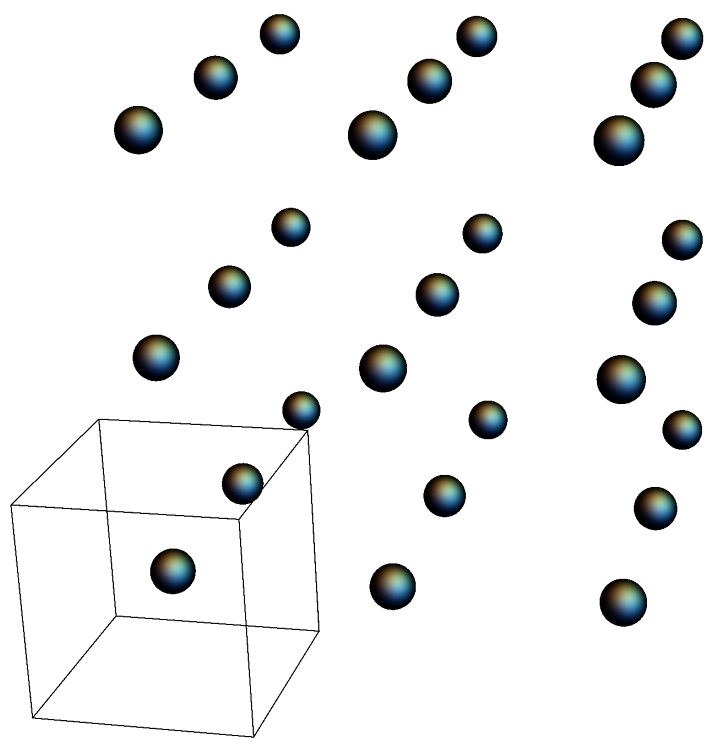
Gauss's law $\longrightarrow$ $E_{\rm grav,int} \approx E_{\rm grav}^{\rm (D)}$

Gravitational energy



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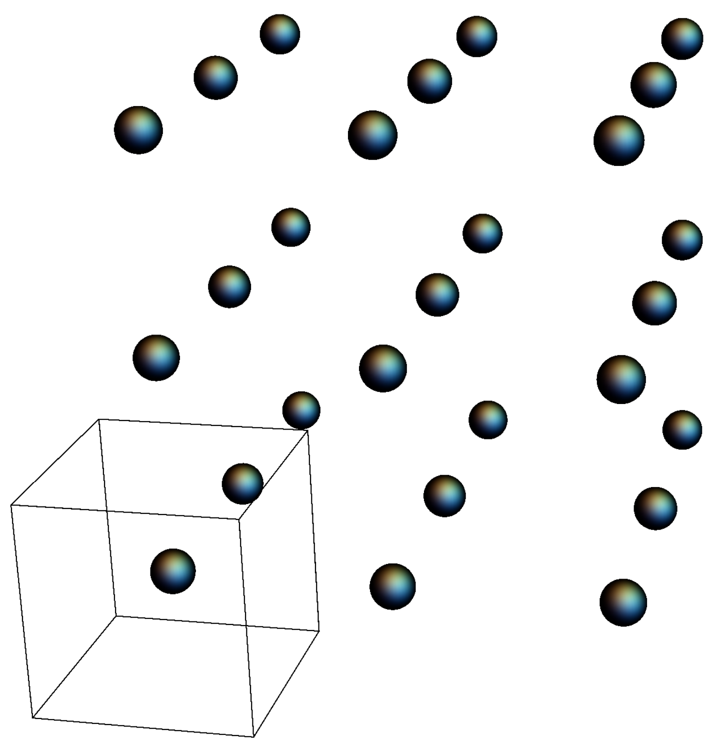
Gauss's law $\longrightarrow$ $E_{\mathrm{grav,int}} \approx E_{\mathrm{grav}}^{(\mathrm{D})}$

Gravitational energy

$$E_{\text{grav}}^{(\text{D})} \approx \left(1 - N^{-2/3}\right) E_{\text{grav}}^{(\text{C})}$$



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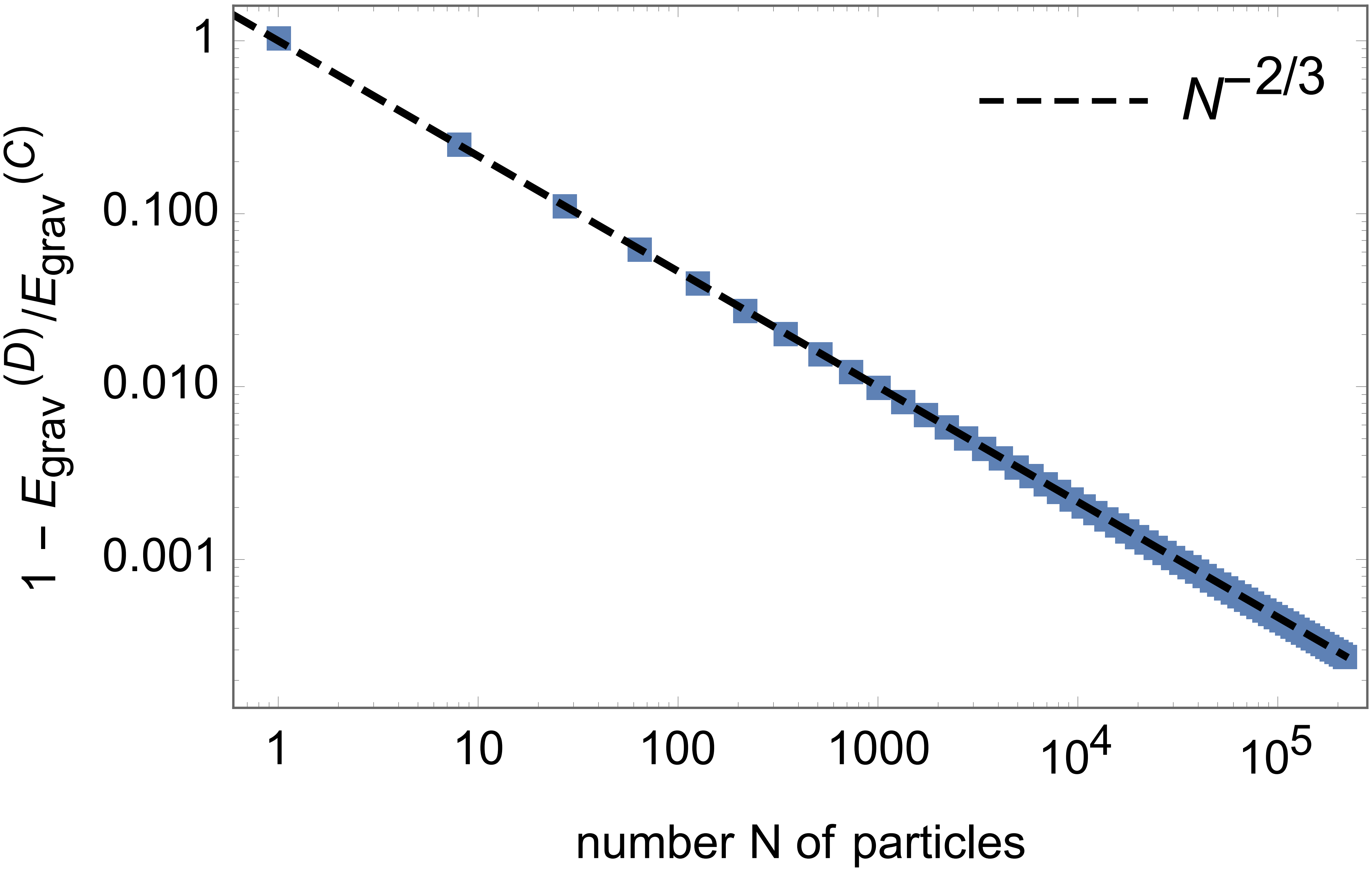
I am not lying to you



