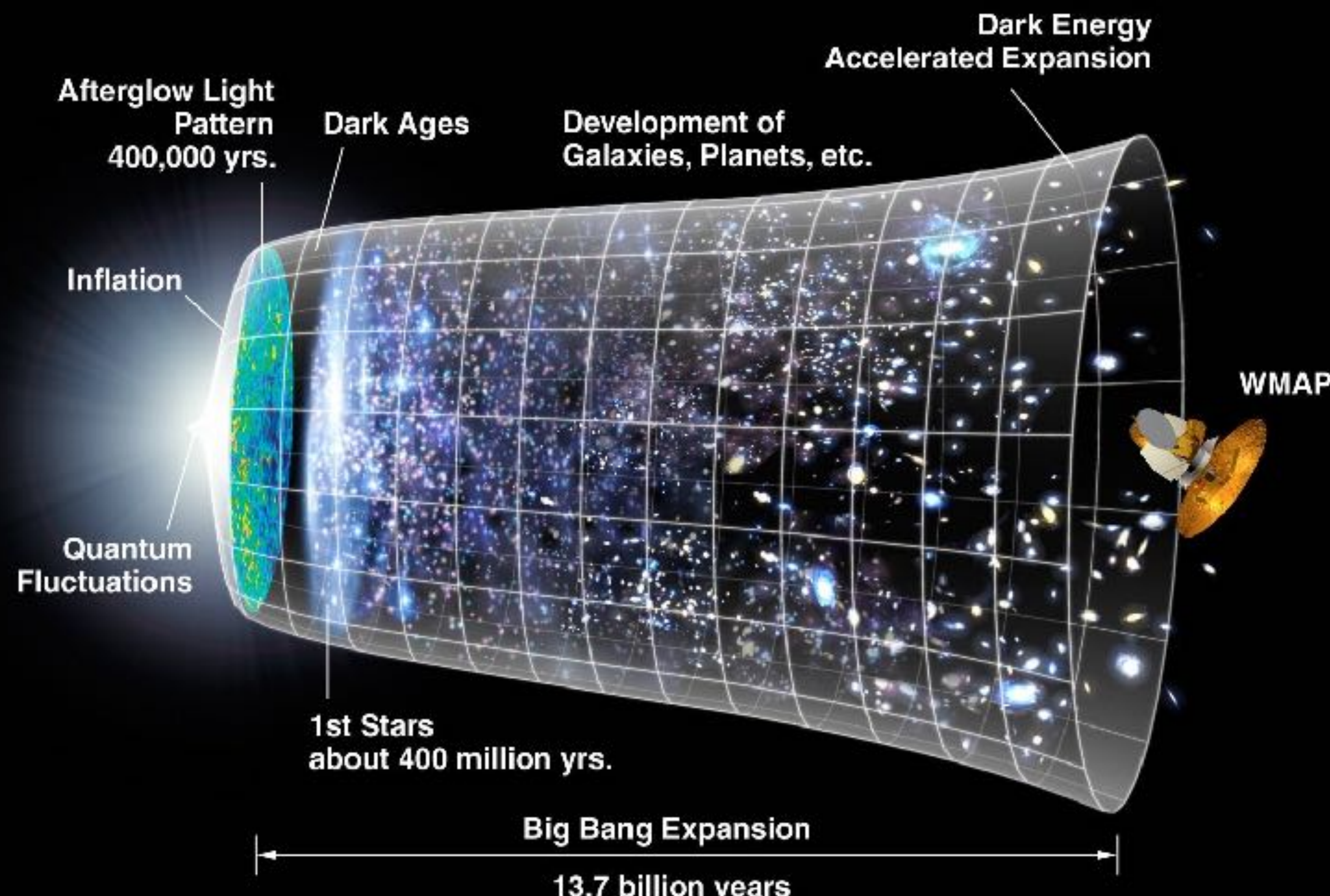
Effects of non-linear structure on cosmological parameter estimation

Nick Kaiser
Ecole Normale Superieure

Marseille backreaction workshop - 2018/05/30

The scope of modern cosmology



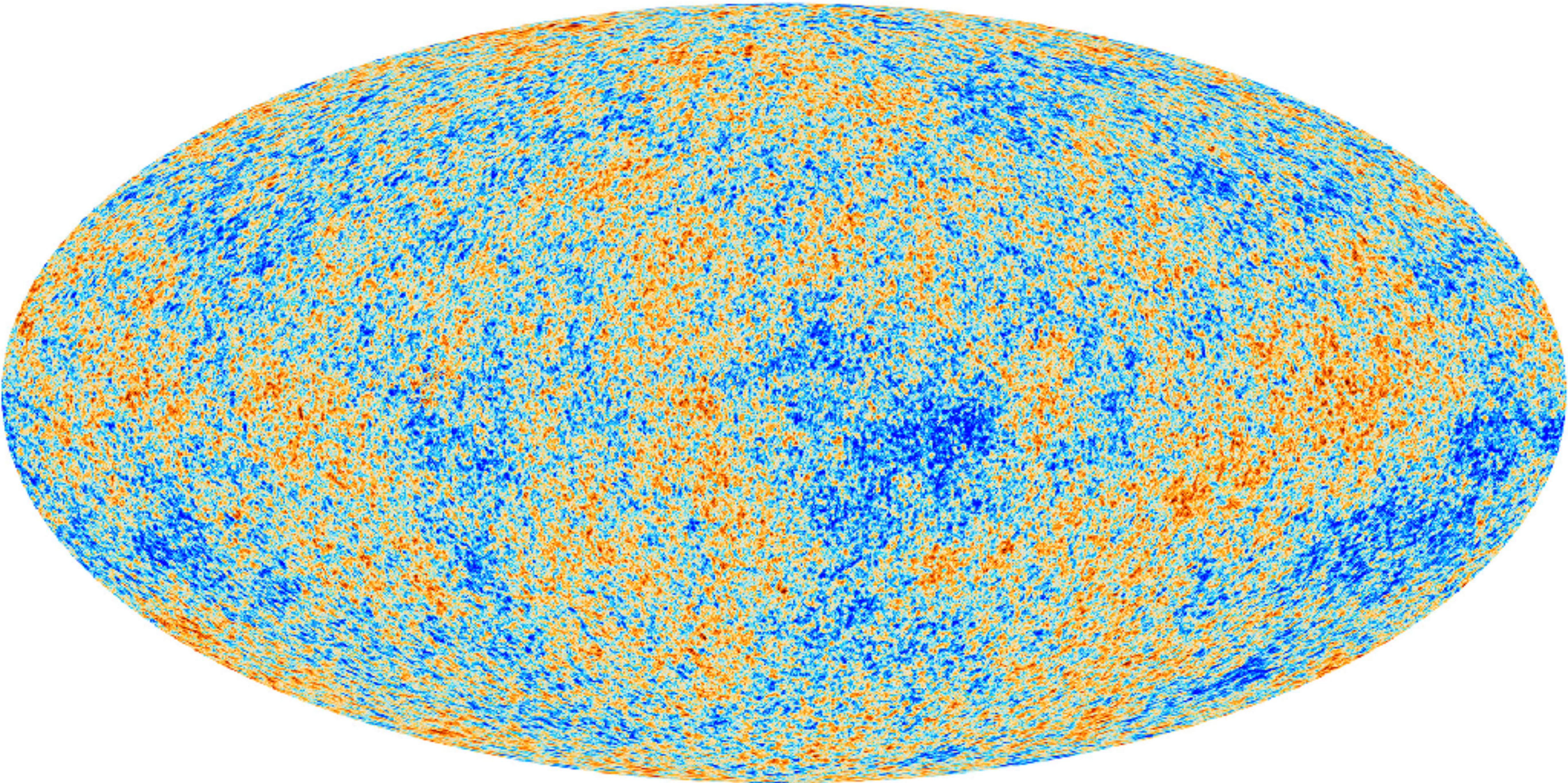
Preamble

- Modern cosmology: an exercise in hubris
  - <80’s - “missing mass” (since ’30-s) - CMB - hot-big-bang model - BBN - non-baryon DM - framework: search for 2 numbers
  - 75-82 - (GUT) inflation + fluctuogenesis -> CDM model
  - 80’s, 90’s growing problems - age, bias, cluster evolution, flatness from CMB, lack of decelleration …. late time inflation - quintessence (or Λ)
- ΛCDM: Remarkably successful
  - Cosmology is the search for 6 numbers?
  - but needs inflaton, DM, quintessence - who ordered those?
- Early universe looks in good shape, but maybe the DE (and DM + other coincidences?) is telling us we are doing something wrong
  - Modified gravity?
  - Conventional gravity but misinterpreted (this meeting)

Outline

- Does lensing by structure <u>bias</u> the distance-redshift relation?
- Backreaction bias in the Hubble diagram
- Some challenges for backreaction

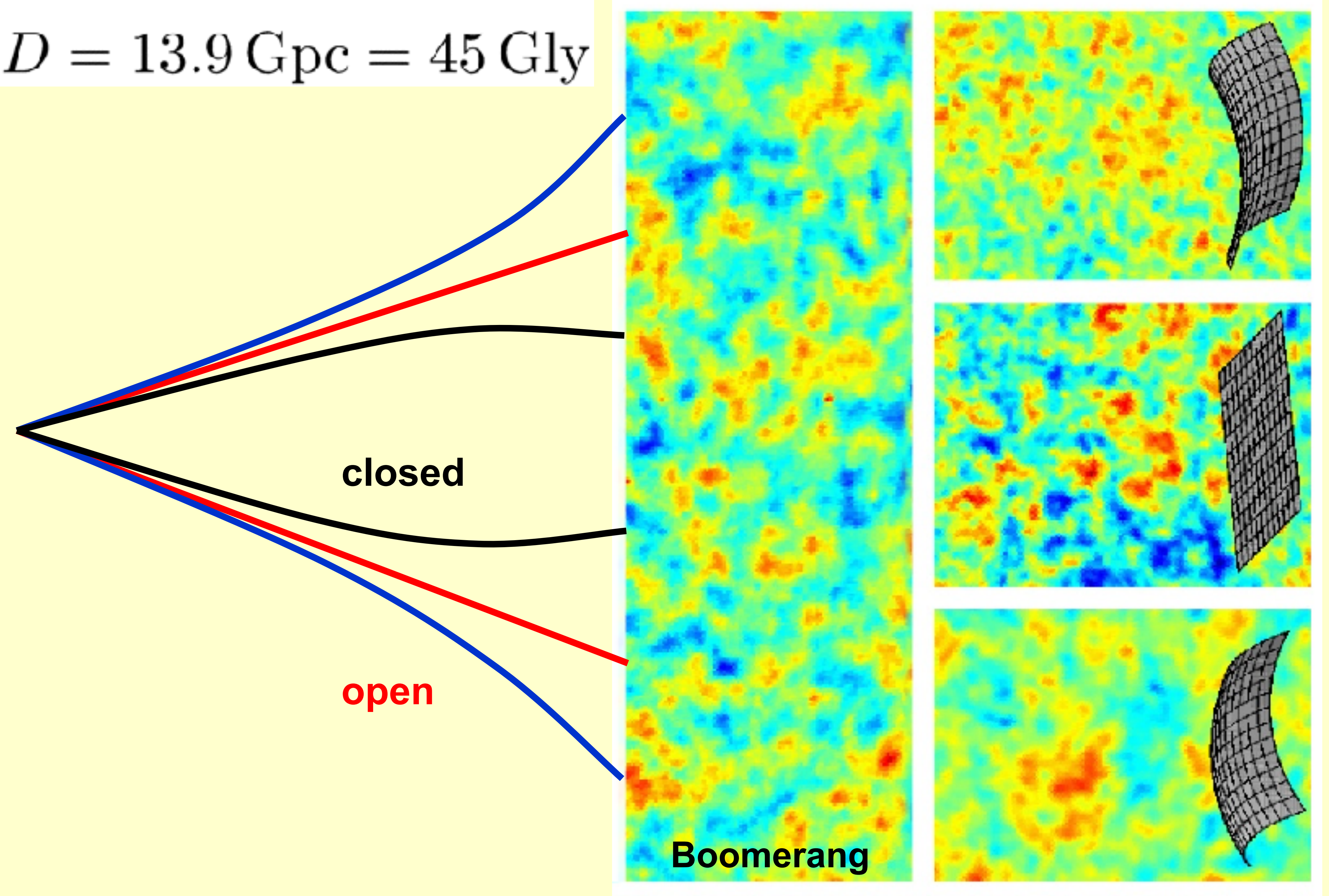
Context: cosmological parameters from the CMB
It is usually assumed that we are looking here at a spherical surface at $z \sim 1100$ with $D = D_0(z=1100)$
But are we?



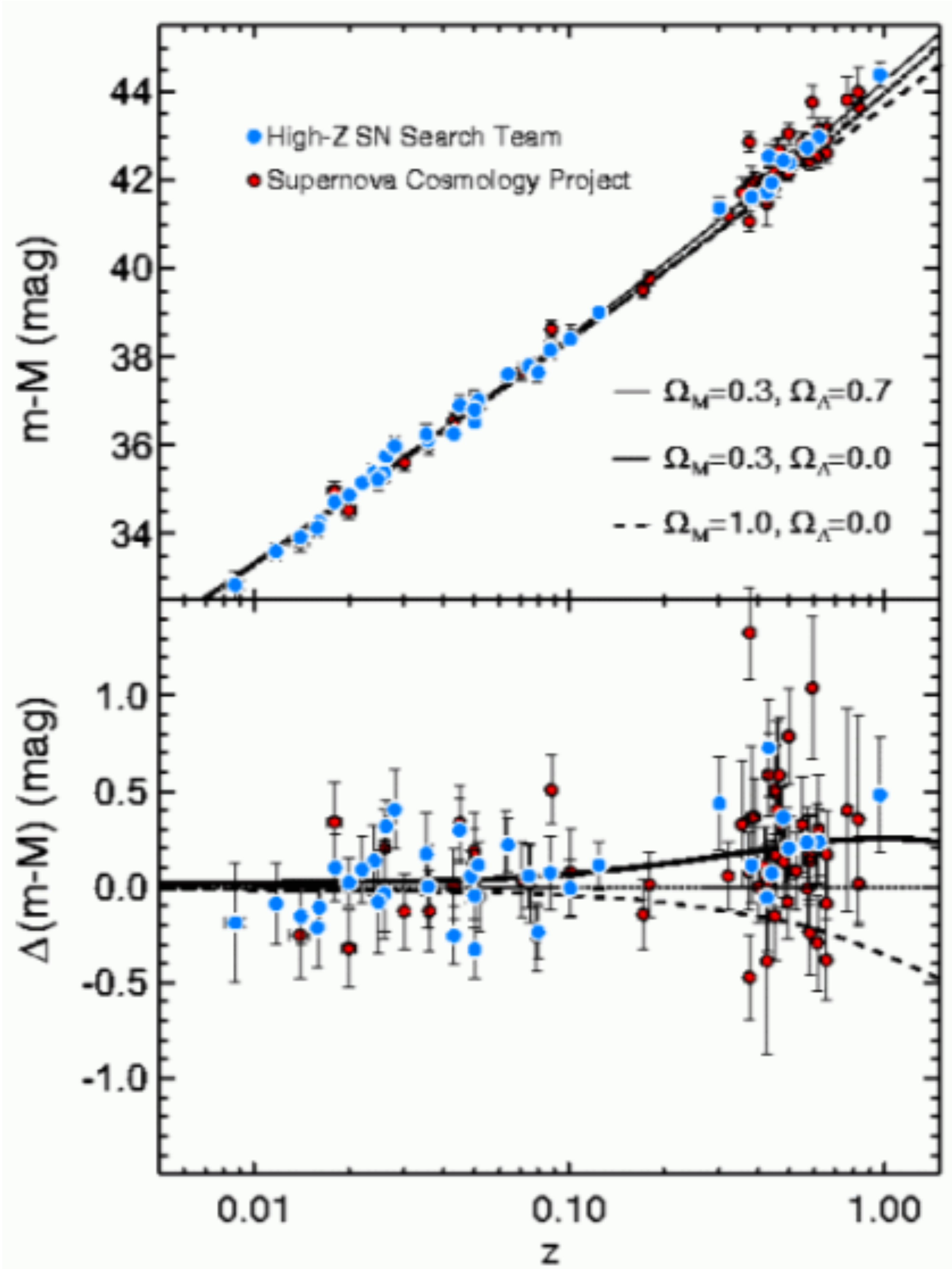
How far away is the CMB?

$$D = \int \frac{c}{H(z)}\, dz$$

$$z = 1080 \Rightarrow D = 13.9\,\mathrm{Gpc} = 45\,\mathrm{Gly}$$



Hubble diagram from SN1a - assumes no flux *bias* from lensing



Outline of talk

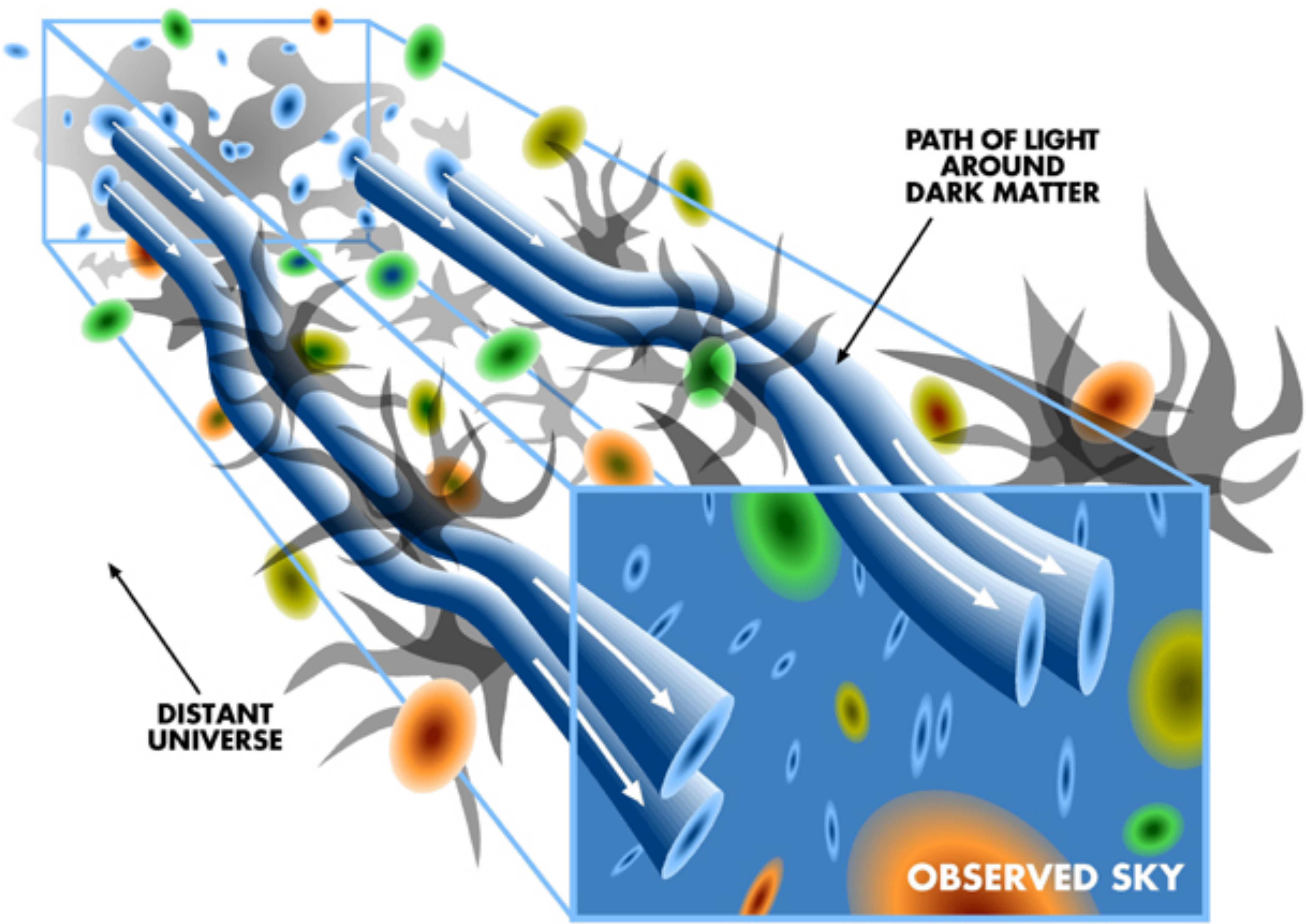
- **Some preliminaries**
  - what do we mean by *distance* in cosmology?
  - basics of gravitational lensing - light deflection, shear & magnification
- **Historical review:**
  - Zel'dovich '63 .... Feynman & Gunn .... Kantowski ... Dyer & Roeder
    - things look fainter in an inhomogeneous universe
  - Weinberg '76 -
    - *no effect* (flux conservation)
  - Schneider et al. ('84..'94): *magnification* and *focusing theorems*
    - things look brighter
  - significant others .... recent studies
- **NK + John Peacock** MNRAS2016
  - we reconcile the above, apparently contradictory, results