Kinetic energy

Kinetic energy

…similar calculations…

We actually get the same scaling law:

$$\boxed{E_{\mathrm{kin}}^{(\mathrm{D})} = \left(1 - N^{-2/3}\right) E_{\mathrm{kin}}^{(\mathrm{C})}}$$

So what happens?

$$\begin{aligned} E^{(\mathrm{D})} &= E_{\mathrm{kin}}^{(\mathrm{D})} + E_{\mathrm{grav}}^{(D)} \\ &= \left(1 - N^{-2/3}\right)\left(E_{\mathrm{kin}}^{(\mathrm{C})} + E_{\mathrm{grav}}^{(\mathrm{C})}\right) \\ &= \left(1 - N^{-2/3}\right) E^{(\mathrm{C})} \end{aligned}$$

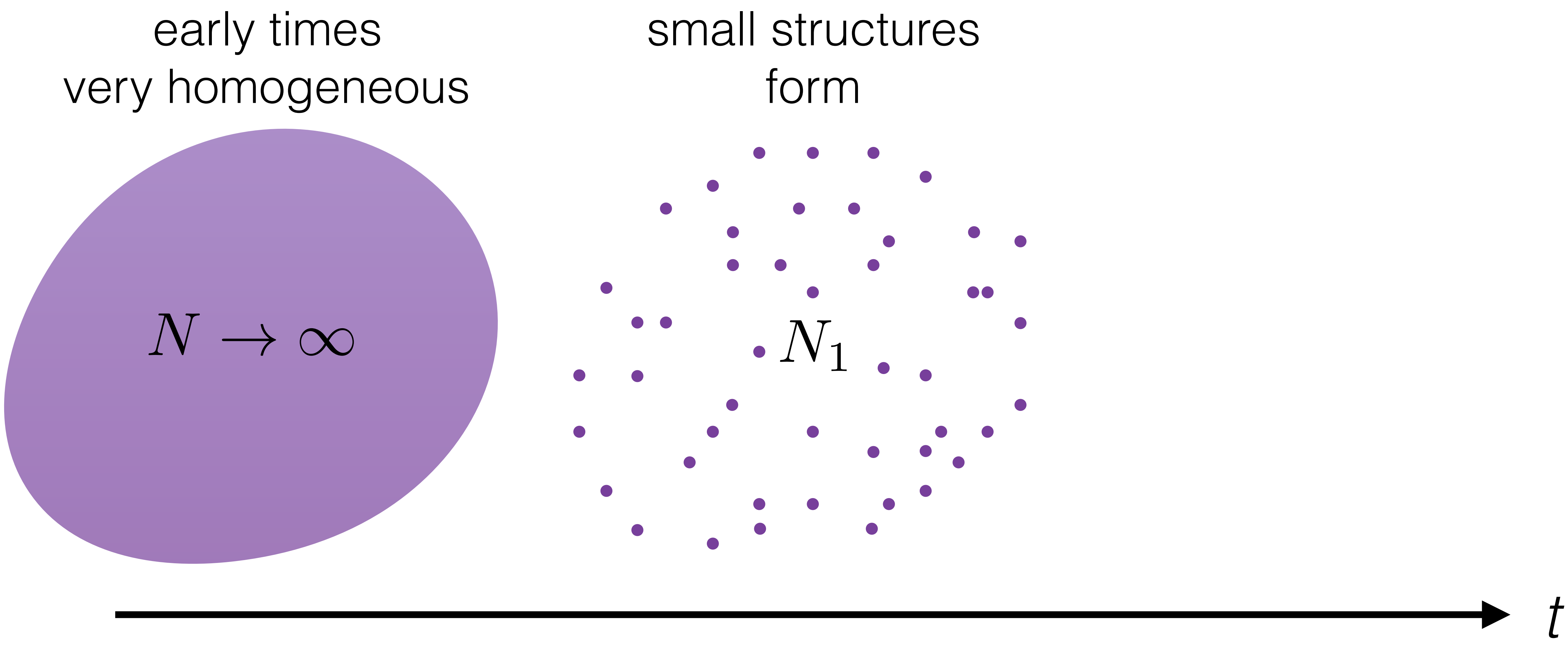
$$K^{(\mathrm{D})} = (1 - N^{-2/3}) K^{(\mathrm{C})}$$

Discrete model: **same dynamics** as the continuous model, but with a renormalized spatial curvature

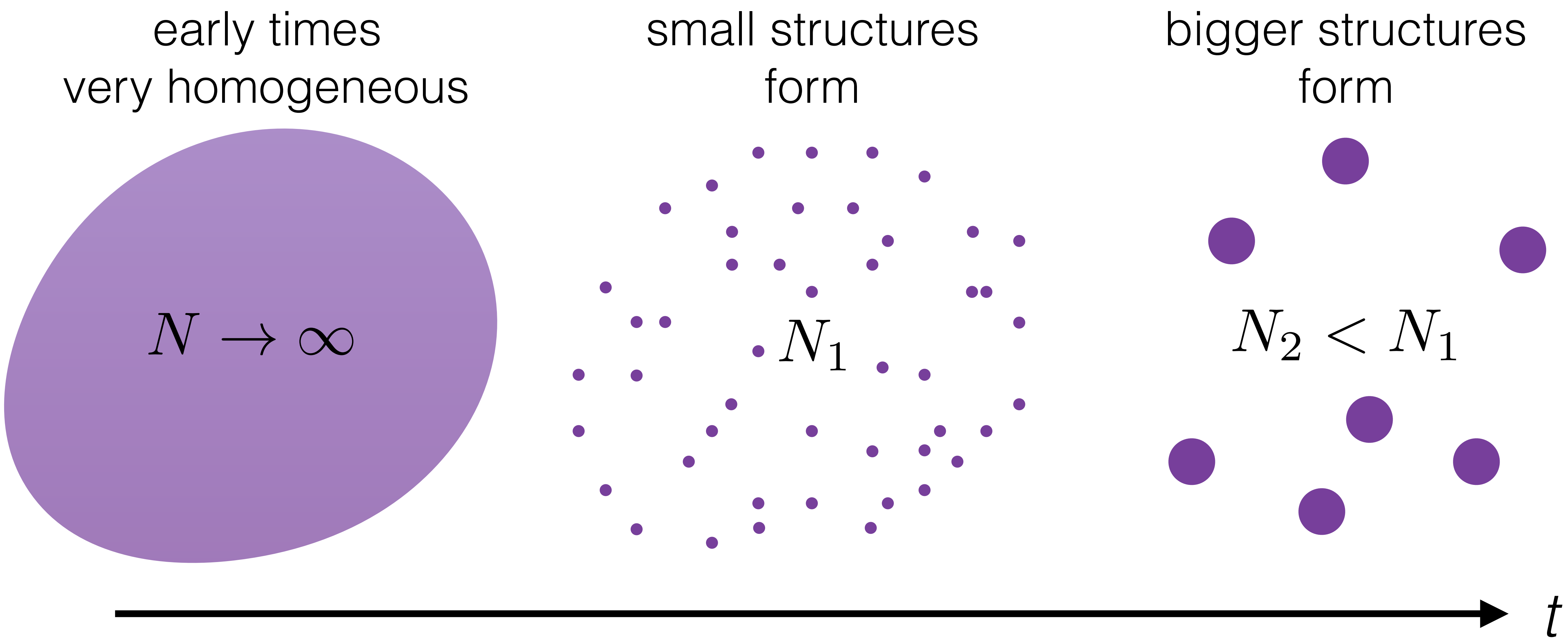
A scenario

early times
very homogeneous

A scenario


early times
very homogeneous
small structures
form
$N \to \infty$
N_1
t

A scenario


early times
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bigger structures
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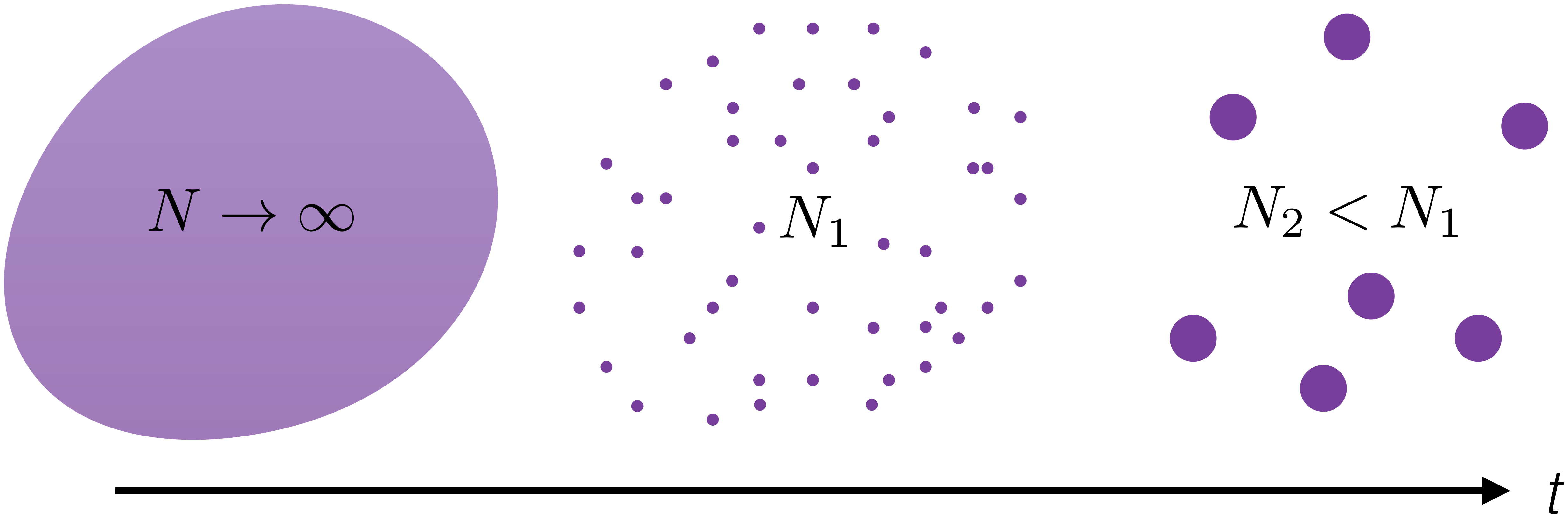
N_1
$N_2 < N_1$
t