What do we mean by "*distance*" in cosmology

- Locally - directly measure distances:
  - radar echoes - parallaxes
- Not useful in cosmology. Instead we have:
  - *redshift* (reflects change in size of the Universe)
  - `*conformal*' or `*comoving*' distance χ - appears in metric
  - **angular diameter distance**: $\theta = d / D_A$
  - **luminosity distance:** $F = L / (4 \pi D_L^2)$
    - apparent distances of "standard candles" or "measuring rods"
- This talk: Lensing magnifies or de-magnifies: changes D_A, D_L:
  - they become random functions of direction
- **Q: does structure *bias* angular sizes or flux densities?**
  - **if it does then we will get the wrong cosmological parameters**

Optical properties of a lumpy universe

- Homogeneous universe: metric: $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$
  - $a(t)$ obeys Friedmann's equations
  - $\mathbf{x}$ is "conformal" coordinate (galaxies have fixed $\mathbf{x}$)
- Lumpiness: $ds^2 = -(1 + 2\,\phi(\mathbf{x}))\,dt^2 + a^2(t)(1 - 2\,\phi(\mathbf{x}))(dx^2 + dy^2 + dz^2)$
  - $\phi(\mathbf{x})$ determined by density fluctuations $\delta\varrho(\mathbf{x})$ (Poisson's equation)
  - very good approximation because velocities are slow
- Light rays are null paths ($ds = 0$)
- Same as light rays in "lumpy glass" with inhomogeneous $n(\mathbf{x})$
  - effective refractive index $n(\mathbf{x}) = (1 - 2\,\phi(\mathbf{x}) / c^2)$
    - $n(\mathbf{x})$ = (coordinate speed of light)$^{-1}$
  - Snell's law: Deflection $\theta_{def} \sim \phi / c^2 \sim G\delta M / r\,c^2$

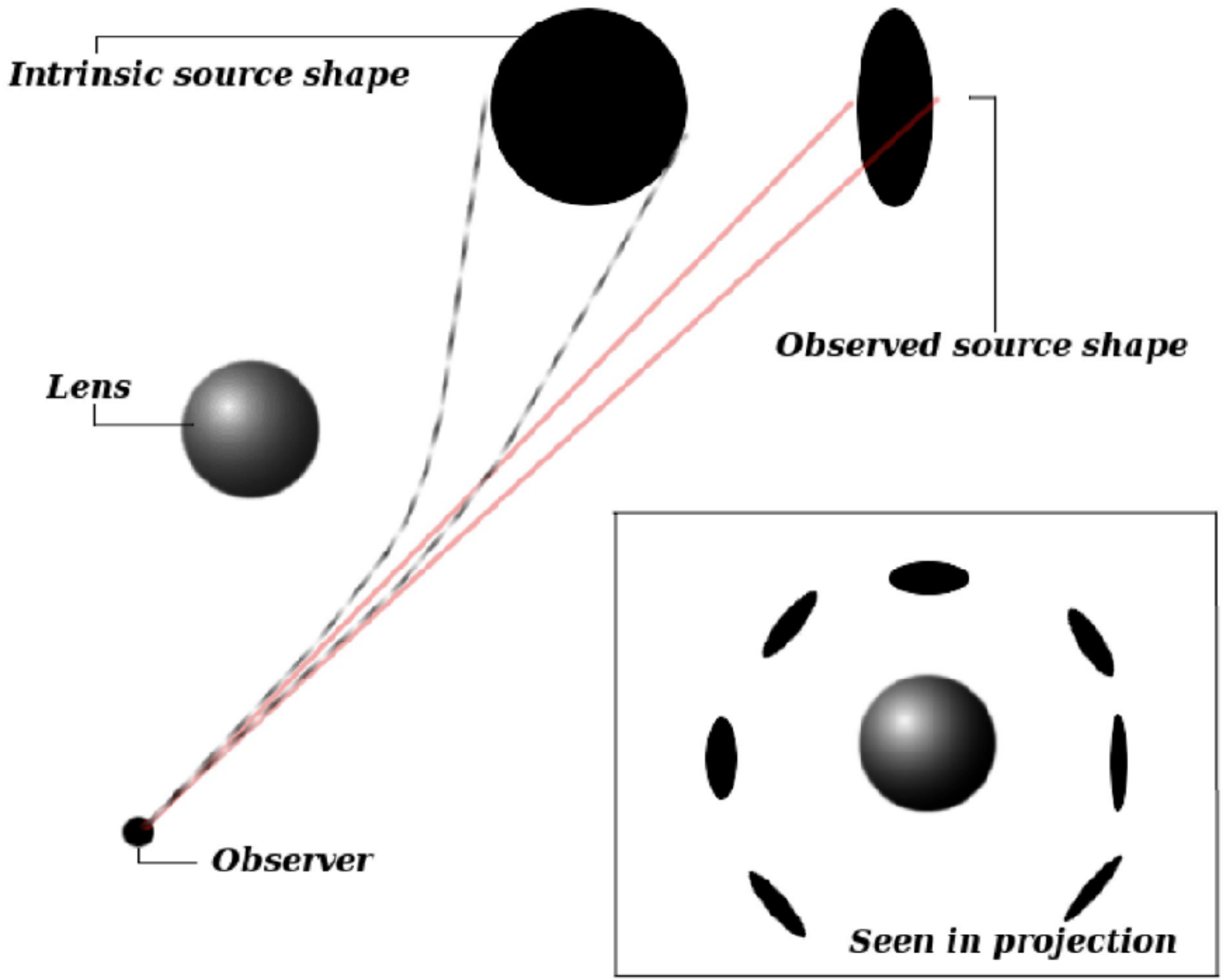

PATH OF LIGHT
AROUND
DARK MATTER
DISTANT
UNIVERSE
OBSERVED SKY

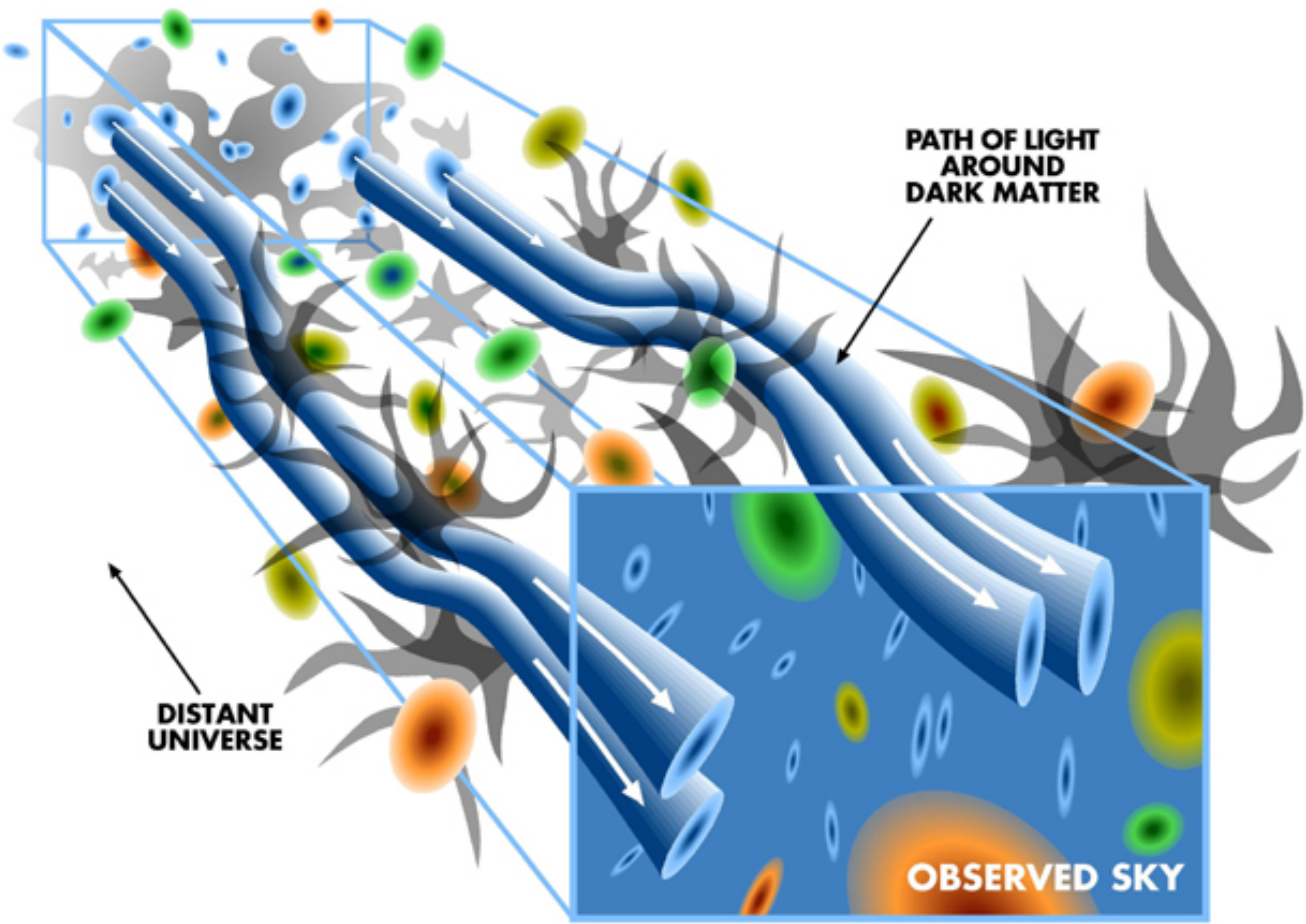
basics of gravitational lensing: Δt, deflection

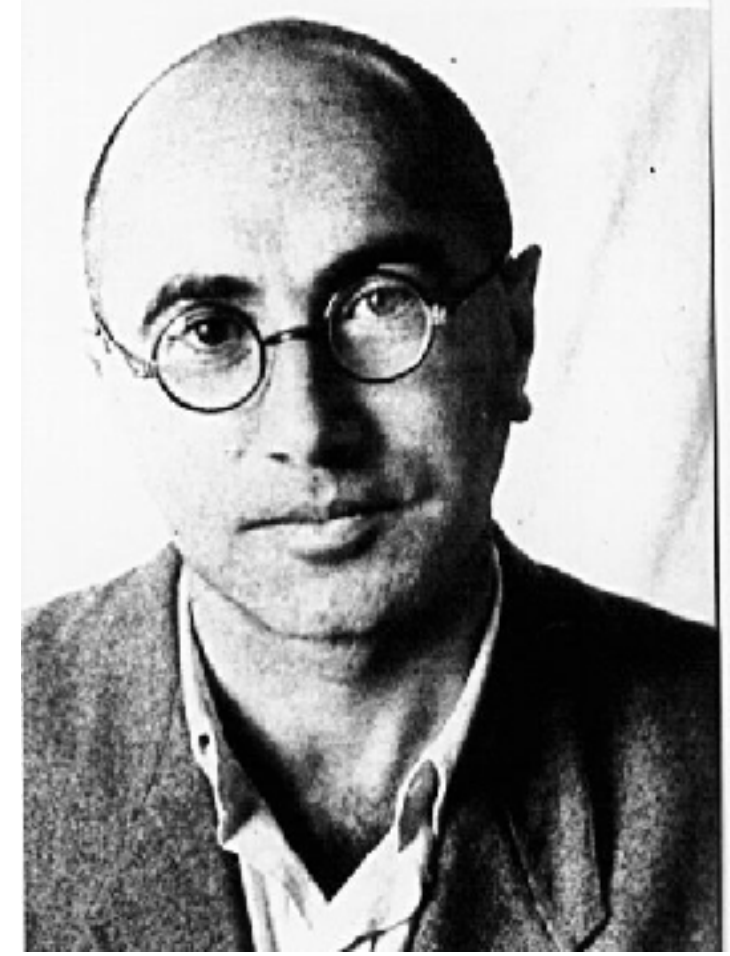
- Gravitational time delay (Shapiro '65): $\Delta t = 2 \int d\lambda \, \Phi/c^2$
  - λ = distance: Φ = gravitational field from $\Delta\varrho/\varrho$
  - measured in "strong lensing" - multiple images of quasars
  - fundamental concept (see Blandford & Narayan '86)
- Light deflection $\theta_1 \sim \int d\lambda \nabla\Phi/c^2 \sim GM/bc^2 \sim (H\lambda/c)^2\Delta$
  - cumulative deflection is a "random walk"
    - $\theta \sim N^{1/2} \theta_1 \sim (H\lambda/c)^{3/2}\Delta$
    - $\Delta = \Delta\varrho/\varrho \sim \xi^{1/2} \sim 1/\lambda$
  - θ dominated by "supercluster" scale structure (~30 Mpc)
    - quite large ~ few arc-minutes ~ 10^{-3} radians at high z
    - but (usually) not directly observable

basics of lensing: Δt, θ_{def} + magnification & shear

- Time delay $\Delta t = 2 \int d\lambda\, \Phi/c$
- Light deflection - cumulative deflection $\theta \sim N^{1/2}\, \theta_1 \sim (H\lambda/c)^{3/2}\Delta$
  - θ dominated by large scale structure (~30 Mpc)
- Weak lensing: observe the *gradient* of the deflection angle
  - described by a 2x2 image distortion tensor
    - trace: $\varkappa$ (kappa) → *magnification* (changes *size* of objects)
    - 2 other components: γ → *image shear* (changes *shapes*)
    - ~1% at ~ degree scales for sources at z ~ 1 (few % @ z=1000)
      - but grows with decreasing angular scale
  - potentially *very large effects* from small-scale lumpiness


Intrinsic source shape
Observed source shape
Lens
Observer
Seen in projection


PATH OF LIGHT AROUND DARK MATTER
DISTANT UNIVERSE
OBSERVED SKY



OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

A local nonuniformity of density due to the concentration of matter of the universe into separate galaxies produces a significant change in the angular dimensions and luminosity of distant objects as compared to the formulas for the Friedman model.

The propagation of light in a homogeneous and isotropic model of the expanding universe (first studied by A. A. Friedman) has been investigated in a number of papers [1, 2, 3].

In these papers expressions were obtained for the observed angular diameter Θ and the observed brightness of an object with a known absolute diameter and absolute brightness as a function of the distance or, strictly speaking, the red shift of the object $\Delta = (\omega_0 - \omega)/\omega_0$.

In particular, there is a remarkable feature in the function $\Theta(\Delta)$, namely, the presence of a minimum when Δ is approximately equal to 1/2. Formula (10) and Fig. 6 in the appendix show the variation of the function $f(\Delta) = rH/c\Theta$ which is inversely proportional to Θ for a given density of matter. Here $\underline{r}$ is the radius of the object, H is Hubble's

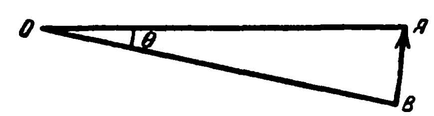


Fig. 1.

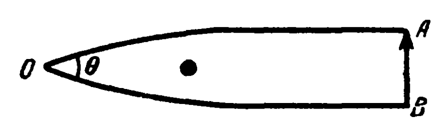
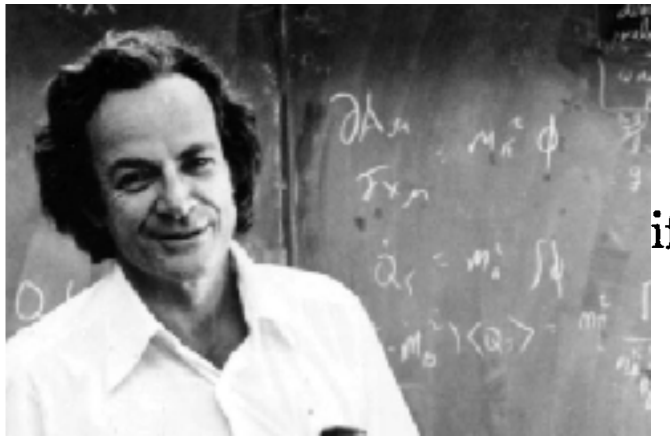


Fig. 2.

A mass situated between these rays bends the latter in such a way that Θ is increased (Fig. 2). What we have in mind is the bending of light rays by the gravitational field predicted by Einstein; this bending amounts to 1.75" for a light ray passing near the limb of the solar disc and has been confirmed by observation.

ON THE PROPAGATION OF LIGHT IN INHOMOGENEOUS COSMOLOGIES. I. MEAN EFFECTS



JAMES E. GUNN
ifornia Institute of Technology and Jet Propulsion Laboratory

Received February 23, 1967; revised May 23, 1967

ABSTRACT

The statistical effects of local inhomogeneities on the propagation of light are investigated, and deviations (including rms fluctuations) from the idealized behavior in homogeneous universes are investigated by a perturbation-theoretic approach. The effect discussed by Feynman and recently by Bertotti of the density of the intergalactic medium being systematically lower than the mean mass density is examined, and expressions for the effect valid at all redshifts are derived.

I. INTRODUCTION

In an unpublished colloquium given at the California Institute of Technology in 1964, Feynman discussed the effect on observed angular diameters of distant objects if the intergalactic medium has lower density than the mean mass density, as would be the case if a significant fraction of the total mass were contained in galaxies. It is an obvious extension of the existence of this effect that luminosities will also be affected, though this was apparently not realized at the time. This realization prompted the conviction that the effect of known kinds of deviations of the real Universe from the homogeneous isotropic models (upon which predictions had been based in the past) upon observable quantities like luminosity and angular diameter should be investigated. The author (1967) has recently made such a study for angular diameters; the present work deals primarily with mean statistical effects upon luminosity. A third paper will deal with possible extreme effects one may expect to encounter more rarely. Some of the results discussed here have been discussed independently by Bertotti (1966) and Zel'dovich (1965).

Kantowski '69

CORRECTIONS IN THE LUMINOSITY-REDSHIFT RELATIONS OF THE HOMOGENEOUS FRIEDMANN MODELS

R. KANTOWSKI*
Southwest Center for Advanced Studies, Dallas, Texas
Received January 22, 1968; revised March 22, 1968

ABSTRACT

In this paper the bolometric luminosity-redshift relations of the Friedmann dust universes ($\Lambda = 0$) are corrected for the presence of inhomogeneities. The "locally" inhomogeneous Swiss-cheese models are used, and it is first shown that the introduction of clumps of matter into Friedmann models does not significantly affect the $R(z)$ or $R(v)$ relations (Friedmann radius versus the redshift or affine parameter) along a null ray. Then, by the use of the optical scalar equations, a linear third-order differential equation is arrived at for the mean cross-sectional area of a light beam as a function of the affine parameter. This differential equation is confirmed by rederiving its small redshift solution from an interesting geometrical point of view. The geometrical argument is then extended to show that "mild" inhomogeneities of a transparent type have no effect on the mean area of a light beam.

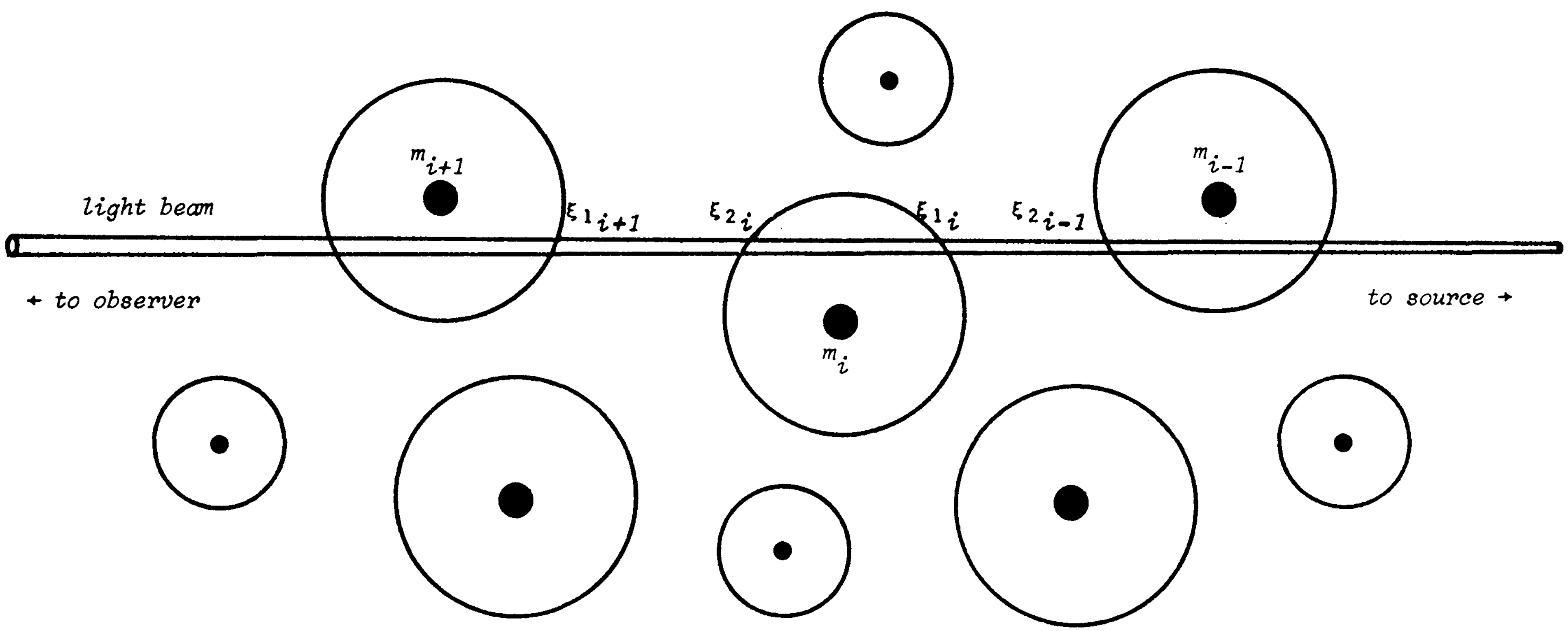


FIG. 1.—Spacelike section of a typical Swiss-cheese universe

Dyer & Roeder '72

THE DISTANCE-REDSHIFT RELATION FOR UNIVERSES WITH NO INTERGALACTIC MEDIUM

C. C. DYER* AND R. C. ROEDER†
Kitt Peak National Observatory,‡ Tucson, Arizona

Received 1972 April 19

ABSTRACT

The distance-redshift relation is derived for model universes in which there is negligible intergalactic matter and in which the line of sight to a distant object does not pass close to intervening galaxies. When fitted to observations, this relation yields a higher value of q_0 than does a homogeneous model.