A scenario

early times
very homogeneous

small structures
form

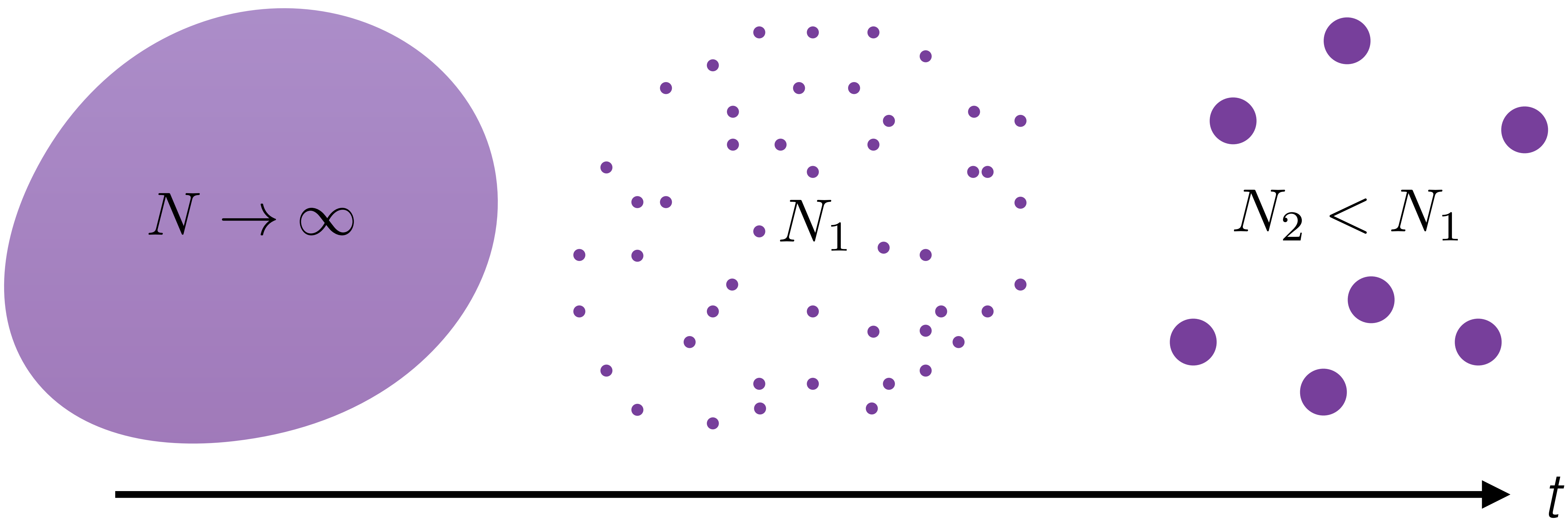
bigger structures
form



$E \propto 1 - N^{-2/3}$ decreases as bound structures form

A scenario

early times very homogeneous | small structures form | bigger structures form

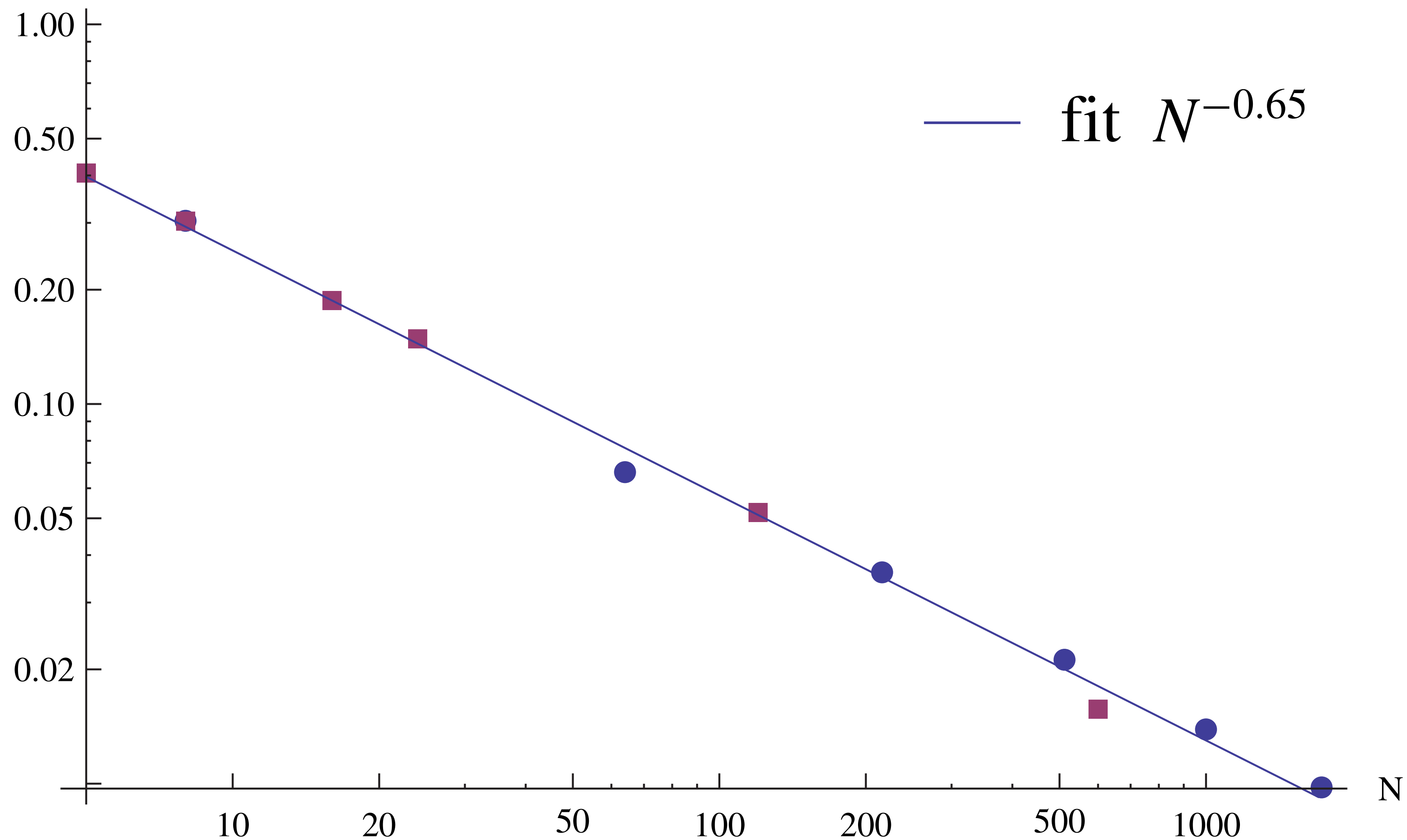


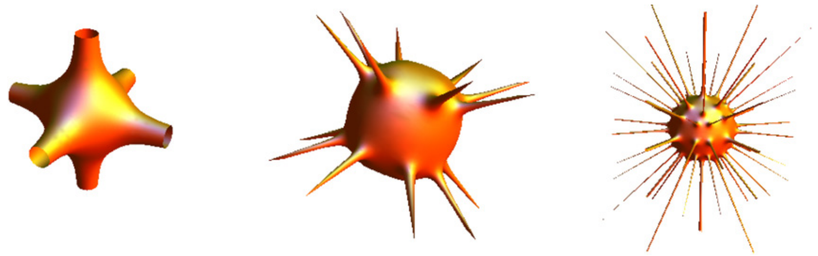
$E \propto 1 - N^{-2/3}$ decreases as bound structures form

Interpretation: energy leaks from the expansion dynamics towards internal degrees of freedom.

Comparison with relativistic results

$$\left|\frac{M_{\rm eff}-M}{M}\right|_{H=0} = \left|\frac{K_{\rm eff}-K}{K}\right|_{H=0}$$





[Korzynski, 2014]
[Clifton, Rosquist, Tavakol, 2012]

Newtonian conclusions

- Positively curved Universe: the formation of bound structures effectively weakens spatial curvature
- Infinite Universe: no effect