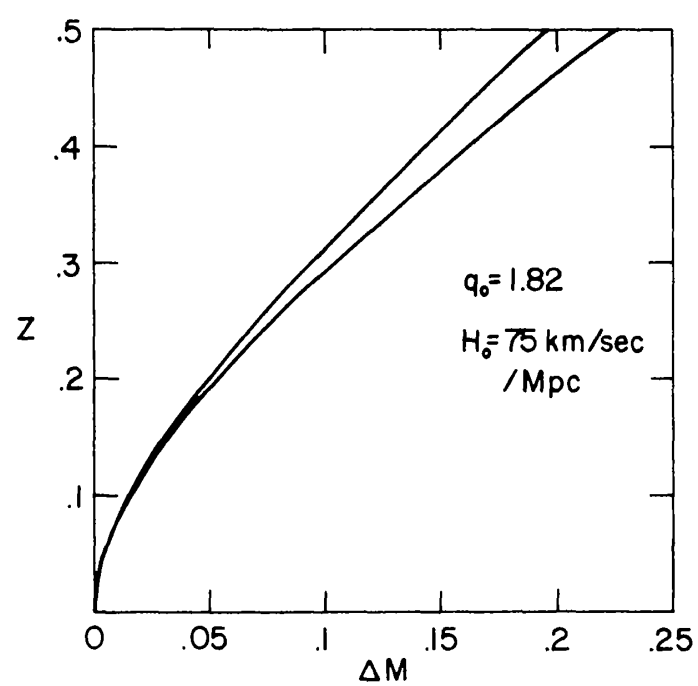
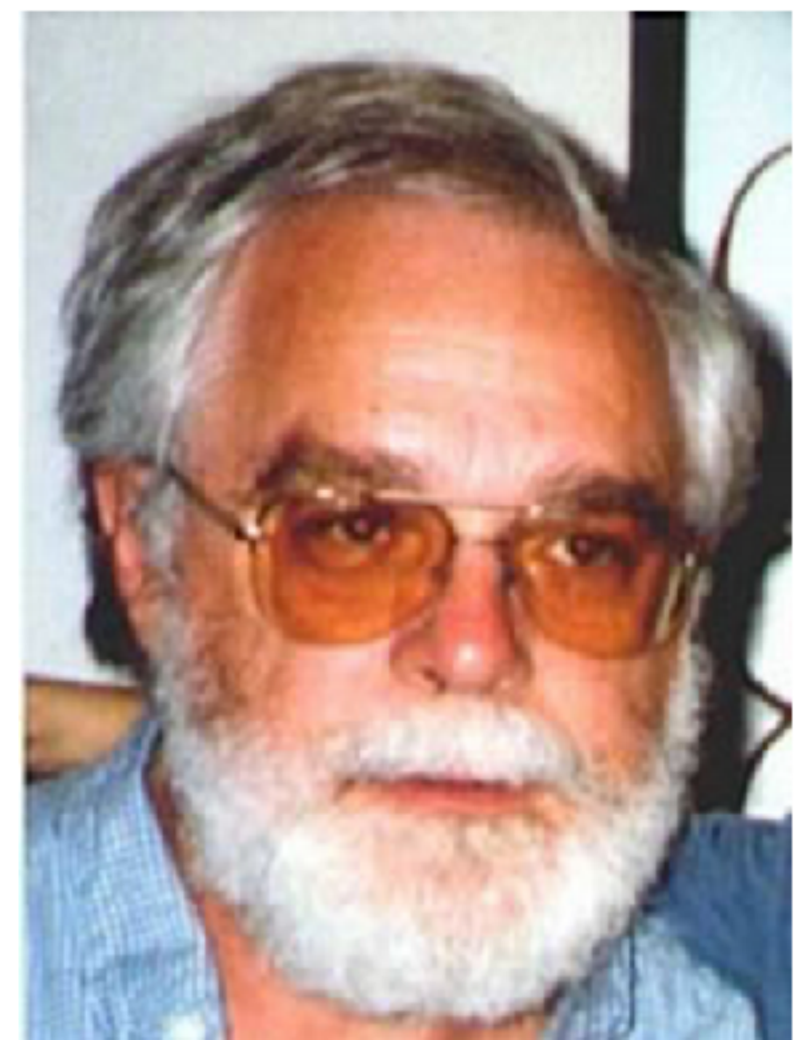


FIG. 1.—The dimming, relative to the homogeneous model, assuming that the beam passes far from any intervening galaxies (*lower curve*) and assuming that the beam passes no closer than 2 kpc to the center of galaxies similar to our own (*upper curve*).



Weinberg 1976 - no effect (flux conservation)

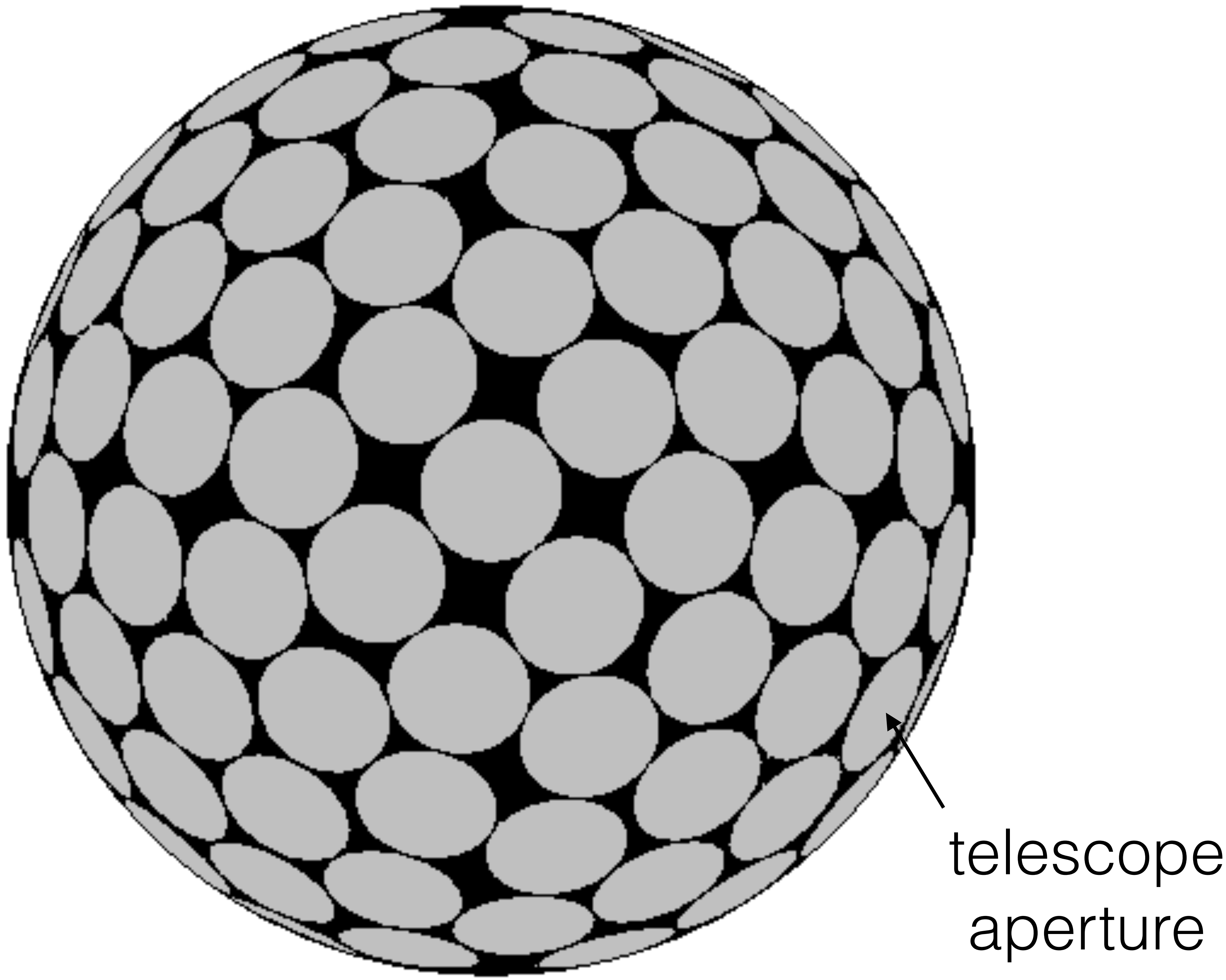
APPARENT LUMINOSITIES IN A LOCALLY INHOMOGENEOUS UNIVERSE

STEVEN WEINBERG
Center for Astrophysics, Harvard College Observatory and Smithsonian Astrophysical Observatory; and
Department of Physics, Harvard University
Received 1976 April 6; revised 1976 May 20

ABSTRACT

Apparent luminosities are considered in a locally inhomogeneous universe, with gravitational deflection by individual clumps of matter taken into account. It is shown that as long as the clump radii are sufficiently small, gravitational deflection by the clumps will produce the same average effect as would be produced if the mass were spread out homogeneously. The conventional formulae for luminosity distance as a function of redshift consequently remain valid, despite the presence of any local inhomogeneities of less than galactic dimensions. For clumps of galactic size, the validity of the conventional formulae depends on the selection procedure used and the redshift of the object studied.
Subject headings: cosmology — galaxies: redshifts — gravitation

Weinberg's argument (that <magnification> = 1)



Lensing and caustic effects on cosmological distances.

EBD '98

G. F. R. Ellis[1], B. A. Bassett[1,2], and P. K. S. Dunsby[1]

1 *Department of Applied Mathematics, University of Cape Town, Rondebosch 7700, Cape Town, South Africa.*

2 *International School for Advanced Studies, SISSA - ISAS Via Beirut 2-4, 34014, Trieste, Italy.*

December 4, 2013

Abstract

We consider the changes which occur in cosmological distances due to the combined effects of some null geodesics passing through low-density regions while others pass through lensing-induced caustics. This combination of effects increases observed areas corresponding to a given solid angle even when averaged over large angular scales, through the additive effect of increases on all scales, but particularly on micro-angular scales; however angular sizes will not be significantly effected on large angular scales (when caustics occur, area distances and angular-diameter distances no longer coincide). We compare our results with other works on lensing, which claim there is no such effect, and explain why the effect will indeed occur in the (realistic) situation where caustics due to lensing are significant. Whether or not the effect is significant for number counts depends on the associated angular scales and on the distribution of inhomogeneities in the universe. It could also possibly affect the spectrum of CBR anisotropies on small angular scales, indeed caustics can induce a non-Gaussian signature into the CMB at small scales and lead to stronger mixing of anisotropies than occurs in weak lensing.

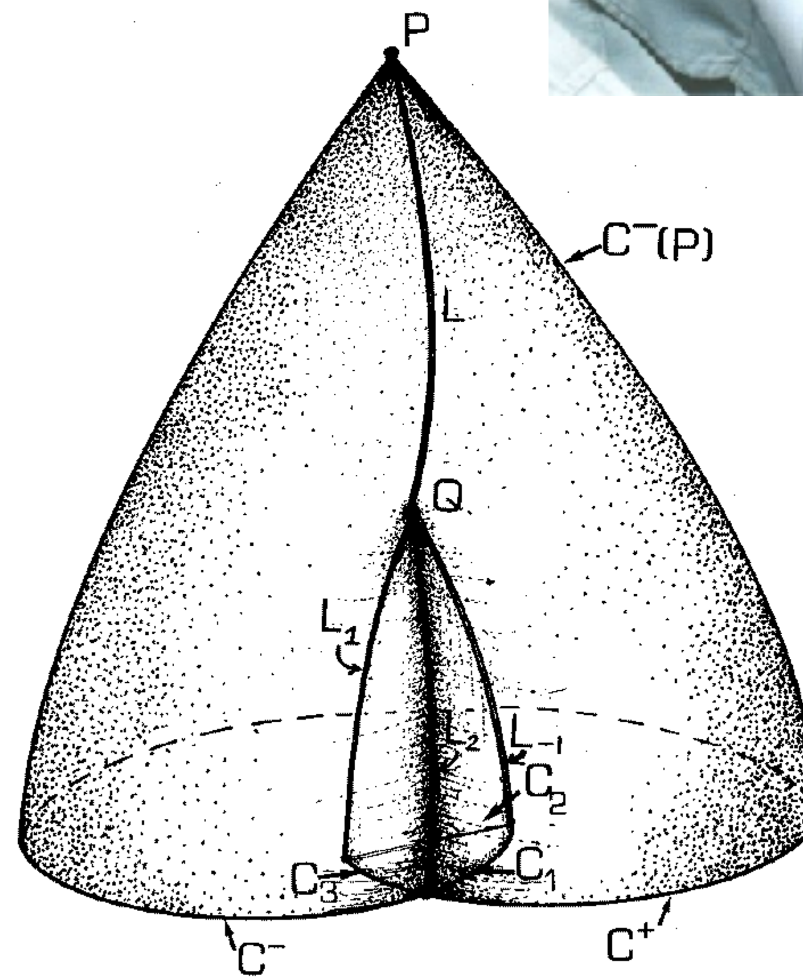
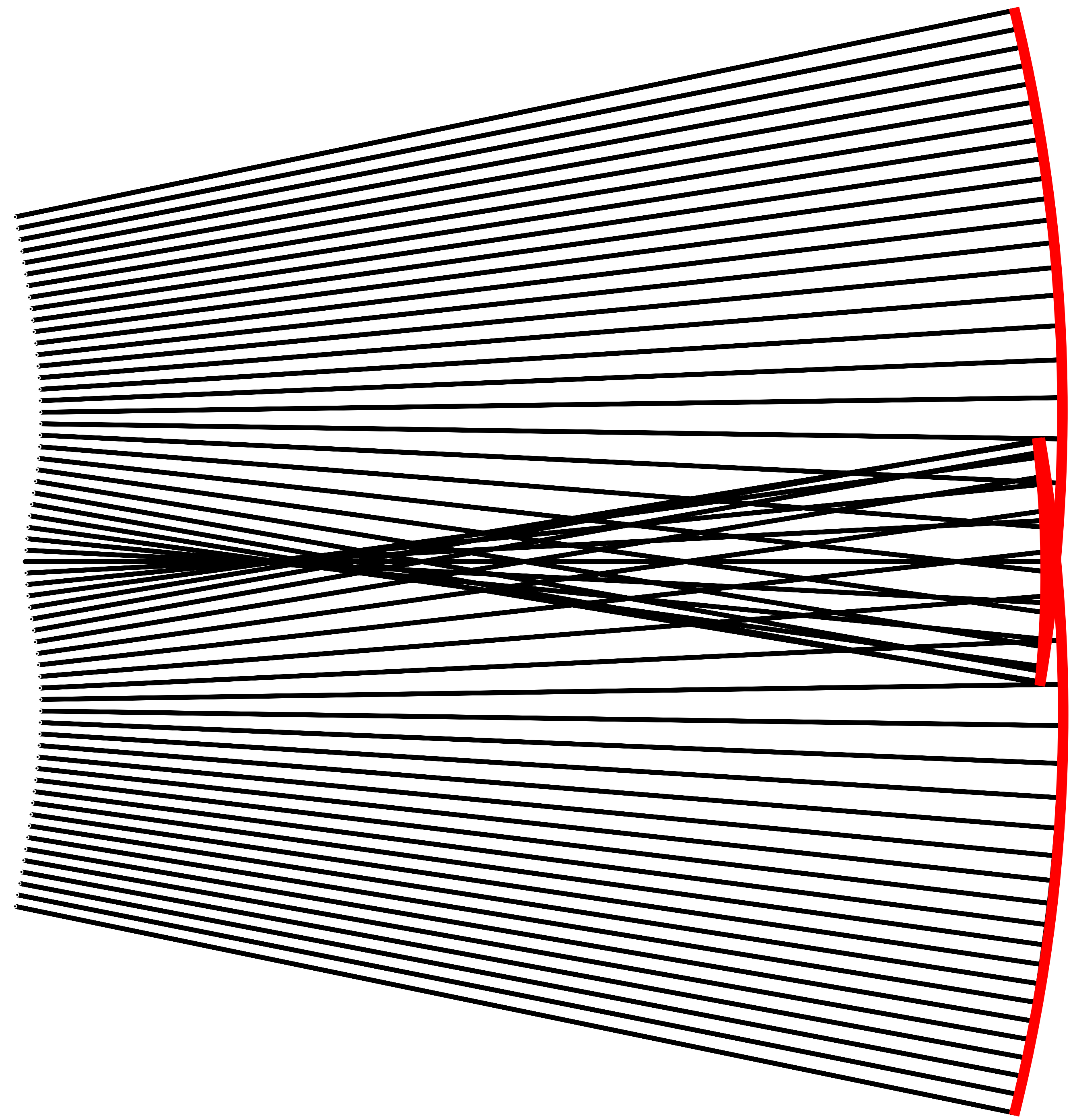


Figure 1: A lens L and resulting caustics on the past light cone $C^-(P)$ (2-dimensional section of the full light cone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point Q. The intersection of the past light cone with a surface of constant time defines exterior segments C^-, C^+ of the light cone together with interior segments C_1, C_2, C_3.

Ellis, Bassett & Dunsby '98 critique of Weinberg '76

- EDB98 make two points:
- Weinberg *assumes* that which is to be proven
  - we agree: W76 assumes that the surface of constant z around a source (or observer) is a sphere
- Small scale strong lensing causes the surface to be folded over on itself so total area greatly enhanced
  - quite possibly true
- Thus Weinberg's claim is disproved
  - we disagree: W76 still applies if multiple images are unresolved



Enter Schneider, Ehlers, Seitz etc... ('80s, '90s)

- Two consistent threads:
  - Lens equation:
    - at least one image is made **brighter**
  - Optical scalar equations (Sachs 1961):
    - -> *focusing theorem* (Seitz et al. 1994)
    - Things viewed through 'clumpiness' are further than they appear...

Seitz, Schneider & Ehlers (1994)

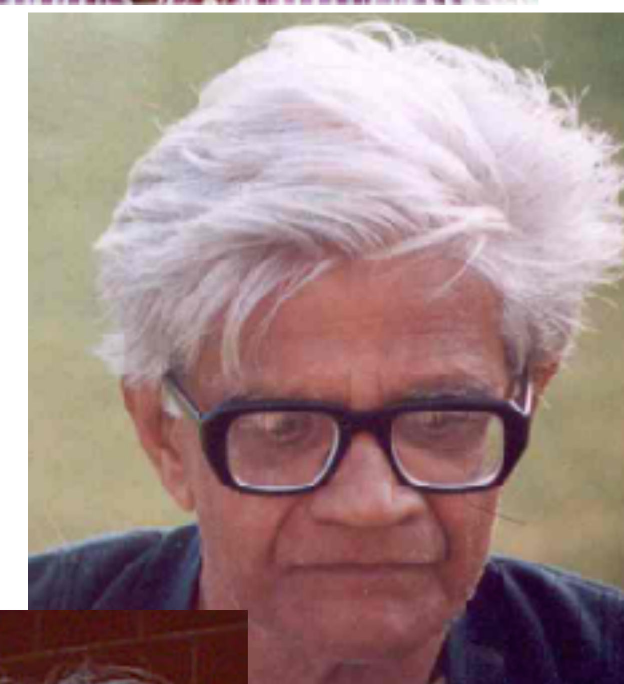
Finally, we have derived an equation for the size of a light beam in a clumpy universe, relative to the size of a beam which is unaffected by the matter inhomogeneities. If we require that this second-order differential equation contains only the contribution by matter clumps as source term, the independent variable is uniquely defined and agrees with the χ-function previously introduced [see SEF, eq. (4.68)] for other reasons. This relative focusing equation immediately yields the result that a light beam cannot be less focused than a reference beam which is unaffected by matter inhomogeneities, prior to the propagation through its first conjugate point. In other words, no source can appear fainter to the observer than in the case that there are no matter inhomogeneities close to the line-of-sight to this source, a result previously demonstrated for the case of one (Schneider 1984) and several (Paper I, Seitz & Schneider 1994) lens planes.

1992). Taking a somewhat different approach, Seitz, Schneider & Ehlers (1994) have used the optical scalars formalism of Sachs (1961) to show that the square root of the proper area of a narrow bundle of rays $D = \sqrt{A}$ obeys the 'focusing equation':

$$\boxed{\ddot{D}/D = -(R + \Sigma^2).} \tag{1}$$

Here $\ddot{D}$ is the second derivative of D with respect to affine distance along the bundle; $R = R_{\alpha\beta}k^\alpha k^\beta/2$ is the local Ricci focusing from matter in the beam, which for non-relativistic velocities is just proportional to the matter density; and Σ^2 is the squared rate of shear from the integrated effect of up-beam Weyl focusing – i.e. the tidal field of matter outside the beam. The resulting *focusing theorem* is that the RHS of (1) is non-positive, so that beams are always focused to smaller sizes, at least as compared to empty space-time,

More on the focusing theorem: $\ddot{D}/D = -(R + \Sigma^2)$



- Derived from Sachs '61 "optical scalars"
- from A.K. Raychaudhuri's equation
  - transport of expansion, vorticity and shear
- $R = R_{ab}k^a k^b$ local effect of matter in beam
- Σ^2 is the cumulative effect of matter *outside* the beam
  - Σ being the *rate* of image shearing
- Like cosmological acceleration equation:
  - $d^2a/dt^2 = - 4\pi G(\varrho+3P/c^2)a$
  - so Σ^2 here plays the role of pressure!
- Also recalls Hawking-Ellis singularity theorem
  - both terms are positive => focusing
- e.g. Narlikar (Introduction to Relativity):
  - "*Thus the normal tendency of matter*
  - *is to focus light rays*"

Narlikar on the focusing theorem

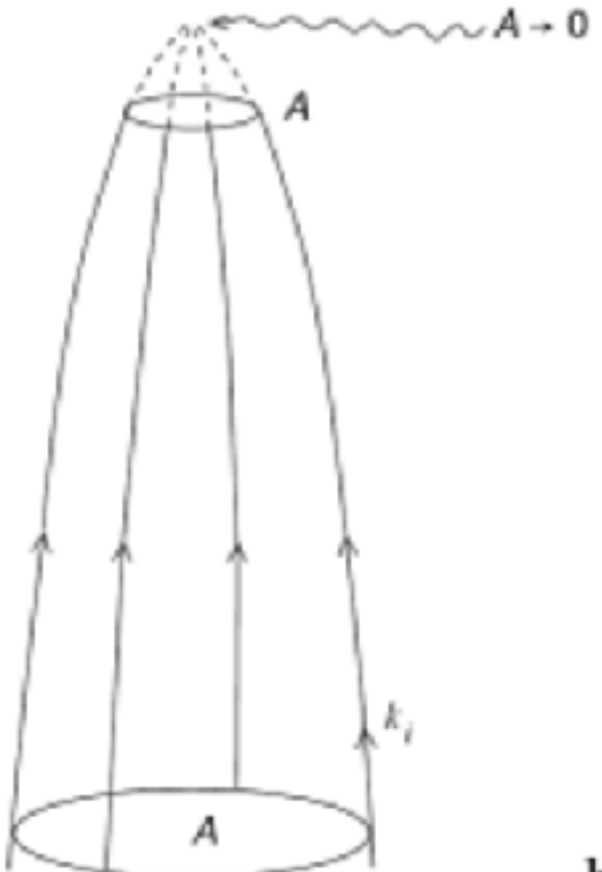


Fig. 18.3. The bundle of geodesics focusses in the future with its cross section A decreasing to zero. This effect was discussed in the context o spacetime singularity by A. K. Raychaudhuri.

The Raychaudhuri equation can be stated in a slightly different form as a *focussing theorem*. In this form it describes the effect of gravity on a bundle of null geodesics spanning a finite cross section. Denoting the cross section by A, we write the equation of the surface spanning the geodesics as $f =$ constant. Define the normal to the cross-sectional surface by $k_i = \partial f/\partial x^i$. Figure 18.3 shows the geometry of the bundle.

Using a calculation similar to that which led to the geodetic deviation equation in Chapter 5, we get the focussing equation as

$$\frac{1}{\sqrt{A}}\frac{\mathrm{d}^2\sqrt{A}}{\mathrm{d}\lambda^2} = \frac{1}{2}R_{im}k^i k^m - |\sigma|^2, \qquad (18.10)$$

Equation (18.10) is similar to the Raychaudhuri equation with $|\sigma|^2$ being the square of the magnitude of shear. With Einstein's equations, we can rewrite (18.10) as

$$\frac{1}{\sqrt{A}}\frac{\mathrm{d}^2\sqrt{A}}{\mathrm{d}\lambda^2} = -4\pi G\left(T_{im} - \frac{1}{2}g_{im}T\right)k^i k^m - |\sigma|^2. \qquad (18.12)$$

For dust we have $T_{im} = \rho u_i u_m$ and this condition is satisfied with the left-hand side equalling $\rho(u_i k^i)^2$. (Remember that k_i is a null vector, so $g_{im}k^i k^m = 0$.) Thus the normal tendency of matter is to focus light rays by gravity.

even more on the focusing theorem: $\ddot{D}/D = -(R + \Sigma^2)$

- What's going on? This seems to conflict with Weinberg!
- Schneider et al are <u>adding</u> lenses - not redistributing matter
  - Does this explain the apparent conflict with flux conservation?
- No. Let $D = D_0 + D_1 + ...$ take the average .... and linearise,
  - gives *averaged focusing theorem*

$$\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$$

- So there <u>is</u> a tendency for structure to focus beams
- decrease of distance - *qualitatively* as found by Clarkson et al. 2014
  - i.e. a big - and possibly even divergent - effect!
- So Weinberg was wrong?

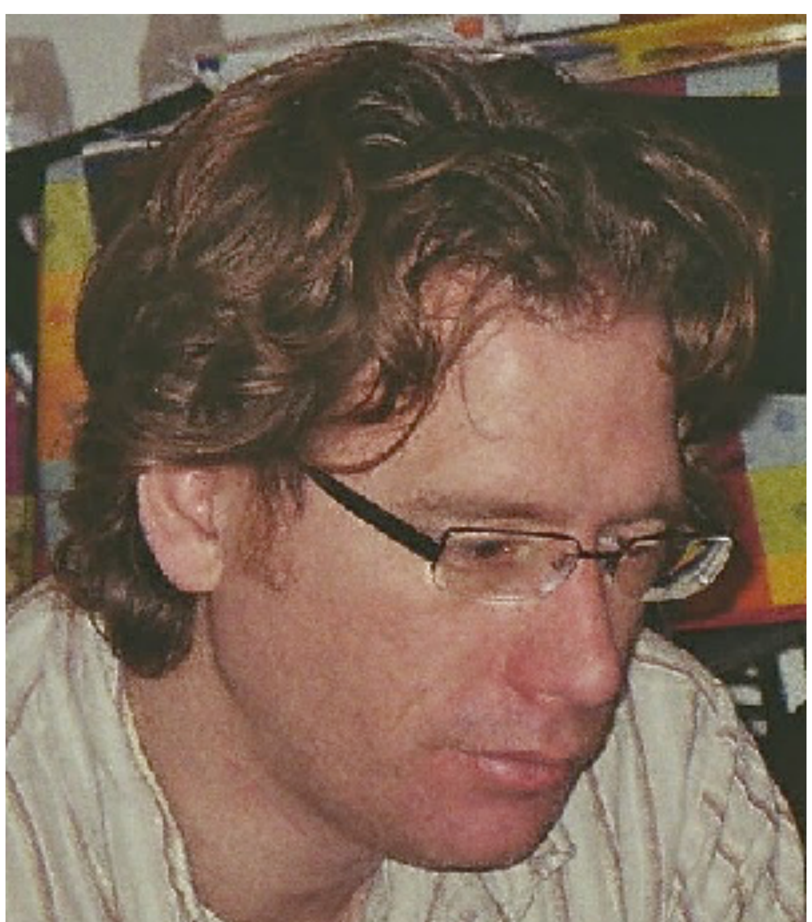
GRAVITATIONAL MAGNIFICATION OF THE COSMIC MICROWAVE BACKGROUND

R. BENTON METCALF AND JOSEPH SILK
Departments of Physics and Astronomy and Center for Particle Astrophysics, University of California, Berkeley, Berkeley, CA 84720
Received 1996 November 6; accepted 1997 June 12

ABSTRACT

Some aspects of gravitational lensing by large-scale structure are investigated. We show that lensing causes the damping tail of the cosmic microwave background (CMB) power spectrum to fall less rapidly with decreasing angular scale than previously expected. This is because of a transfer of power from larger to smaller angular scales, which produces a fractional change in power spectrum that increases rapidly beyond $\ell \sim 2000$. We also find that lensing produces a nonzero mean magnification of structures on surfaces of constant redshift if weighted by area on the sky. This is a result of the fact that light rays that are evenly distributed on the sky oversample overdense regions. However, this mean magnification has a negligible affect on the CMB power spectrum. A new expression for the lensed power spectrum is derived, and it is found that future precision observations of the high-ℓ tail of the power spectrum will need to take lensing into account when determining cosmological parameters.

Subject headings: cosmic microwave background — gravitational lensing



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Departments of Physics and Astronomy and Center for Particle Astrophysics, University of California, Berkeley, Berkeley, CA 84720
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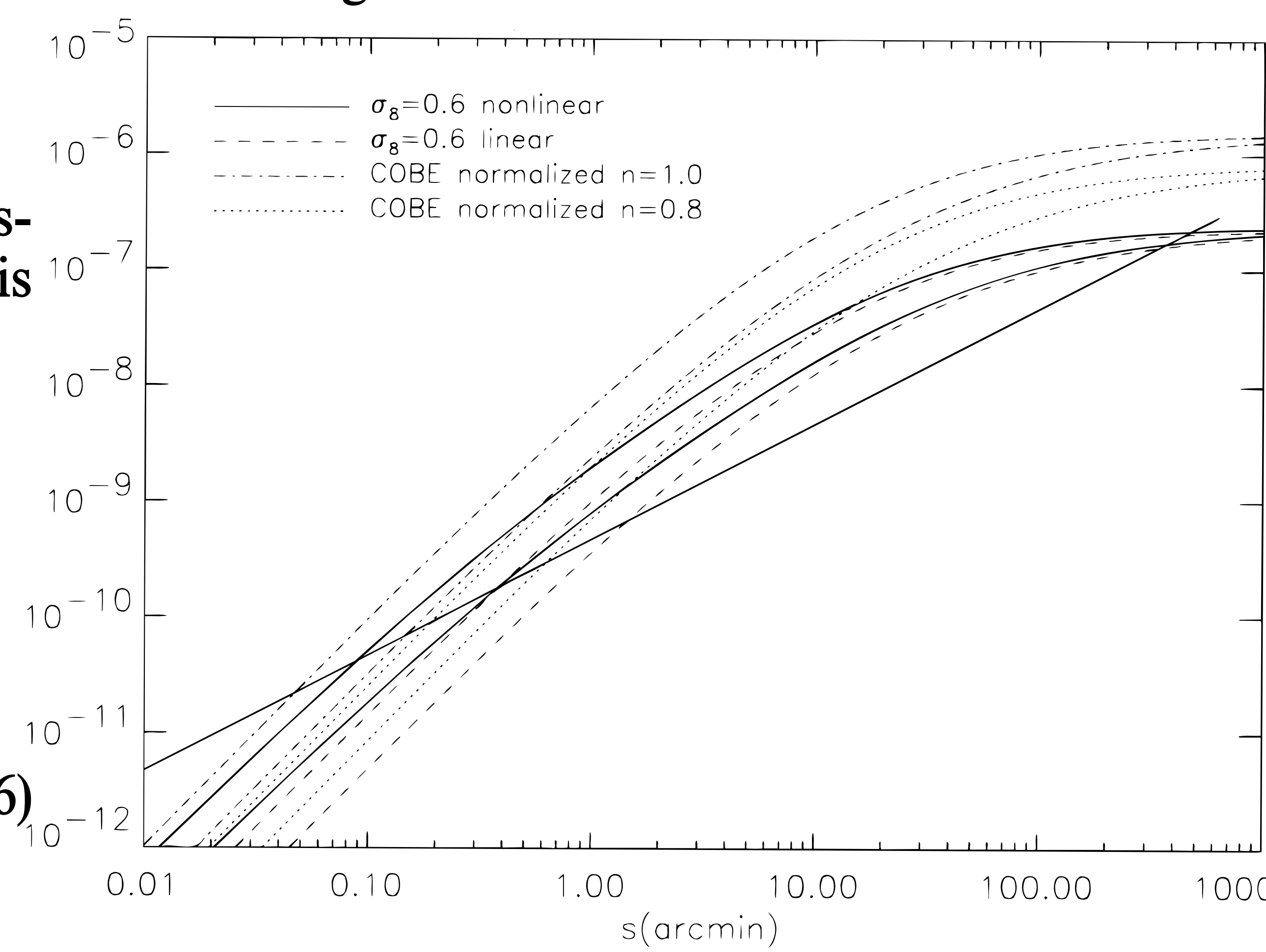
unperturbed path. The shear tensor that measures the distortion and expansion of an infinitesimally thin beam is then

$$\Phi_{ij} \equiv \frac{\partial \delta\theta_i}{\partial \theta_j} = \Phi^o_{ij} + \Delta\Phi_{ij} = \frac{-2}{g(r)} \int_0^r dr' g(r') g(r - r') \phi_{,ij}(r')$$
$$+ \frac{4}{g(r)} \int_0^r dr' \int_0^{r'} dr'' g(r - r') g(r' - r'')$$
$$\times \left[g(r') \phi_{,k}(r'') \phi_{,ijk}(r') + g(r'') \phi_{,jk}(r'') \phi_{,ik}(r') \right] . \qquad (6)$$



Kibble & Lieu (2005)

AVERAGE MAGNIFICATION EFFECT OF CLUMPING OF MATTER

T. W. B. Kibble
Blackett Laboratory, Imperial College, London SW7 2AZ, UK; kibble@imperial.ac.uk

AND

Richard Lieu
Department of Physics, University of Alabama at Huntsville, Huntsville, AL 35899; lieur@cspar.uah.edu
Received 2004 December 9; accepted 2005 June 20

ABSTRACT

The aim of this paper is to reexamine the question of the average magnification in a universe with some inhomogeneously distributed matter. We present an analytic proof, valid under rather general conditions, including clumps of any shape and size and strong lensing, that as long as the clumps are uncorrelated, the average "reciprocal" magnification (in one of several possible senses) is precisely the same as in a homogeneous universe with an equal mean density. From this result, we also show that a similar statement can be made about one definition of the average "direct" magnification. We discuss, in the context of observations of discrete and extended sources, the physical significance of the various different measures of magnification and the circumstances in which they are appropriate.

Subject headings: cosmology: miscellaneous — distance scale — galaxies: distances and redshifts — gravitational lensing

Kibble & Lieu 2005

There is another important distinction to be made. We may choose at random one of the sources at redshift z, or we may choose a random direction in the sky and look for sources there. These are not the same; the choices are differently weighted. If one part of the sky is more magnified, or at a closer angular-size distance, the corresponding area of the constant-z surface will be smaller, so fewer sources are likely to be found there. In other words, choosing a source at random will give on average a smaller magnification or larger angular-size distance.

- Weinberg: $\langle\mu\rangle = 1$ when averaged over *sources*
- Kibble & Lieu: $\langle 1/\mu\rangle = 1$ when averaged over *directions on the sky*
  - latter is more relevant for CMB observations
    - strictly only valid in weak lensing regime

Recap of historical review

- Zel'dovich '63 .... Feynman & Gunn .... Kantowski ... Dyer & Roeder
  - structure makes things look *fainter* on average
- Weinberg '76 - *no effect* for transparent lenses (flux conservation)
- Schneider et al. ('84..'94) (from Raychaudhuri, Sachs, Narlikar):
  - *magnification* and *focusing theorems*
  - structure makes things look *nearer* (i.e. *brighter*)- a big effect
- Ellis, Bassett & Dunsby '97 - critique of Weinberg '76
- Metcalf and Silk '97: negligible ($O(\theta^2) \sim 10^{-6}$) effect on the CMB
- Kibble & Lieu '05 - distinguished between *source* and *direction* averages
  - Weinberg: $\langle\mu\rangle = 1$ averaged over *sources* (or area on source sphere)
  - K+L: $\langle 1/\mu\rangle = 1$ when averaged over *directions* (as e.g. for CMB)
- Outstanding questions:
  - How do we make sense of these apparently conflicting results?
  - What is the relation to recent results from 2nd order Pert^{n} Theory?

Recent developments...

- Backreaction: "have cosmologists erred in failing to take into account the inherent non-linearity of Einstein's equations?"
  - cosmologists tend to do linear theory calculations
  - but Einstein's equations (metric <-> matter) are non-linear
  - averaging and non-linearity "do not commute"
  - so is *dark energy* a mirage?
- requires calculations in 2nd order perturbation theory (v. technical)
- now mostly accepted that effects are too small to explain acceleration
- but maybe there are still interesting percent level effects:
  - Clarkson, Ellis++ '12 - large ($O(\kappa^2)$) source magnification
  - Clarkson++ '14 - similarly large z-surface *area* increase
    - violates Weinberg's assumption
  - "backreaction" strikes back?
  - and the size of the effect is qualitatively consistent with expectation of the *focusing theorem* (Seitz, Schneider & Ehlers)