- Explains the discrepancies between previous results in the literature (finite vs infinite lattices)

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Key ingredient: Gauss's law

What about alternative theories of gravity?

Example: scalar tensor

$$S = S_{\mathrm{EH}}[g_{\mu\nu}] + S_\phi[\phi] + S_{\mathrm{m}}[\psi, C^2(\phi) g_{\mu\nu}]$$

Example: scalar tensor

$$S = \underbrace{S_{\rm EH}[g_{\mu\nu}]}_{\text{general relativity}} + S_\phi[\phi] + S_{\rm m}[\psi, C^2(\phi)g_{\mu\nu}]$$

Example: scalar tensor

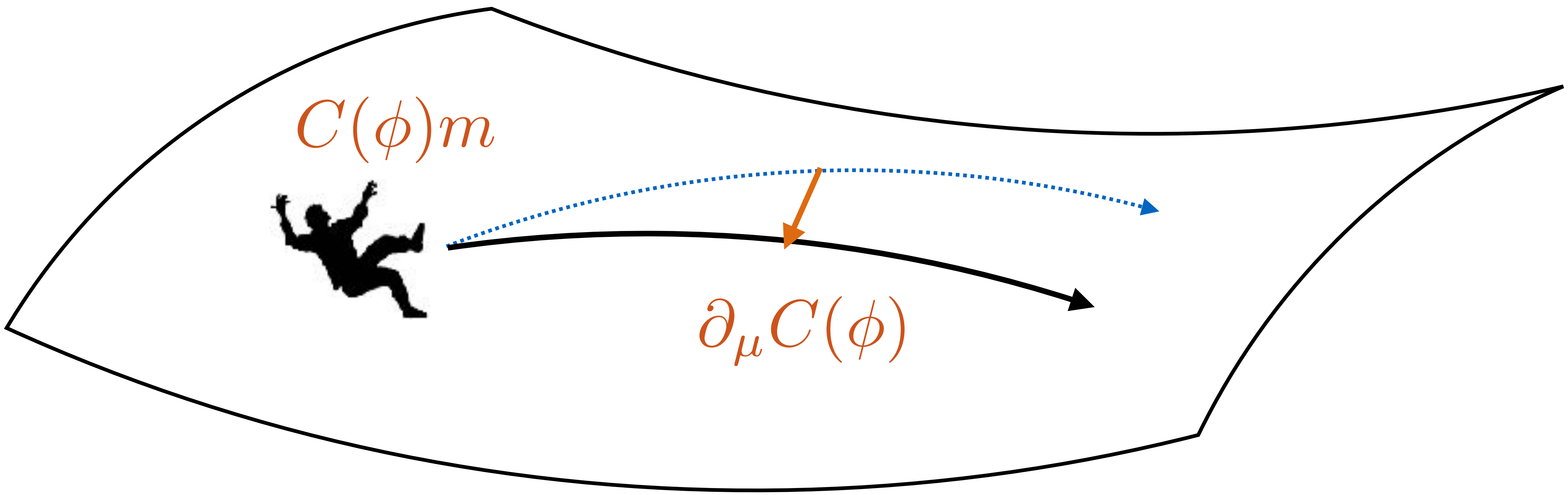
$$S = S_{\mathrm{EH}}[g_{\mu\nu}] + S_\phi[\phi] + S_{\mathrm{m}}[\psi, C^2(\phi) g_{\mu\nu}]$$

general relativity

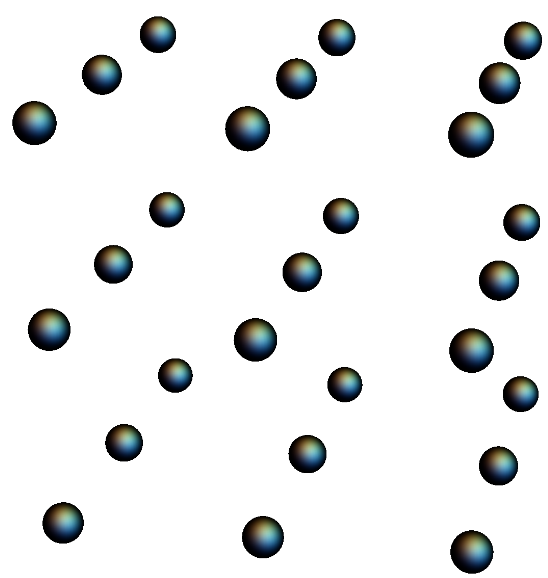
fifth force

Example: scalar tensor

$$S = \underbrace{S_{\mathrm{EH}}[g_{\mu\nu}]}_{\text{general relativity}} + \underline{S_\phi[\phi]} + S_{\mathrm{m}}[\psi, \underbrace{C^2(\phi)}_{\text{fifth force}} g_{\mu\nu}]$$