$$
\begin{aligned}
\hat{D}_A \;=\;& a(\chi_s)\chi_s \Big\{ 1 + \left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_o + \frac{1}{2}\Big[\left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_o^{(2)} - \left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\omega_{\parallel o} - \left(\frac{\mathcal{H}'}{\mathcal{H}^2} - 7\right)\Phi_o^2\Big] - \left(2 - \frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_s \\
&+\frac{1}{2}\Big[-\Psi_s^{(2)} - \left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_s + \frac{1}{2}\left(1 - \frac{2}{\mathcal{H}_s\chi_s}\right)\omega_{\parallel s} - \frac{1}{2}h_{\parallel s} - \left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{2}{\chi_s\mathcal{H}_s} - 7\right)\Phi_s^2 - 2\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_s\Phi_s' \\
&-2\chi_s\Phi_s\nabla_\parallel\Phi_s\Big] + \frac{1}{\mathcal{H}_s\chi_s}\nabla_\parallel v_o + \frac{1}{2}\Big[\frac{1}{\mathcal{H}_s\chi_s}\nabla_\parallel v_o^{(2)} - \frac{1}{\mathcal{H}_s\chi_s}v_{\parallel o} - \left(\frac{\mathcal{H}'}{\mathcal{H}^2} - 2\right)\nabla_\parallel v_o\nabla_\parallel v_o + \frac{1}{\chi_s\mathcal{H}_s}\nabla_{\perp i}v_o\nabla_\perp^i v_o\Big] \\
&+\left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\parallel v_s + \frac{1}{2}\Big[\left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\parallel v_s^{(2)} + \left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)v_{\parallel s} + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{2}{\chi_s\mathcal{H}_s} - 1\right)\nabla_\parallel v_s\nabla_\parallel v_s \\
&+2\chi\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_s\left(\nabla_\parallel v_s' - \nabla_\parallel^2 v_s\right) + \left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_{\perp i}v_s\nabla_\perp^i v_s - 2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{1}{\chi_s\mathcal{H}_s} + 4\right)\Phi_s\Phi_o \\
&+2\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_o\Phi_s' - 2\chi\Phi_o\nabla_\parallel\Phi_s - 2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_s\nabla_\parallel v_o - 2\chi_s\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_o\left(\nabla_\parallel v_s' - \nabla_\parallel^2 v_s\right) \\
&-2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{1}{2\chi_s\mathcal{H}_s} + \frac{11}{2}\right)\Phi_o\nabla_\parallel v_o - 2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{1}{\chi_s\mathcal{H}_s} + 3\right)\Phi_s\nabla_\parallel v_s + 2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} - 1\right)\Phi_o\nabla_\parallel v_s + 2\chi_s\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\Phi_s'\nabla_\parallel v_s \\
&+2\chi_s\nabla_\parallel\Phi_s\nabla_\parallel v_s - 2\chi_s\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\Phi_s(\nabla_\parallel v_s' - \nabla_\parallel^2 v_s) - 2\chi_s\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\Phi_s'\nabla_\parallel v_o - 2\chi_s\nabla_\parallel\Phi_s\nabla_\parallel v_o \\
&+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} + 2\right)\Phi_s\nabla_\parallel v_o + 2\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_o(\nabla_\parallel v_s' - \nabla_\parallel^2 v_s)\Big] + \frac{2}{\chi_s}\int_\chi\Phi + \frac{1}{2}\Big[\frac{1}{\chi_s}\int_\chi(\Phi^{(2)} + \Psi^{(2)}) \\
&-\frac{1}{\chi_s}\int_\chi\frac{(\chi - \chi_s)}{\chi}\omega_\parallel + \frac{1}{\chi_s}\int_\chi 3\frac{(\chi - \chi_s)}{\chi}h_\parallel\Big] - 2\left(1 - \frac{1}{\mathcal{H}_s\chi}\right)\int_\chi\Phi' + \frac{1}{2}\Big[- \left(1 - \frac{1}{\mathcal{H}_s\chi_s}\right)\left(\int_\chi\left(\Phi'_{(2)} + \Psi'_{(2)}\right)\right) \\
&-\int_\chi\left(1 - \frac{1}{\mathcal{H}_s\chi_s} + \frac{(2\chi - \chi_s)}{2\chi_s}\right)\omega_\parallel^{(2)\prime} + \int_\chi\left(1 - \frac{1}{\mathcal{H}_s\chi_s} - \frac{(2\chi - \chi_s)}{\chi_s}\right){h_\parallel}'\Big] + \int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi \\
&+\frac{1}{2}\Big[\frac{1}{2}\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2(\Phi^{(2)} + \Psi^{(2)}) - \frac{1}{2}\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\omega_\parallel - \frac{1}{2}\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2 h_\parallel\Big] + \frac{1}{2}\Big\{ - 2\Phi_o\Big[\left(1 + \frac{2}{\chi_s\mathcal{H}_s}\right)\frac{2}{\chi}\int_\chi\Phi \\
&+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{3}{\chi_s\mathcal{H}_s} - 4\right)\int_\chi\Phi' - \left(2 + \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi + \left(9 + \frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi + 4\int_\chi\frac{\chi}{\chi_s}\Phi' \\
&+4\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\Phi' - 4\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\Phi''\Big] + 2\Phi_s\Big[\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\frac{4}{\chi}\int_\chi\Phi + 2\left(\frac{\mathcal{H}'}{\mathcal{H}^2} - \frac{2}{\chi_s\mathcal{H}_s} + 2\right)\int_\chi\Phi' \\
&-\left(2 + \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi + 2\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big] + 2\nabla_\parallel v_o\Big[\frac{2}{\chi}\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi - 2\int_\chi\Phi' \\
&-\left(2 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi + 2\left(2 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big] - 2\nabla_\parallel v_s\Big[\frac{2}{\chi_s}\left(5 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi - 4\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_{\lambda_s}\Phi' \\
&-\left(2 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi + \left(3 - \frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big] + 4\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\perp v_s\int_\chi\nabla_\perp^i\Phi \\
&+4\Big[\frac{2}{\chi_s}\left(3 - \frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{1}{\chi_s\mathcal{H}_s} - 2\right)\int_\chi\Phi' - \left(2 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi + \left(3 - \frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big]\int_\chi\Phi' \\
&-4\chi\left(1 - \frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_s'\int_\chi\Phi' - 4\chi_s\nabla_\parallel\Phi_s\int_\chi\Phi' - 4\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\left[\chi\left(\nabla_\parallel v_s' - \nabla_\parallel^2 v_s\right)\right]\int_\chi\Phi' + 16\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\Phi\Phi' \\
&+\frac{2}{\chi_s}\int_\lambda\Phi^2 - 4\int_\chi\frac{1}{\chi\chi_s}\Phi\int_{\tilde\chi}\Phi(\tilde\chi) + 8\int_\chi\Phi'\int_{\tilde\chi}\Phi'(\tilde\chi) + 8\int_\chi\Phi\Phi' - 4\int_\chi\frac{(\chi - \chi_s)}{\chi\chi_s}\left[\frac{1}{\chi}\Phi\int_{\tilde\chi}\Phi(\tilde\chi) + \Phi^2 - 2\Phi'\int_{\tilde\chi}\Phi(\tilde\chi)\right] \\
&-4\int_\chi\frac{1}{\chi}\Phi\int_{\tilde\chi}\frac{(\tilde\chi - \chi)\tilde\chi}{\chi_s}\nabla_\perp^2\Phi(\tilde\chi) - 4\int_\chi\frac{(\chi - \chi_s)}{\chi^2\chi_s}\Phi\int_{\tilde\chi}(\tilde\chi - \chi)\tilde\chi\nabla_\perp^2\Phi(\tilde\chi) + 4\int_\chi\frac{(\chi - \chi_s)^2}{\chi_s}\Phi\nabla_\perp^2\Phi \\
&-4\int_\chi\Phi\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^2\Phi(\tilde\chi) + 14\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\Phi\nabla_\perp^2\Phi + 2\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\Phi\int_{\tilde\chi}\tilde\chi\nabla_\perp^2\Phi(\tilde\chi) + 4\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\nabla_\perp^2\Phi\int_{\tilde\chi}\Phi(\tilde\chi) \\
&+4\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\Phi'\int_{\tilde\chi}\tilde\chi\nabla_\perp^2\Phi(\tilde\chi) + 4\int_\chi\frac{(\chi - \chi_s)}{\chi}\Phi\int_{\tilde\chi}\frac{(\tilde\chi - \chi)\tilde\chi}{\chi}\nabla_\perp^2\Phi(\tilde\chi) - 8\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^i\Phi(\tilde\chi) \\
&-8\int_\chi\frac{\chi}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi) - \frac{8}{\chi_s}\int_\chi\int_{\tilde\chi}\nabla_{\perp i}\Phi(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi)\,, +12\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_{\perp i}\Phi\nabla_\perp^i\Phi \\
&-8\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\nabla_{\perp i}\Phi'\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi) - 8\left(1 - \frac{1}{\chi_s\mathcal{H}_s}\right)\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^i\Phi'(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi) \\
&-4\int_\chi\frac{(\chi - \chi_s)\chi}{\chi_s}\int_{\tilde\chi}\nabla_{\perp\langle i}\nabla_{\perp j\rangle}\Phi(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^{\langle i}\nabla_\perp^{j\rangle}\Phi(\tilde\chi) - 2\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\nabla_\perp^2\Phi\int_{\tilde\chi}(\tilde\chi - \chi)\tilde\chi\nabla_\perp^2\Phi(\tilde\chi) \\
&+8\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\int_{\tilde\chi}\nabla_{\perp i}\Phi(\tilde\chi)\int_{\tilde\chi}\frac{\tilde\chi^2}{\chi}\nabla_\perp^i\nabla^2\Phi(\tilde\chi) - 4\int_\chi\frac{(\chi - \chi_s)}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\frac{(\tilde\chi - \chi)\tilde\chi^2}{\chi}\nabla_\perp^i\nabla_\perp^2\Phi(\tilde\chi)\Big\}\Big\}\,. \qquad \text{(B1)}
\end{aligned}
$$




What is the distance to the CMB?
How relativistic corrections remove the tension with local H_0 measurements

Chris Clarkson[1], Obinna Umeh[2], Roy Maartens[2,3] and Ruth Durrer[4]

[1]*Astrophysics, Cosmology & Gravity Centre, and, Department of Mathematics & Applied Mathematics, University of Cape Town, Cape Town 7701, South Africa.*
[2]*Physics Department, University of the Western Cape, Cape Town 7535, South Africa*
[3]*Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom*
[4]*Département de Physique Théorique & Center for Astroparticle Physics, Université de Genève, Quai E. Ansermet 24, CH-1211 Genève 4, Switzerland.*

The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Clarkson et al. 2014

$$\langle \Delta \rangle \simeq \frac{3}{2} \left\langle \left(\frac{\delta d_A}{\chi_s} \right)^2 \right\rangle = \frac{3}{2} \left\langle \kappa^2 \right\rangle , \tag{1.5}$$

where κ is the usual linear lensing convergence. This is actually the leading contribution to the expected change to large distances. We prove this remarkably simple and important result in a variety of ways in several appendices. It implies that the total area of a sphere of constant redshift will be larger than in the background. Physically this is because a sphere about us in redshift space is not a sphere in real space — lensing implies that this 'sphere' becomes significantly crumpled in real space, and hence has a larger area. When interpreted

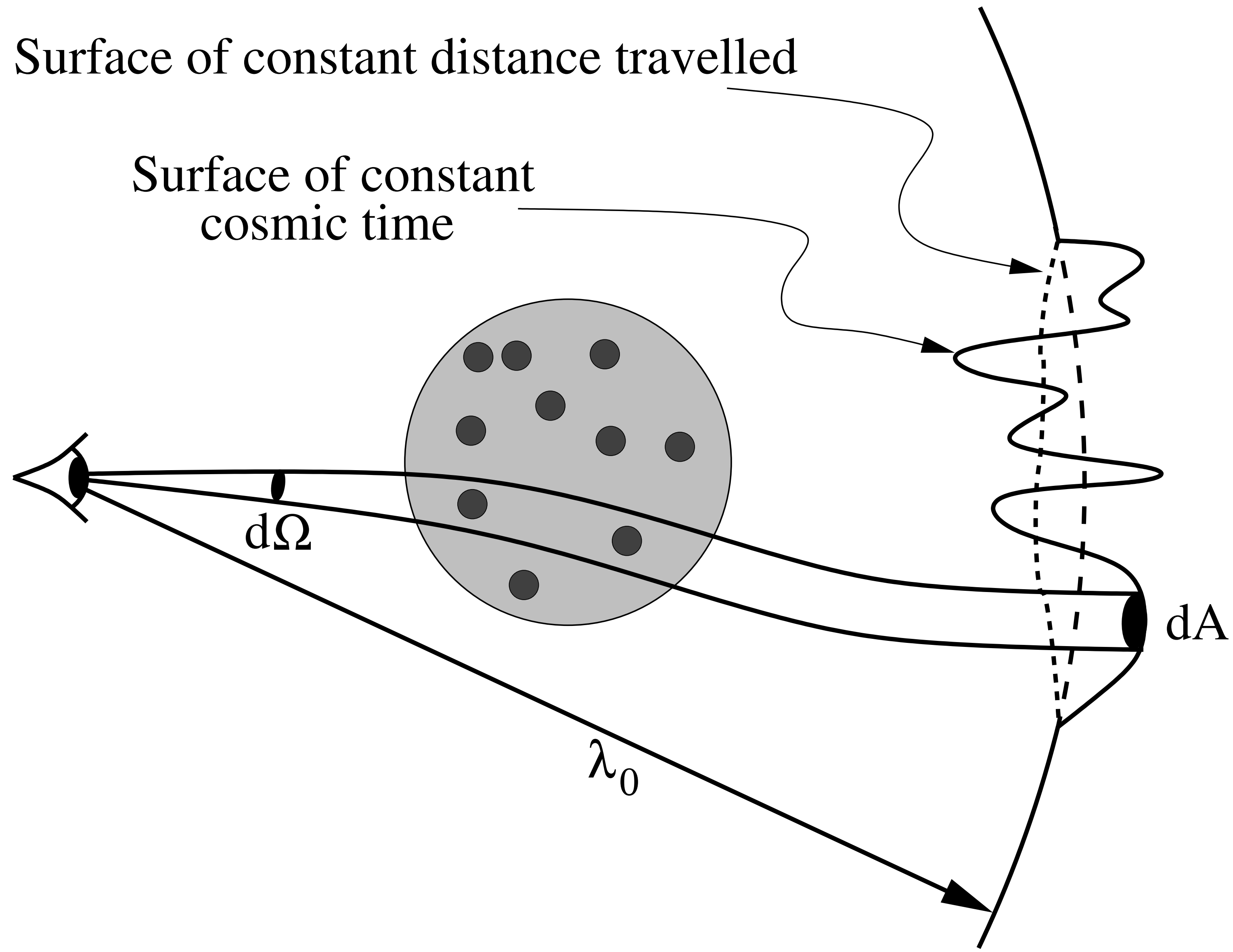
4 Conclusions

We have demonstrated an important overall shift in the distance redshift relation when the aggregate of all lensing events is considered, calculated by averaging over an ensemble of universes. This result is a consequence of flux conservation at second-order in perturbation theory. This is a purely relativistic effect with no Newtonian counterpart — and it is the first quantitative prediction for a significant change to the background cosmology when averaging over structure [21]. The extraordinary amplification of aggregated lensing comes mainly from the integrated lensing of structure on scales in the range 1–100 Mpc, although structure down to 10kpc scales contributes significantly. We have estimated the size of the effect using

NK + Peacock 2015

- Weinberg *assumes* that the area of a surface of constant redshift is unperturbed by lensing by intervening structures
  - same assumption is made by Kibble & Lieu
  - seems reasonable since *static* lenses do not affect redshift
  - and leads to conservation of e.g. source-averaged flux density
    - but not strictly true and breaks down at some level
- What *is* the change in the area of the constant-z surface (or cosmic photosphere) caused by structure?

KP2015: closing the loophole in Weinberg's argument



2 effects:

1) wiggly lines don't get as far as straight lines
2) wrinkly surface has more area than a smooth one

but both effects are ~(bending angle)^2 ~ 10^{-6}

What is the area of a wavy surface?

Or a lumpy sphere?

Key features of KP15 calculation of area of photosphere

- Calculations are rather technical, some key features are:
  - Weak field assumption:
    - we model the metric as weak field limit of GR
      - but we allow for non-rel motion of sources
        - these have negligible effects
      - similarly for gravitational waves
        - "photons can't surf a gravitational wave"
      - going beyond 1st order can be estimated and is tiny effect
    - *the problem is isomorphic to light propagation in "lumpy glass"*
  - Boundary conditions:
    - Perturbation theory calculations assume photosphere is constant z
    - Not true. It is more realistically a surface of constant cosmic time
    - Pert. theo. results may be qualitatively OK, but fail quantitatively
  - Final result for perturbation to the area of the photosphere is

$$\langle \Delta A \rangle / A_0 = \frac{1}{\lambda_0^2} \int_0^{\lambda_0} d\lambda \, (2\lambda(\lambda_0 - \lambda) + \lambda^2) J(\lambda).$$

where

$$J \equiv -8 \int_{-\infty}^{0} dy \, \xi'_\phi(y)/y = 2\pi \int k \, \Delta_\phi^2(k) \, d\ln k,$$

but $J = d\langle\theta^2\rangle/d\lambda$ and $J\lambda$ is on the order of 10^{-6}

NK + Peacock 2015 - 2nd point

- Perturbation to the *area* is on the order of the mean squared cumulative deflection angle
- This is a one-part-in-a-million effect
  - dominated by large-scale structure
- Relativistic perturbation theory, *focussing theorem* etc. give perturbation to the distance that is on the order of the mean squared convergence
  - much larger
  - dominated by small-scale structure (possibly divergent)
- All claims for large effects are *purely statistical effects*:
  - The mean flux magnification μ of a source is unity
    - so $\langle \Delta\mu \rangle_{source} = 0$
  - but μ is a fluctuating quantity
  - so any non-linear function of μ (e.g. $D/D_0 = 1 / \sqrt{\mu}$) will *not* average to unity

KP15: Statistical biases...

- Example: Source averaged distance bias:
  - $D/D_0 = \mu^{-1/2} = (1 + \Delta\mu)^{-1/2} = 1 - \Delta\mu / 2 + 3(\Delta\mu)^2/8 + \ldots$
  - so $\langle D/D_0 \rangle_{source} = 1 + 3\langle(\Delta\mu)^2\rangle/8 + \ldots = 1 + 3\langle\kappa^2\rangle/2 + \ldots$
- Similarly for source averaged mean inverse magnification
  - $\langle D^2/D_0^2 \rangle_{source} = 1 + 4 \langle\kappa^2\rangle + \ldots$
- *These are precisely the results for the mean perturbation to the distance and distance squared found by Clarkson et al. 2014*
- But e.g. the latter is not the perturbation to the constant z surface area
  - that would be the average over *directions* rather than over sources
- Similarly, Clarkson et al. 2012 claim mean source averaged flux magnification is $\langle\mu\rangle = 1 + \langle 3\kappa^2 + \gamma^2\rangle + \ldots = 1 + \langle 4\kappa^2\rangle + \ldots.$
  - but this is the *direction* averaged magnification
- *These come from non-commutativity of averaging and non-linearity*
  - $\langle f(x)\rangle$!= $f(\langle x\rangle)$ if x is a fluctuating quantity
  - and have nothing to do with the non-linearity of Einstein's equations

What about the "*focusing theorem*"? $\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$

- 2 lessons from foregoing:
  - 1) The theorem applies to a bundle of rays fired along a given direction
    - i.e. a *direction* - not *source*-averaged quantity
    - and paths to sources avoid over-densities
    - so care is needed in interpreting this
  - 2) D is a non-linear function of A
    - so, because A is a fluctuation quantity, we automatically expect a statistical bias in D
    - and the size of the effect is ~ $\langle \varkappa^2 \rangle$
- So is there a "normal tendency of matter to focus light rays"?
  - as inferred from the averaged focusing theorem
- or is this simply a statistical effect?

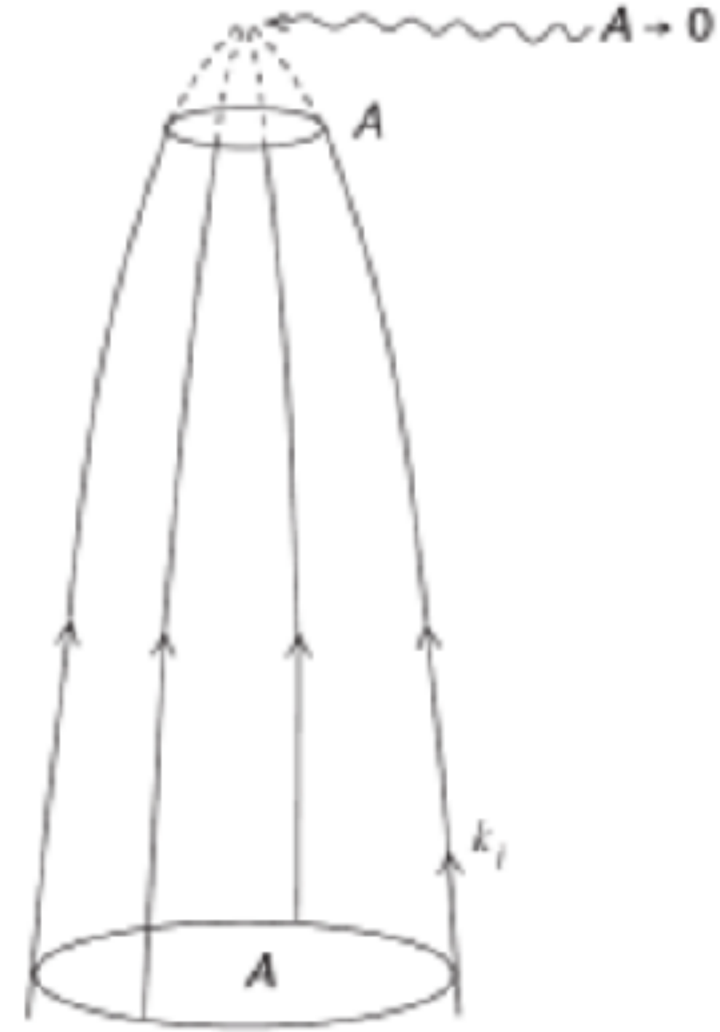


Fig. 18.3. The bundle of geodesics focusses in the future with its cross section *A* decreasing to zero. This effect was discussed in the context of spacetime singularity by A. K. Raychaudhuri.

KP15 on the "*focusing theorem*"? $\langle \ddot{D} \rangle / D_0 = -\langle \Sigma^2 \rangle < 0.$

- We have developed the optical scalar transport equations in a form appropriate when one wishes to specify the metric fluctuations as a stochastic random field (with zero mean for k=0 component)
  - interesting subtlety: one should *not* assume $\langle \delta R \rangle = 0$
  - in inflationary context, small scale space-time curvature fluctuations have to accommodate themselves within the (flat-space) boundary conditions imposed when the larger regions accelerate outside of horizon
- We have solved these to obtain the ensemble average of the perturbation to the *area* of a beam of specified solid angle fired off from the observer and propagating back to the source surface.
- We perform a double expansion, working to second order in δ(metric) and to lowest order in the inverse of "coherence scale"/Hubble scale
- Cancellation: Not just "Born level", but 1st "beyond Born" also
- We were only able to solve for the case where metric fluctuations are non-evolving (like in Einstein - de Sitter) but were able to obtain the "un-focusing theorem": $\langle \Delta A/A \rangle = - 2J\lambda/3 + ...$
  - this is consistent with the more general result (variable J) found by more straightforward approach.
- An exactly analogous calculation for $\langle \Delta D/D \rangle$ does not show cancellation and results in much larger ($O(\varkappa^2)$) result. *But just the statistical bias*. QED

Optical scalars (in weak-field GR or lumpy glass)

$\ddot{\mathbf{r}} = \boldsymbol{\nabla}_\perp \tilde{n}$ Geodesic equation

$$n = [(1 - 2\phi(\mathbf{r})/c^2)/(1 + 2\phi(\mathbf{r})/c^2)]^{1/2}$$

Optical *tensor* transport equation:

$$\dot{\mathbf{K}} = (\boldsymbol{\nabla}_\mathbf{x}\boldsymbol{\nabla}_\mathbf{x} - \mathbf{K}\partial_z)\tilde{n} - \boldsymbol{\nabla}_\mathbf{x}\tilde{n}\boldsymbol{\nabla}_\mathbf{x}\tilde{n} - \mathbf{K}\cdot\mathbf{K}$$

Optical scalar transport equations:

$$\dot{\theta} = \left(\frac{{\nabla_\perp}^2}{2} - \theta\partial_\lambda\right)\tilde{n} - |\boldsymbol{\nabla}_\perp\tilde{n}|^2/2 - \theta^2 - \Sigma^2$$

$$\dot{\boldsymbol{\Sigma}} = (\{\boldsymbol{\nabla}_\perp\boldsymbol{\nabla}_\perp\} - \boldsymbol{\Sigma}\partial_\lambda)\tilde{n} - \{\boldsymbol{\nabla}_\perp\tilde{n}\boldsymbol{\nabla}_\perp\tilde{n}\} - 2\theta\boldsymbol{\Sigma}$$

Solve for θ

The solution of $\dot{A}/2A = \theta(\lambda) = \lambda^{-1} + \Delta\theta(\lambda)$ is

$$A = \Omega\lambda^2 \exp\left(2\int_0^\lambda d\lambda'\,\Delta\theta(\lambda')\right)$$

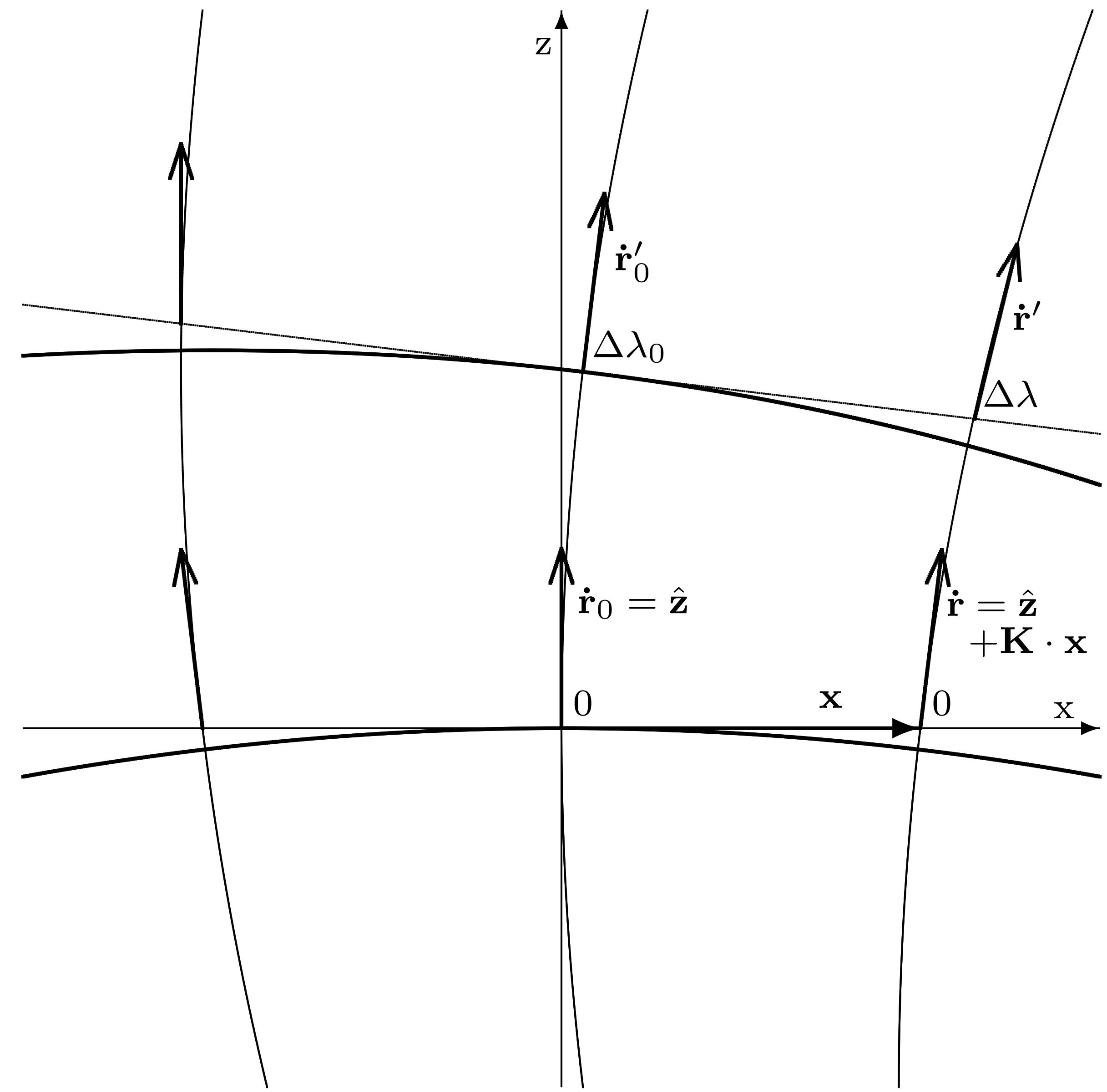


Figure D1. Illustration of a bundle of rays (thin curves) and associated wave-fronts (thick curves) and ray direction vectors $\dot{\mathbf{r}} = d\mathbf{r}/d\lambda$ (arrows). The base of each arrow is labelled by distance (physical for lumpy glass, background conformal for perturbed FRW) along the path. Close to the guiding ray the ray vectors will vary linearly with transverse displacement. The optical tensor $\mathbf{K}$ is the derivative of the ray direction with respect to coordinates $\mathbf{x}$ on the plane that is tangent to the wavefront at the location of the guiding ray. The optical tensor transport equation tells us how $\mathbf{K}$ evolves as the bundle propagates through any metric or refractive index fluctuations. Since rays are perpendicular to the

Part I: Concluding comments....

- The problem of how lensing by cosmic structure affects the mean *distance-redshift relation* (or the mean area of a surface of constant redshift) goes back for at least 50 years
    - Interesting problem....
    - many people played with it...
    - potentially important for "*precision cosmology*" with SN1a and CMB
- A conflict arose in the '80s between Weinberg's flux conservation argument and the contrary indications from the focussing theorem
- This remained unresolved and resurfaced recently in results of relativistic 2nd order perturbation theory.

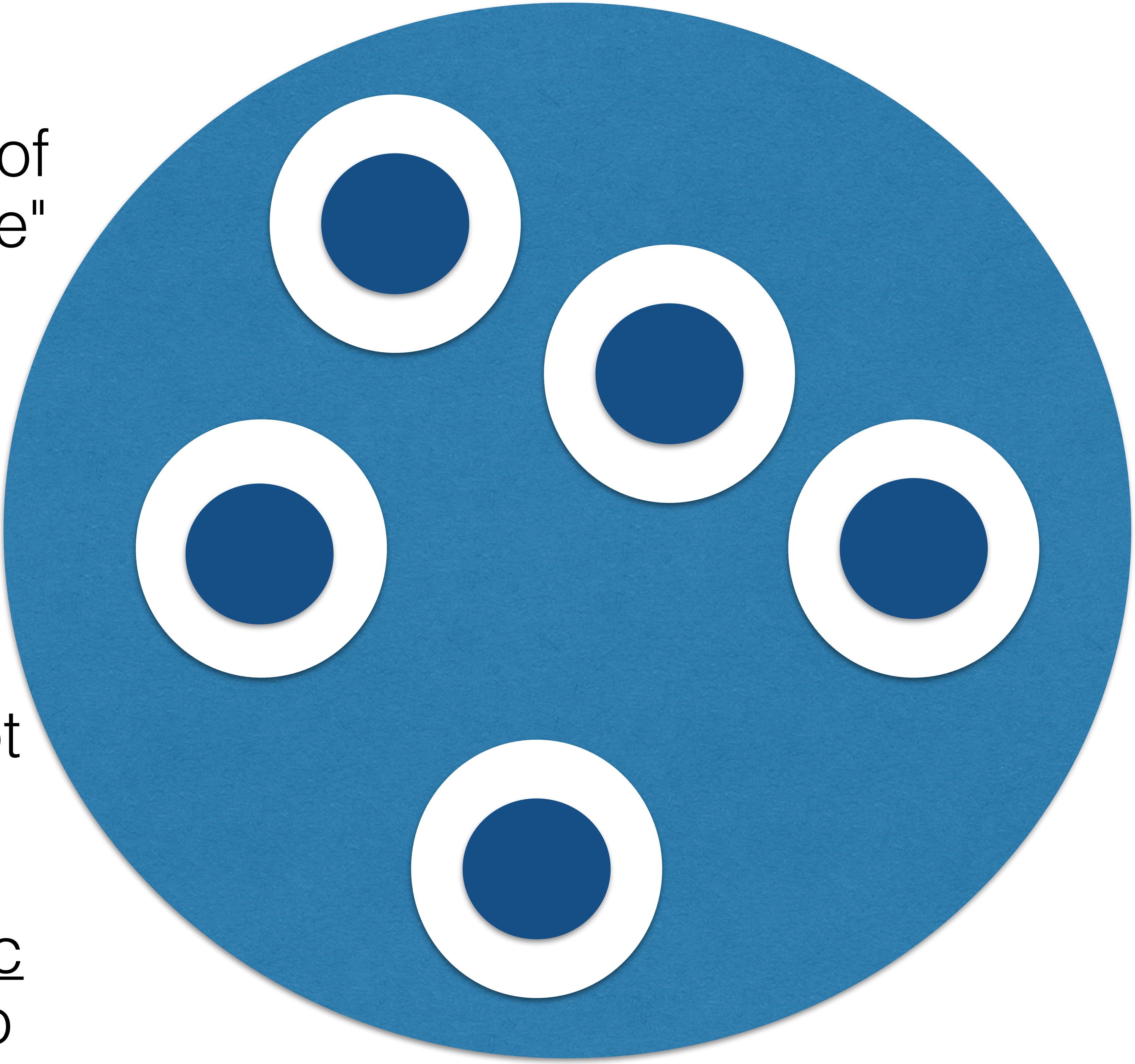
Part I: Concluding comments continued...

- John Peacock and I have reconciled the conflicts
- We support Weinberg:
  - lensing affects individual source flux densities in a random way
  - but averaged flux density of sources is *almost exactly unperturbed*
- and pay tribute to Kibble and Lieu
  - emphasised the distinction between source and direction averaging
- Our main results:
  - Relativistic studies have misinterpreted statistical biases.
  - there *is* a bias in the area of constant z or photosphere surfaces - but it is very, very small ~ 10^{-6}
  - we have shown that the celebrated "*focusing theorem*", despite its name, does not imply any intrinsic tendency for bundles of rays to be focused as they wend their wiggly way through the lumpy cosmos
- Implication: conventional methods for analysing the CMB & SN1a (mostly) are valid.
- ΛCDM lives to fight another day!

Part I:Backreaction issues

- Comments on Raychaudhuri:
  - Powerful tool, but dangerous
  - The 2nd order terms (shear^2 etc) depend on the “measure”
  - Equation for D = sqrt(A) -> focussing theorem
  - Equation for A -> non-focussing theorem
  - Same is true for time-like geodesics
    - Different “backreaction” terms (Buchert’s Qs) for different measures.
    - Need to carefully choose the appropriate measure (here A)
- Setting up the physical model:
  - For inflation need to model metric as background + perturbations
  - Different result if you model curvature

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- <u>proper mass</u> perturbation does not average to zero
- Need to model <u>metric</u> perturbations as zero mean process



2) Bias in H_0 from 2nd order pert^n theory

PHYSICAL REVIEW D **78,** 083531 (2008)

Scale dependence of cosmological backreaction

Nan Li* and Dominik J. Schwarz+

Fakultät für Physik, Universität Bielefeld, Universitätsstraße 25, D-33615 Bielefeld, Germany

(Received 2 November 2007; published 23 October 2008)

Because of the noncommutation of spatial averaging and temporal evolution, inhomogeneities and anisotropies (cosmic structures) influence the evolution of the averaged Universe via the cosmological backreaction mechanism. We study the backreaction effect as a function of averaging scale in a perturbative approach up to higher orders. We calculate the hierarchy of the critical scales, at which 10% effects show up from averaging at different orders. The dominant contribution comes from the averaged spatial curvature, observable up to scales of ~200 Mpc. The cosmic variance of the local Hubble rate is 10% (5%) for spherical regions of radius 40 (60) Mpc. We compare our result to the one from Newtonian cosmology and Hubble Space Telescope Key Project data.

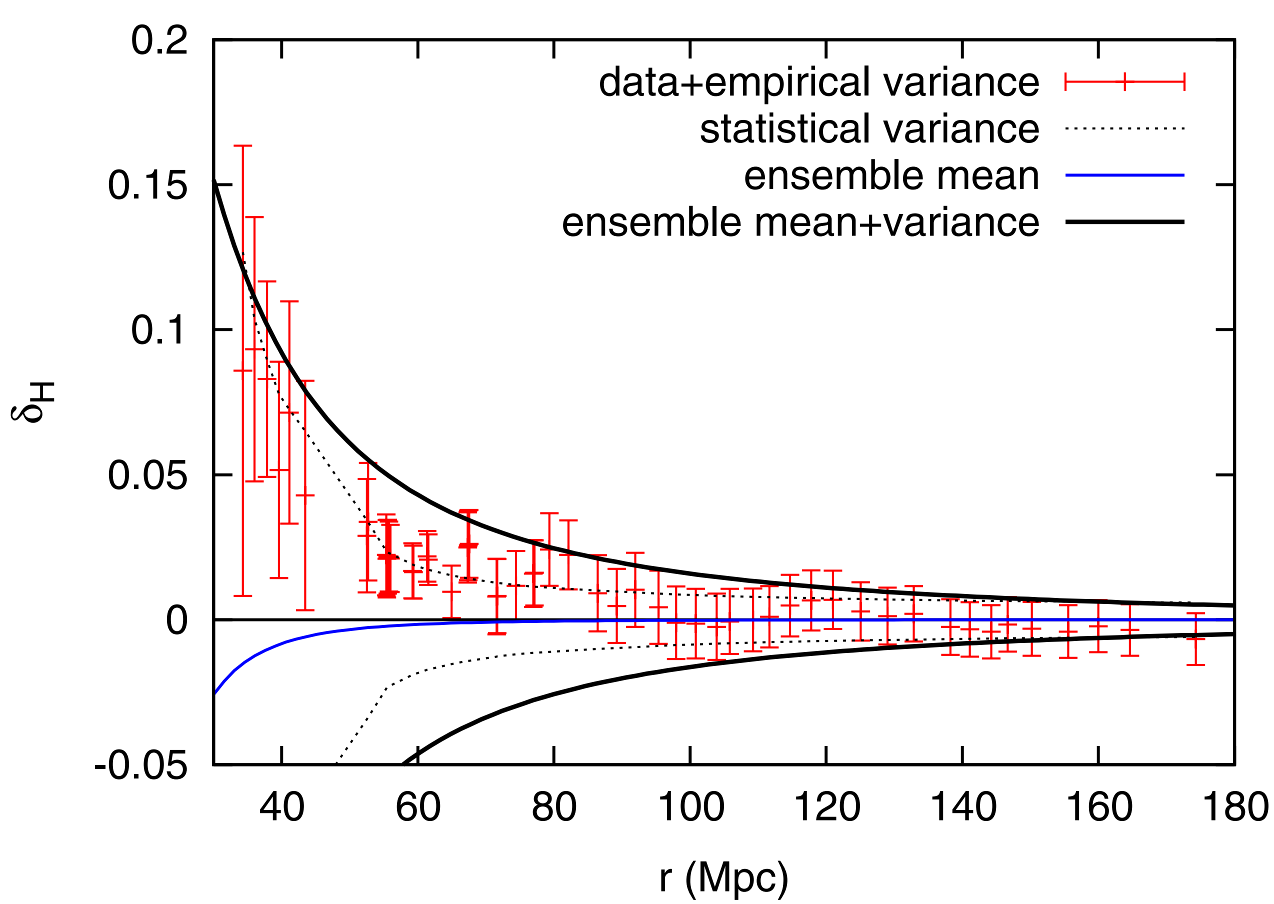


FIG. 2 (color online). Relative fluctuation of the Hubble rate from cosmological backreaction and its cosmic variance band (thick lines) compared to the empirical mean and variance of δ_H obtained from the HST Key Project data [5] as a function of averaging radius. The thin line shows the ensemble mean of δ_H. The band enclosed by the thick lines indicates the effect of the inhomogeneities ($\propto 1/r^2$), and the dashed lines are the effect from sampling with given measurement errors in an otherwise perfectly homogeneous Universe.

PHYSICAL REVIEW D **80,** 083525 (2009)

Influence of structure formation on the cosmic expansion

Chris Clarkson,[*] Kishore Ananda,[†] and Julien Larena[‡]

Cosmology & Gravity Group, Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa

(Received 4 August 2009; published 23 October 2009)

We investigate the effect that the average backreaction of structure formation has on the dynamics of the cosmological expansion, within the concordance model. Our approach in the Poisson gauge is fully consistent up to second order in a perturbative expansion about a flat Friedmann background, including a cosmological constant. We discuss the key length scales which are inherent in any averaging procedure of this kind. We identify an intrinsic homogeneity scale that arises from the averaging procedure, beyond which a residual offset remains in the expansion rate and deceleration parameter. In the case of the deceleration parameter, this can lead to a quite large increase in the value, and may therefore have important ramifications for dark energy measurements, even if the underlying nature of dark energy is a cosmological constant. We give the intrinsic variance that affects the value of the effective Hubble rate and deceleration parameter. These considerations serve to add extra intrinsic errors to our determination of the cosmological parameters, and, in particular, may render attempts to measure the Hubble constant to percent precision overly optimistic.

The Hubble rate in averaged cosmology

Obinna Umeh, Julien Larena and Chris Clarkson

Astrophysics, Cosmology and Gravity Center and
Department of Mathematics and Applied Mathematics, University of Cape Town,
Rondebosch 7701, South Africa

E-mail: umeobinna@gmail.com, julien.larena@gmail.com, chris.clarkson@uct.ac.za

Received November 23, 2010
Revised January 27, 2011
Accepted March 3, 2011
Published March 21, 2011

Abstract. The calculation of the averaged Hubble expansion rate in an averaged perturbed Friedmann-Lemaître-Robertson-Walker cosmology leads to small corrections to the background value of the expansion rate, which could be important for measuring the Hubble constant from local observations. It also predicts an intrinsic variance associated with the finite scale of any measurement of H_0, the Hubble rate today. Both the mean Hubble rate and its variance depend on both the definition of the Hubble rate and the spatial surface on which the average is performed. We quantitatively study different definitions of the averaged Hubble rate encountered in the literature by consistently calculating the backreaction effect at second order in perturbation theory, and compare the results. We employ for the first time a recently developed gauge-invariant definition of an averaged scalar. We also discuss the variance of the Hubble rate for the different definitions.

Keywords: cosmic flows, cosmological perturbation theory, dark energy theory

JCAP03(2011)029

The second-order luminosity-redshift relation in a generic inhomogeneous cosmology

Ido Ben-Dayan,[a,b] **Giovanni Marozzi,**[c,d] **Fabien Nugier**[e] **and Gabriele Veneziano**[c,f]

Published November 22, 2012

Abstract. After recalling a general non-perturbative expression for the luminosity-redshift relation holding in a recently proposed "geodesic light-cone" gauge, we show how it can be transformed to phenomenologically more convenient gauges in which cosmological perturbation theory is better understood. We present, in particular, the complete result on the luminosity-redshift relation in the Poisson gauge up to second order for a fairly generic perturbed cosmology, assuming that appreciable vector and tensor perturbations are only generated at second order. This relation provides a basic ingredient for the computation of the effects of stochastic inhomogeneities on precision dark-energy cosmology whose results we have anticipated in a recent letter. More generally, it can be used in connection with any physical information carried by light-like signals traveling along our past light-cone.