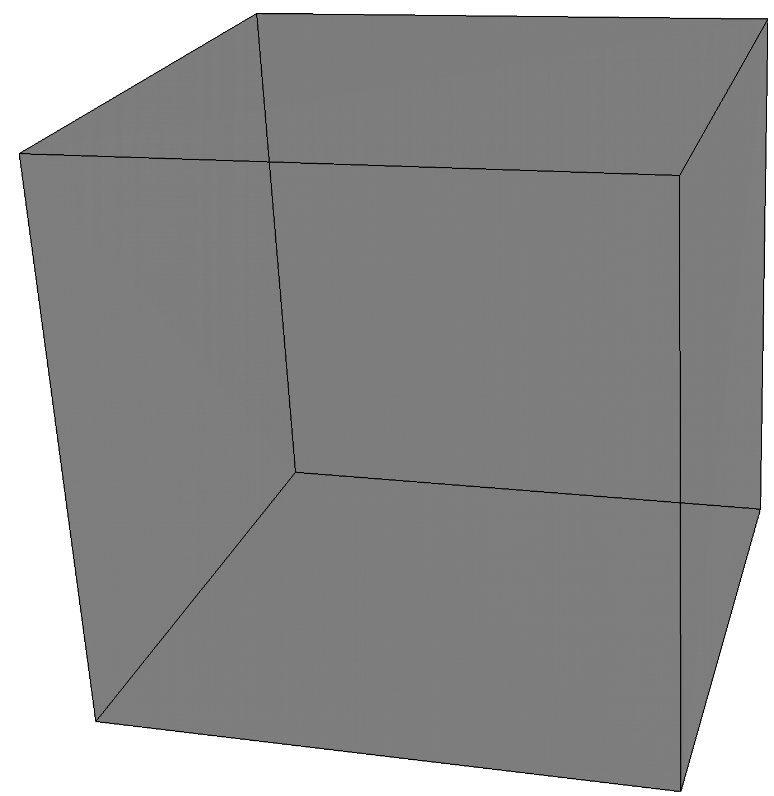


Discrete and continuous



$\phi(t, \boldsymbol{x})$

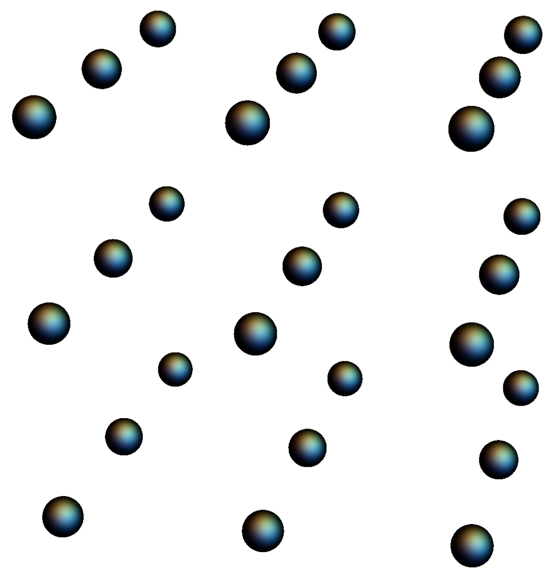
fifth force?



$\phi(t)$

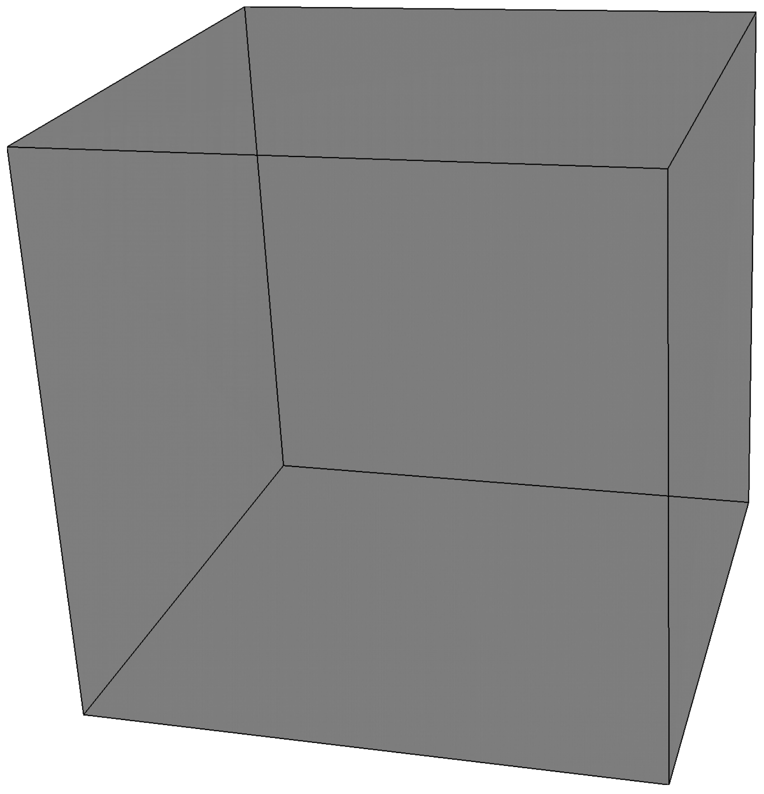
no additional force

Discrete and continuous



$\phi(t, \boldsymbol{x})$

fifth force?



$\phi(t)$

no additional force

If the scalar field has no potential, the expansion dynamics is **unchanged** **[Sanghai & Clifton 2017]**

Escaping Gauss's law
(in modified gravity)

- modification of gravity with a potential (Yukawa-like fifth force)
- screening mechanisms
- compact dark matter (PBH, …)

Conclusion

- In Newtonian gravity / GR, discreteness causes backreaction in positively curved universes
- The correction is realistically small, and vanishes in an infinite Universe
- Gauss's law is central in this mechanism
- In alternative theories of gravity, the amplitude of the corrections must be evaluated

Back-up slides

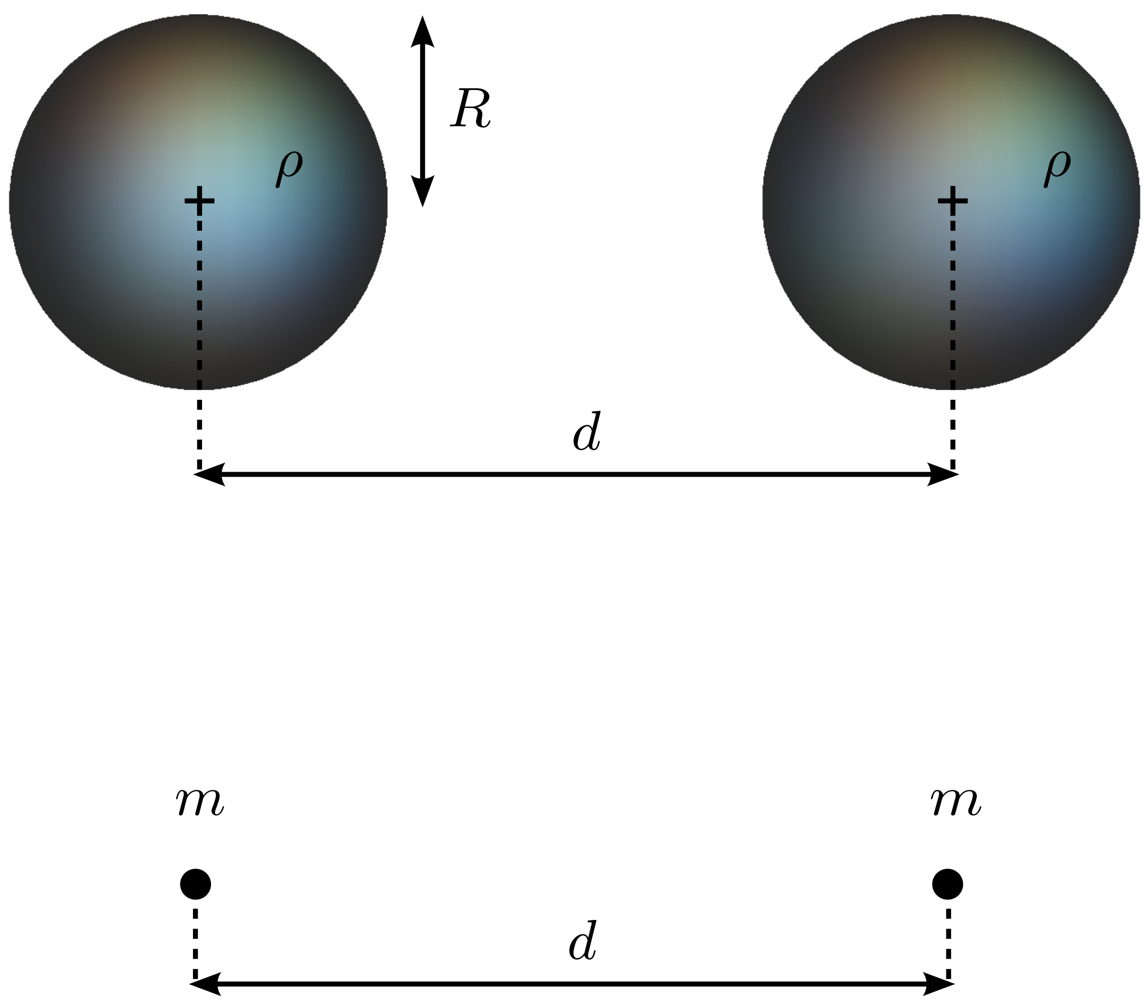
Yukawa gravity

$$\Delta\Phi - \lambda^{-2}\Phi = 4\pi G\rho$$

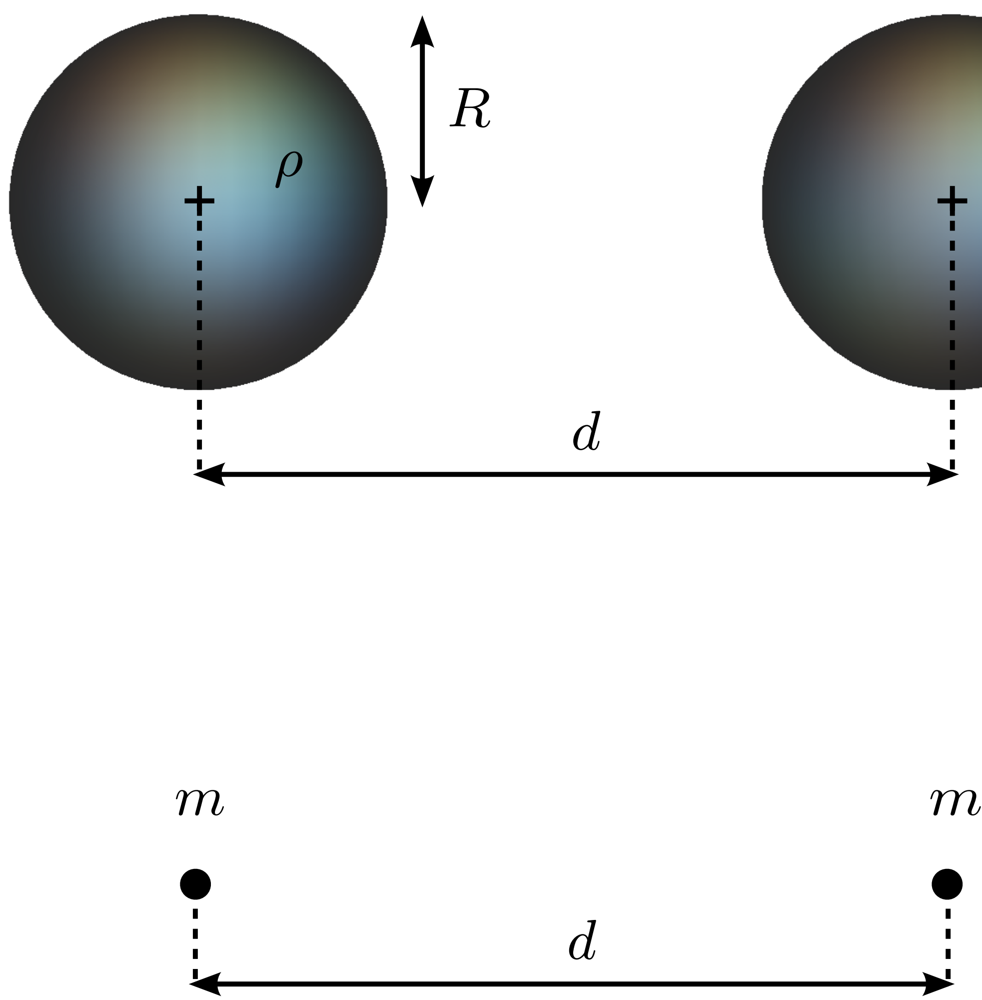
$$\int_{\partial\mathcal{D}} \boldsymbol{g}\cdot\boldsymbol{n}\,\mathrm{d}S = -4\pi G M_{\mathcal{D}} - \lambda^{-2}\int_{\mathcal{D}} \Phi\,\mathrm{d}V$$