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Backreaction on the luminosity-redshift relation from gauge invariant light-cone averaging

I. Ben-Dayan,[a,b] M. Gasperini,[c,d] G. Marozzi,[e] F. Nugier[f] and G. Veneziano[e,g]

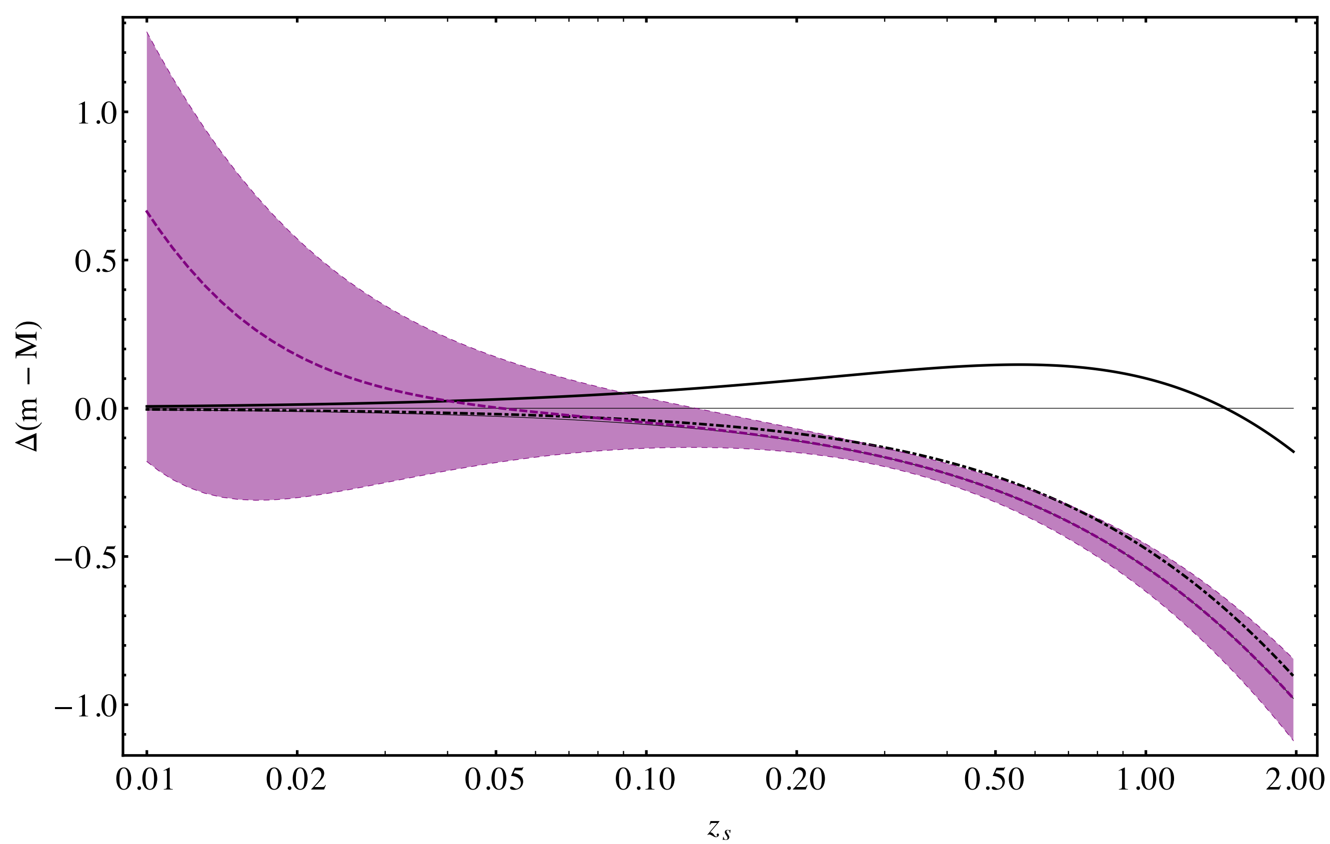


Figure 4. The distance-modulus difference of eq. (6.3) is plotted for a pure CDM model (thin line), for a CDM model including the contribution of IBR_2 (dashed blue line) plus/minus the dispersion (coloured region), and for a ΛCDM model with $\Omega_\Lambda = 0.73$ (thick line) and $\Omega_\Lambda = 0.1$ (dashed-dot thick line). We have used for all backreaction integrals the cut-off $k = 1\,\mathrm{Mpc}^{-1}$.

JCAP04(2012

Average and dispersion of the luminosity-redshift relation in the concordance model

I. Ben-Dayan,[a] M. Gasperini,[b,c] G. Marozzi,[d,e] F. Nugier[f] and G. Veneziano[d,g,h]

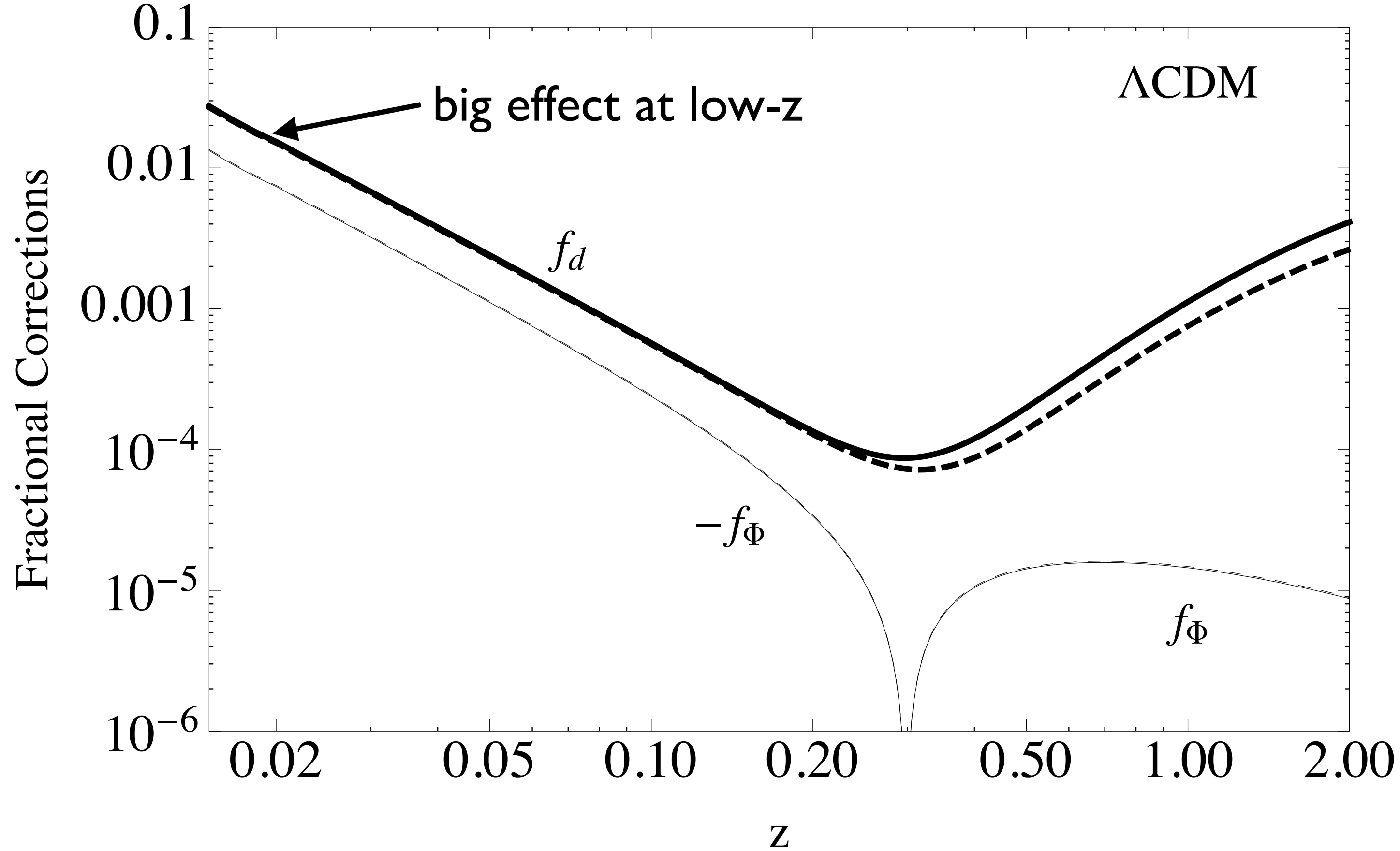


Figure 6. The fractional correction to the flux (f_Φ, thin curves) and to the luminosity distance (f_d, thick curves), for a perturbed ΛCDM model with $\Omega_{\Lambda 0} = 0.73$. Unlike in figure 3, we have taken into account the non-linear contributions to the power spectrum given by the HaloFit model of [17] (including baryons), and we have used the following cutoff values: $k_{UV} = 10h\,\mathrm{Mpc}^{-1}$ (dashed curves) and $k_{UV} = 30h\,\mathrm{Mpc}^{-1}$ (solid curves).

JCAP06(2013

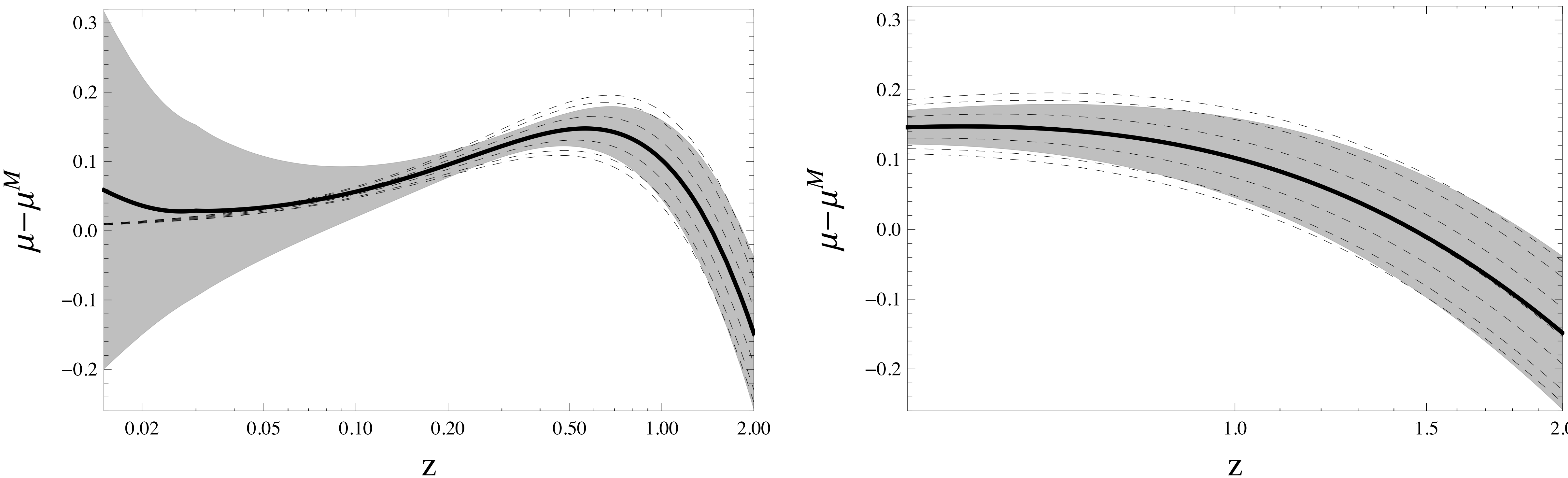


Figure 7. The averaged distance modulus $\overline{\langle\mu\rangle} - \mu^M$ of eq. (3.6) (thick solid curve), and its dispersion of eq. (3.9) (shaded region), for a perturbed ΛCDM model with $\Omega_{\Lambda 0} = 0.73$. Unlike figure 4, we have taken into account the non-linear contributions to the power spectrum given by the HaloFit model of [17] (including baryons), and used the cut-off $k_{UV} = 30h\,\mathrm{Mpc}^{-1}$. The averaged results are compared with the homogeneous values of μ predicted by unperturbed ΛCDM models with (from bottom to top) $\Omega_{\Lambda 0} = 0.68$, 0.69 0.71, 0.73, 0.75, 0.77, 0.78 (dashed curves). The right panel simply provides a zoom of the same curves, plotted in the smaller redshift range $0.5 \leq z \leq 2$.

Do Stochastic Inhomogeneities Affect Dark-Energy Precision Measurements?

I. Ben-Dayan,[1,2] M. Gasperini,[3,4] G. Marozzi,[5] F. Nugier,[6] and G. Veneziano[5,7]

The effect of a stochastic background of cosmological perturbations on the luminosity-redshift relation is computed to second order through a recently proposed covariant and gauge-invariant light-cone averaging procedure. The resulting expressions are free from both ultraviolet and infrared divergences, implying that such perturbations cannot mimic a sizable fraction of dark energy. Different averages are estimated and depend on the particular function of the luminosity distance being averaged. The energy flux being minimally affected by perturbations at large z is proposed as the best choice for precision estimates of dark-energy parameters. Nonetheless, its irreducible (stochastic) variance induces statistical errors on $\Omega_\Lambda(z)$ typically lying in the few-percent range.

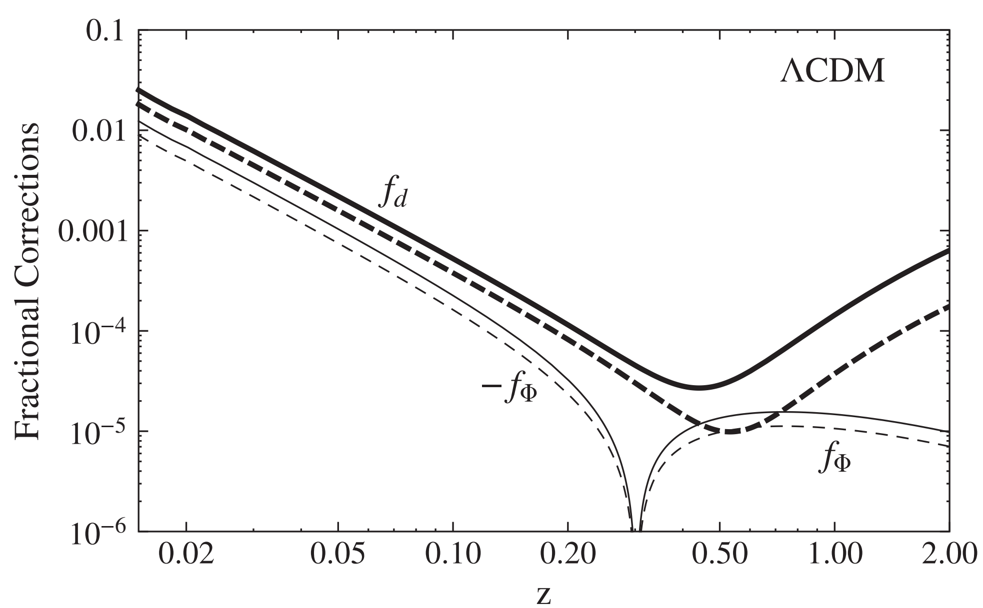


FIG. 2. The fractional correction to the flux f_Φ of Eq. (7) (thin curves) is compared with the fractional correction to the luminosity distance f_d of Eq. (13) (thick curves) for a ΛCDM model with $\Omega_\Lambda = 0.73$. We have used two different cutoff values: $k_{\rm UV} = 0.1\ {\rm Mpc}^{-1}$ (dashed curves) and $k_{\rm UV} = 1\ {\rm Mpc}^{-1}$ (solid curves). The spectrum is the same as that of Fig. 1 adapted to ΛCDM.

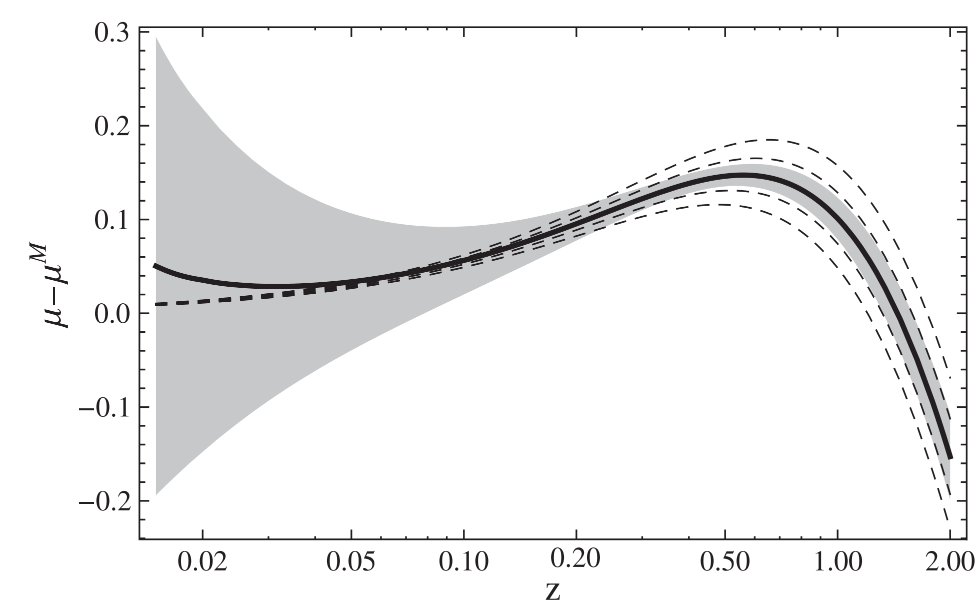


FIG. 3. The averaged distance modulus $\overline{\langle\mu\rangle} - \mu^M$ (thick solid curve) and its dispersion of Eq. (15) (shaded region) are computed for $\Omega_\Lambda = 0.73$ and compared with the homogeneous value for the unperturbed ΛCDM models with $\Omega_\Lambda = 0.69, 0.71, 0.73, 0.75, 0.77$ (dashed curves). We have used $k_{\rm UV} = 1\ {\rm Mpc}^{-1}$ and the same spectrum as in Fig. 2.

Value of H_0 in the Inhomogeneous Universe

Ido Ben-Dayan,[1] Ruth Durrer,[2] Giovanni Marozzi,[2] and Dominik J. Schwarz[3]
[1]*Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany*
[2]*Université de Genève, Département de Physique Théorique and CAP, 24 quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland*
[3]*Fakultät für Physik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany*
(Received 5 February 2014; published 6 June 2014)

Local measurements of the Hubble expansion rate are affected by structures like galaxy clusters or voids. Here we present a fully relativistic treatment of this effect, studying how clustering modifies the mean distance- (modulus-)redshift relation and its dispersion in a standard cold dark matter universe with a cosmological constant. The best estimates of the local expansion rate stem from supernova observations at small redshifts ($0.01 < z < 0.1$). It is interesting to compare these local measurements with global fits to data from cosmic microwave background anisotropies. In particular, we argue that cosmic variance (i.e., the effects of the local structure) is of the same order of magnitude as the current observational errors and must be taken into account in local measurements of the Hubble expansion rate.

$$\overline{\langle d_L^{-2}\rangle}(z) = (d_L^{\rm FL})^{-2}[1 + f_\Phi(z)], \tag{4}$$

where for $z \ll 1$,

$$f_\Phi(z) \simeq -\left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \overline{\langle(\vec{v}_s\cdot\vec{n})^2\rangle}. \tag{5}$$

would nearly double the effect in Eq. (5). The dominant peculiar velocity contribution at low redshift gives

$$f_\Phi(z) \simeq -\left(\frac{1}{\mathcal{H}(z)\Delta\eta}\right)^2 \frac{\tau^2(z)}{3}\int_{H_0}^{k_{\rm UV}} \frac{dk}{k} k^2 \mathcal{P}_\psi(k), \tag{6}$$

The brightness of supernovae is typically expressed in terms of the distance modulus μ. Because of the nonlinear function relating μ and Φ, one obtains different second order contributions,

$$\overline{\langle\mu\rangle} - \mu^{\rm FL} = -\frac{2.5}{\ln(10)}\left[f_\Phi - \frac{1}{2}\overline{\langle(\Phi_1/\Phi_0)^2\rangle}\right], \tag{7}$$

where at $z \ll 1$, we also find

$$\overline{\langle(\Phi_1/\Phi_0)^2\rangle} \simeq -4 f_\Phi. \tag{8}$$

Bias in H_0 from 2nd order pert^n theory

- Backreaction causes systematic bias in H measurement
  - very large effects on DM??
  - interesting bias in flux density, distance etc at low-z

Bias in H_0 from 2nd order pert^n theory

- Backreaction causes systematic bias in H measurement
  - very large effects on DM??
  - interesting bias in flux density, distance etc at low-z
- But isn't this just the residual "homogeneous Malmquist bias" in "inverse + type II" method?

Malmquist bias?

- Objects in a region of estimated distance space will have a distance that is biased
  - because of (large) uncertainty in distance
- But "Schechter's method" largely avoids that
  - don't measure velocity as a function of distance
  - do it the other way round
  - small scatter in distance for objects same redshift
- but not completely free from bias
  - analysed by Lynden-Bell '92 and Willick & Strauss '97

DLB '92

Eddington–Malmquist Bias, Streaming Motions, and the Distribution of Galaxies

D. Lynden-Bell

ABSTRACT Schechter's method of eliminating Malmquist bias is reviewed and presented in the context of the $D_n-\sigma$ relationship for elliptical galaxies. A Malmquist-like correction occurs which is dependent on the dispersion in the velocity field of galaxies; however, this correction does not increase with distance so it is much less important than the normal Malmquist bias that this method eliminates. The method is applied to a bulk flow model of the ellipticals and gives almost identical results to those found using the other reduction method which employs the Malmquist corrections. Ways of using the method to model the density and velocity fields out to 10,000 km/sec are briefly indicated.

is already small.

Solving for R, we obtain the value R_m at which the maximum occurs

$$R_m = \tfrac{1}{2}\left\{w + \sqrt{w^2 + 4\sigma_v^2\left[3 + \frac{d\ln(n/\sigma_v)}{d\ln r}\right][1+u'(v)]^{-2}}\right\}. \qquad (9.16)$$

Equations (9.16) and (9.15) constitute our solution for R_m. Notice that when $w >> \sigma_v$, then

$$R_m = w\left\{1 + \frac{\sigma_v^2}{w^2}\left[3 + \frac{d\ln(n/\sigma V)}{d\ln r}\right][1+u'(v)]^{-2}\right\} \qquad (9.17)$$

Willick et al 1997 (astro-ph vs ApJ)

2.2.2. Further discussion of the VELMOD *likelihood*

The physical meaning of the VELMOD likelihood expressions is clarified by considering them in a suitable limit. If we take σ_v to be "small," in a sense to be made precise below, the integrals in Eqs. (11) and (12) may be approximated using standard techniques. If in addition we neglect sample selection ($S = 1$) and density variations ($n(r) =$ constant), and assume that the redshift-distance relation is single-valued, we find for the forward relation:

10 = 2*5

$$P(m|\eta, cz) \simeq \frac{1}{\sqrt{2\pi}\sigma_e} \exp\left\{ -\frac{1}{2\sigma_e^2} \left(m - \left[M(\eta) + 5\log w + \frac{10}{\ln 10}\Delta_v^2 \right] \right)^2 \right\} , \qquad (15)$$

$$P(m\,|\,\eta,\, cz) \simeq \frac{1}{\sqrt{2\pi\sigma_e}} \exp\left[\!\!\left[-\frac{1}{2\sigma_e^2} \left\{ m - \left[M(\eta) + 5 \log w + 3 \times \frac{5}{\ln 10}\, \Delta_v^2 \right] \right\}^2 \right]\!\!\right] , \qquad (15)$$

We thank Marc Davis, Carlos Frenk, and Amos Yahil for extensive discussions of various aspects of this project, as well as the support of the entire Mark III team: David Burstein, Stéphane Courteau, and Sandra Faber. We also thank the referee, Alan Dressler, for an insightful report that improved the quality of the paper. J. A. W. and M. A. S. are grateful for the

- KH15: The "3" here comes from the standard formula for HMB.
- The right answer is 1.5
  - as found by the relativistic backreaction folks!

Why there is no Newtonian backreaction

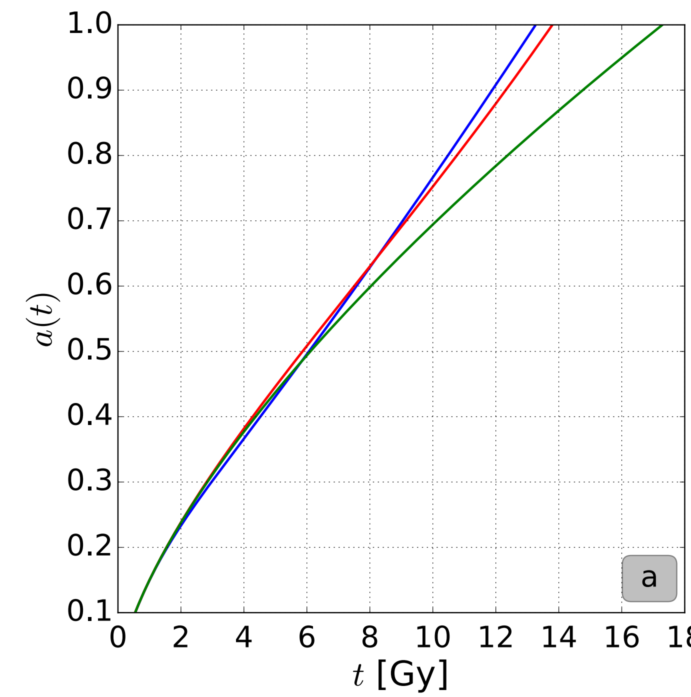
arXiv:1703.08809

Conventional Framework for Cosmological Dynamics

- Homogeneous background with scale factor a(t)
  - $a'' = -(4\pi/3)\, G\, \rho_b\, a$ ($' = d/dt$) Friedmann eq
- Structure (in e.g. N-body calc.) obeys
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi / a^2 = 0$ where
    - $\mathbf{x} = \mathbf{r} / a(t)$ are "conformal" coords, and
    - $\boldsymbol{\nabla}^2\phi = 4\pi\, G\, (\rho - \rho_b)\, a^2$
- No feedback (or "backreaction") of $\delta\rho$ on evolution of a(t)
- G.F.R. Ellis (1984...): is this legitimate?
  - explored by Buchert & Ehlers '97 plus many others

Racz et al 2017: Modified N-body calculations

- They assume the conventional <u>structure</u> equations:
    - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi\,/\,a^2 = 0$
    - $\nabla^2\phi = 4\pi\,G\,(\rho - \rho_b)\,a^2$
- but evolve a(t) according to $a \rightarrow a + a'\delta t$



    - with a' obtained by averaging <u>local expansion</u>: $\langle a'/a \rangle$ invoking "separate universe" approximation
    - "Strong backreaction" based on Newtonian physics
- Big effect: a(t) very similar to ΛCDM concordance model
    - "<u>concordance cosmology without dark energy</u>"

Racz et al. world view

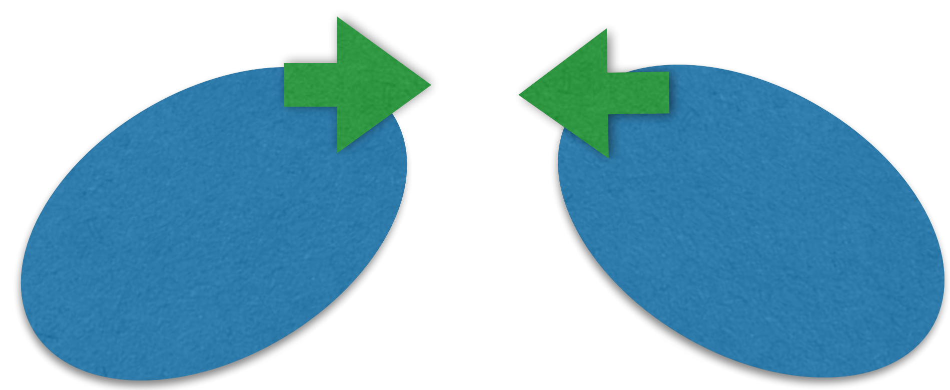
- *"N-body simulations integrate Newtonian dynamics with a changing GR metric that is calculated from averaged quantities"*
- *"changing GR metric"*: FRW metric: expansion factor a(t)
  - a(t) comes from strong-field GR physics
    - so we don't really understand it except in highly idealised (e.g. homogeneous) situations
    - hence legitimate to propose alternative ansatz?
  - a(t) - the "expansion of space" - affects the small-scale dynamics of structure

Is it legitimate to modify the Friedmann equation?

- Does emergence of structure really "backreact" on a(t)?
- Can address this in <u>Newtonian</u> gravity. Relevant as:
  - Accurate description of the local universe (v << c)
    - aside from effects from BHs
  - this is where we observe e.g. H_0 = 70 km/s/Mpc!
    - not H_0 ~ 35 km/s/Mpc expected w/o dark energy, Ω_k
  - At z = 0.1 relativistic corrections ~ 0.01
- If backreaction is important at > 1% level Newtonian analysis should show it

Why we might expect backreaction - tidal torques

- Neighbouring structures exert torques on each other
  - happens as structures reach $\delta \sim 1$
  - a non-linear (2nd order) effect
  - purely Newtonian
  - explains spin of galaxies
- can this affect expansion?
  - it does in the local group
  - do internal degrees of freedom couple to (i.e. exchange energy with) universal expansion



Inhomogeneous Newtonian cosmology

- Lay down particles on a uniform grid in a big uniformly expanding sphere ($\mathbf{v} = H\mathbf{r}$)
- Perturb the particles off the grid $\mathbf{r} \to \mathbf{r} + \delta\mathbf{r}$
  - plus related velocity perturbations to generate "growing mode" of structure
- $\mathbf{g}(\mathbf{r})$ can be decomposed into:
  - homogenous field sourced by mean density ρ
  - inhomogeneous field sourced by $\delta\rho$ (little dipoles)
- equations of motions $\mathbf{r}'' = \mathbf{g}$ can be re-scaled
  - gives the equations that are solved in N-body codes

Newtonian gravity in re-scaled coordinates

N-particles of mass m: $$\ddot{\mathbf{r}}_i = Gm\sum_{j\neq i}\frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}.$$

With **r** = a(t) **x** for arbitrary a(t)

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i = \frac{Gm}{a^3}\sum_{j\neq i}\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^3} - \frac{\ddot{a}}{a}\mathbf{x}_i.$$

initial conditions: $\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$

Defining $n(\mathbf{x}) \equiv \sum_i \delta(\mathbf{x}-\mathbf{x}_i)$ and $\delta n \equiv n - \overline{n}$

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3}\int d^3x\ \delta n(\mathbf{x})\frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^3}$$
$$= -\left(\frac{\ddot{a}}{a} + \frac{4\pi Gm\overline{n}}{3a^3}\right)\mathbf{x}_i.$$

Exactly equivalent to the usual equations in **r**-coords

Newtonian cosmology:

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3}\int d^3x\ \delta n(\mathbf{x})\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3} = -\left(\frac{\ddot{a}}{a} + \frac{4\pi G m \overline{n}}{3a^3}\right)\mathbf{x}_i.$$

with ICs

$$\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$$

- 3N equations for N particles
  - <u>there is no extra equation of motion for a(t)</u>
- But we may <u>choose</u> a(t) to obey Friedmann equation
  - an "auxiliary relation"
- Gives conventional expansion + structure equations
  - <u>a(t) suffers no backreaction from structure emergence</u>
  - a(t) is just a "book-keeping" factor - no physical effect

Part 1: summary/conclusions

- A different perspective on the conventional equations for structure growth (Dmietriev & Zel'dovich '63)
  - fully non-linear & exact (but Newtonian) description
- a(t) is arbitrary, but extra terms appear in equations of motion if a(t) does not obey Friedmann's equation
  - physical quantities invariant under choice of a(t)
- No coupling of expansion to internal structure via tidal torques
  - can also be understood from scaling with radius/mass

Relation to Buchert & Ehlers '97 "kinematic BR"

- Matter modelled as pressure-free Newtonian fluid
  - unrealistic, but maybe a useful "toy model"
- Consider a specific volume $V = a^3$ containing mass M
- Raychaudhuri equation (expansion θ, vorticity ω, shear σ)
  - $a''/a + (4\pi/3)\, GM/a^3 = Q$
    - with $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2-\sigma^2\rangle$
    - 2nd order - no linear effect!
- Naively a big effect (individual terms in $Q \sim G\rho$)
  - but...

Buchert & Ehlers '97

- "<u>Generalised Friedmann equation</u>": $a''/a + GM/a^3 = Q$
  - $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2-\sigma^2\rangle$
  - Q=0 is *"conspiracy assumption"*
- But *"Q is a divergence"*: $Q = V^{-1}\int \mathbf{dA.}(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u})$
  - so no global effect for periodic BCs - *"by construction"*
- No surprise that $a''/a \neq -GM/a^3$ for an individual region
  - fluctuations affect acceleration a'' and M
  - but <u>local</u>, not "backreaction of $\delta\rho$ on global expansion"
- If $\langle Q\rangle_{V\to\infty}$!= 0 would imply a conflict - this is <u>not</u> the case

Do B&E claim Newtonian backreaction?

- Q = 0 requires *"conspiracy"* - but *"the average motion may be approximately given by the Friedmann equation on a scale which is larger than the largest existing inhomogeneities"*

- Later works: E.g. Buchert & Rasanen 2011 review

  - *"..linear theory ... effect vanishes by construction ... in Newtonian ... true also in non-perturbative regime"*

  - *"When we impose periodic BCs .... Q is strictly zero"*

  - but *"If backreaction is substantial then current Newtonian simulations (and analytic studies) are inapplicable".*

- So the absence of backreaction is a consequence of assuming (falsely, one presumes) periodicity.

- How big is Q in reality?

How large is $Q = (3\, a''/a + 4\pi GM/a^3)$?

- $Q = Q_1 + Q_2 = V^{-1}\int \mathbf{dA.}(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}) - (3/2V^2)(\int \mathbf{dA.u})^2$
  - **u** is peculiar velocity wrt global H
- If structure is a stat. homog. and isotropic random process (i.e. random vector field)
  - $\langle\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}\rangle_{\text{ensemble}} = 0$ (Monin and Iaglom, 1975)
  - so Q_1 is pure fluctuation
  - $|Q_1| \sim \langle u^2\rangle/r^2$ independent of coherence length λ
- Second term is systematic: $Q_2 \sim \langle u^2\rangle\, \lambda^2 / r^4$
- Both are very small ($\ll H^2$) for large V

Is there relativistic backreaction?

- Claims: *"GR backreaction"* is non-zero - and large
- But local universe should be accurately Newtonian
  - errors ~ $v^2/c^2 \rightarrow$ ~1% accuracy within $z = 0.1$
  - and that's where we measure H_0
  - so very hard to believe there are >> 1% effects
- Q: Are there even very small effects on expansion history coming from non-relativistic effects?

Is there *relativistic* backreaction?

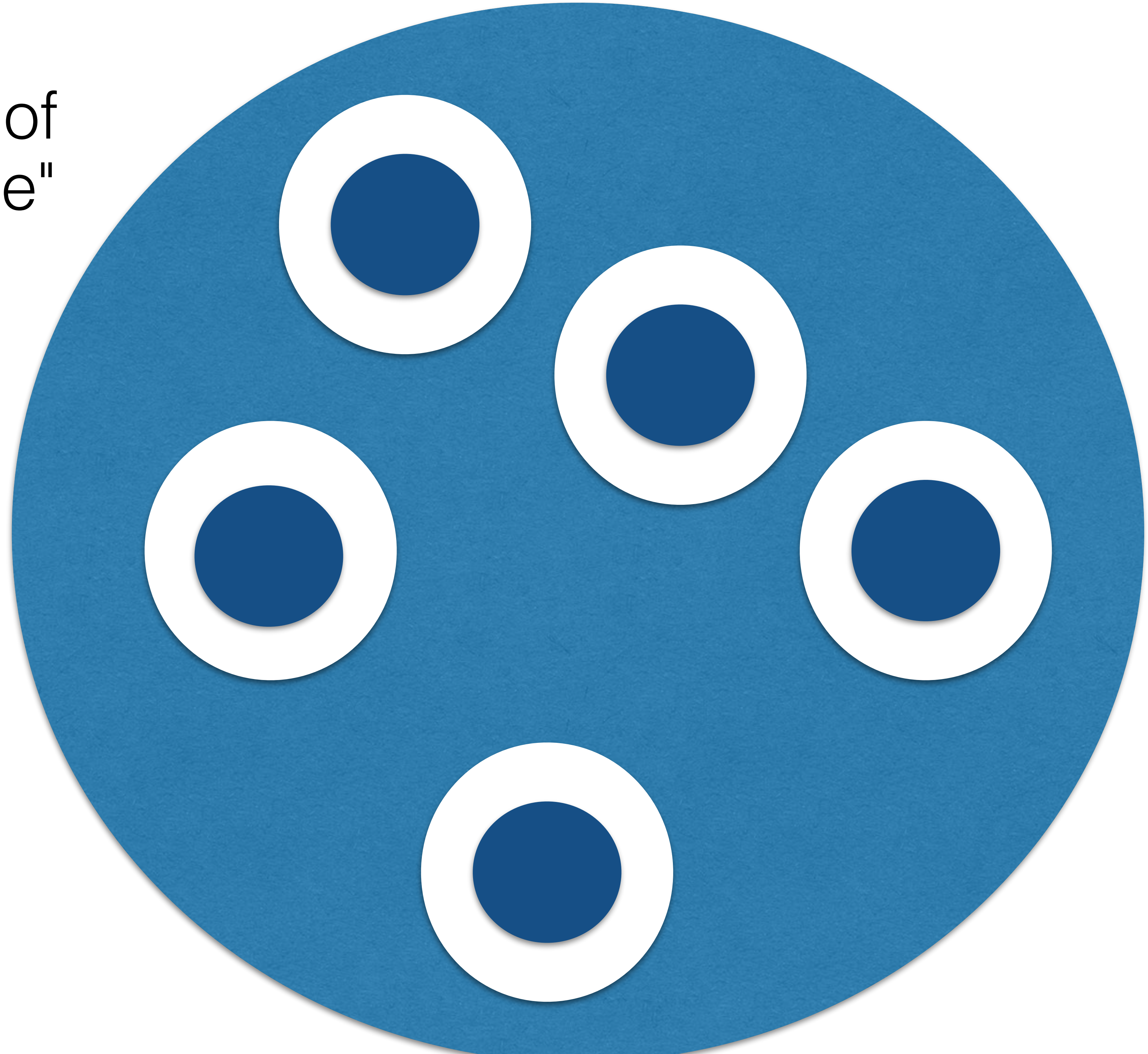
- Averaging of Einstein equations: **G** = **T**
- FRW: metric **g** -> **G** and **T** = diag(ρ, P, P, P) are diagonal
  - **G** = **T** and **∇.T** = 0 -> Friedmann equations
- with inhomogeneity < **G** > = < **T** >?
  - *"averaging problem"* widely discussed in BR literature
- what about internal pressure P of clusters?
  - or internal pressure in stars, other compact objects
- Do those give Friedmann equations with non-zero P?
  - and hence deviation from Newtonian expansion law?

Averaging of Einstein equations: $< \mathbf{G} > = < \mathbf{T} >$?

- Consider e.g. <u>stars</u> with internal pressure P
  - does that give Friedmann equations with non-zero P?
- No. Stars have Schwarzschild exterior with mass m
    - space integral of the <u>stress pseudo-tensor</u>
      - includes rest mass, motions, P, binding energy
      - but is independent of time
- Conservation of stars implies $\rho \sim a^{-3}$
  - which demands P = 0 in the Friedmann equations

Relativistic BR from large-scale structure?

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- Interesting pert^n to e.g. proper mass
- but background expansion is exactly unperturbed
- small effects on D(z)



Backreaction from inter-galactic pressure

- Stars & DM ejected from galaxies by merging SMBHs
  - intergalactic pressure $P = n\, m\, \sigma_v^2$
  - and P in the background of GWs emitted
- Homogeneous (in conformal coords) pressure is a <u>flux of energy</u> with non-zero divergence in real space
  - 1st law ... PdV work .... : $\rho' = -(\rho + P/c^2)\, V' / V$
    - but a <u>very small effect</u>
- relies on pressure being extended throughout space
  - no effect from internal pressure in bound systems that are surrounded by empty space

Summary

- A different perspective on the DZ equations. There is no dynamical equation for $a(t)$. $a(t)$ is arbitrary. But there is no freedom to modify F-equation w/o changing structure eqs. Conventional system of equations is exact.

- Clarification of "generalised Friedmann equation". Periodic BCs is not the issue. $|Q_1| \sim \langle v^2 \rangle / r^2$ and $\langle Q_1 \rangle = 0$ (Monin and Iaglom). $\langle Q_2 \rangle \sim \langle v^2 \rangle \lambda^2 / r^4$. Both are v. small and tend to zero for large r.

- Discussion of relativistic backreaction. Averaging of stress-energy for systems with internal pressure does not introduce non-zero P in Freidmann equations. Exact non-linear solutions show no backreaction. Intergalactic P does backreact, but P is weak and positive.