I violate Gauss

A consequence



A consequence



$$\frac{E^{\bigcirc - \bigcirc}_{\mathrm{grav,int}}}{E^{\bullet - \bullet}_{\mathrm{grav,int}}} = \Gamma^2 \left(\frac{R}{\lambda} \right)$$

with

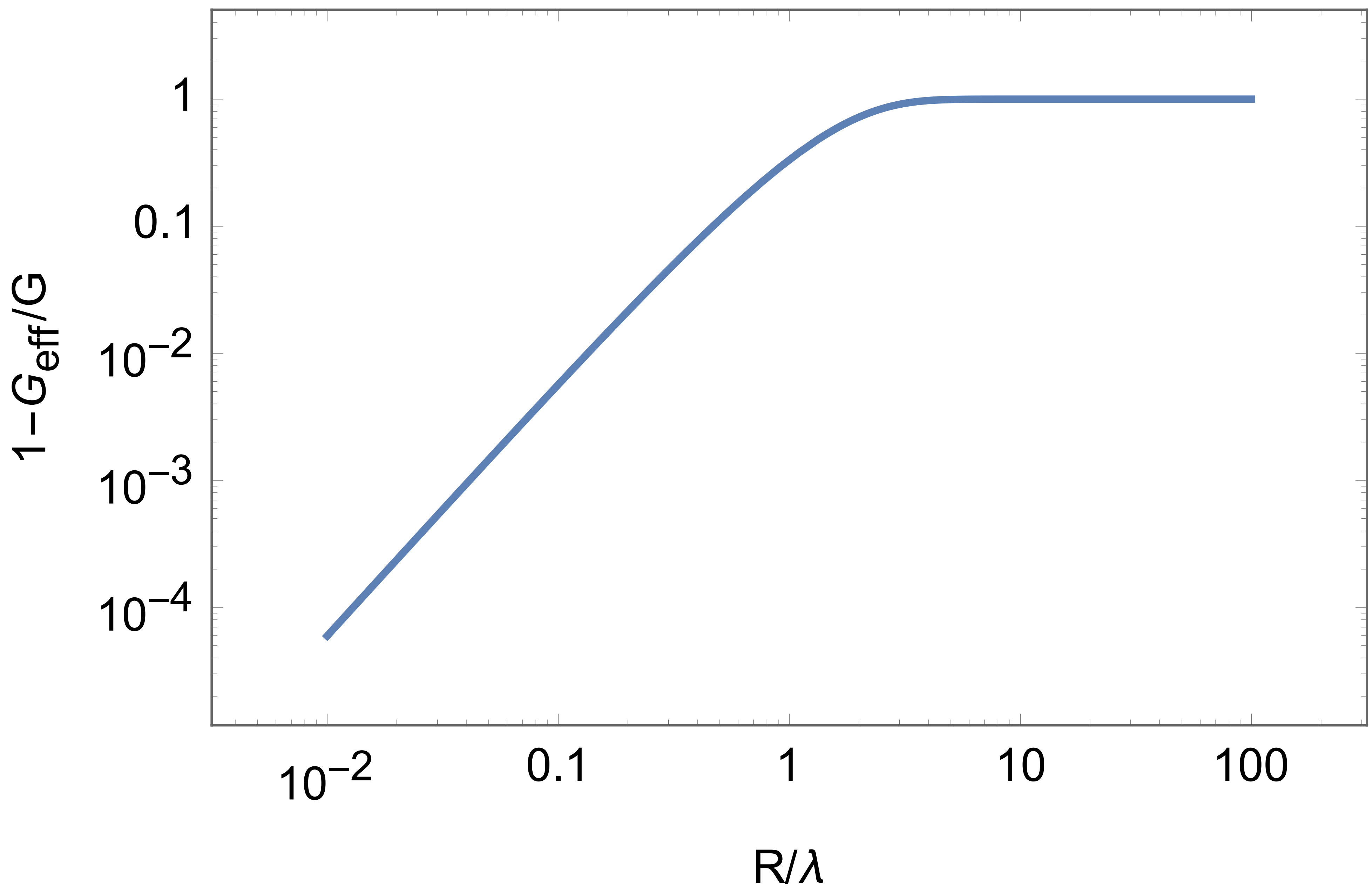
$$\Gamma(x) \equiv \frac{3}{x^3} \left(x \cosh x - \sinh x \right)$$

Backreaction effect

Contrary to the Newtonian case:

- forming bound structures does **not** have the same effect on kinetic and gravitational energies;
- a correction **persists** even in an infinite universe;
- it is equivalent to renormalizing Newton's constant:

$$\frac{G_{\rm eff}}{G} = \frac{1}{\Gamma^2(R/\lambda)}\left[1 - \frac{2}{5}\left(\frac{R}{\lambda}\right)^2 \Delta\left(\frac{R}{\lambda}\right)\right]$$



λ : Compton wavelength of the graviton

R: radius that the largest bound structures should have if they had the mean density of the Universe