Do cosmological data rule out $f(R)$ with $w \neq -1$?

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Outline

Why modify gravity

- ΛCDM model works well, but
- Naturalness and coincidence problem
- Based on General Relativity
- Will the ΛCDM model survive with more accurate data?

How can we modify General Relativity

Lovelock's Theorem

- Uniqueness of Einstein Tensor in 4D
  - Symmetric
  - Divergenceless
  - Linear function of the second derivatives of the metric tensor

Evade Lovelock's theorem

- Extra fields
  - Scalar-Tensor gravity
  - Vector-Tensor gravity
  - Tensor-Tensor gravity
  - Scalar-Vector-Tensor gravity
- Higher dimensions
- Higher derivatives
- Massive gravity
- Non-locality (inverse d'Alembertian)
- Lovelock's gravity
- ...

Horndeski scalar-tensor action

- Most general model with second order equations of motion
- It depends on four free functions
- It encompasses many models studied in literature

f(R) model

$$\mathcal{L} = R + f(R)$$

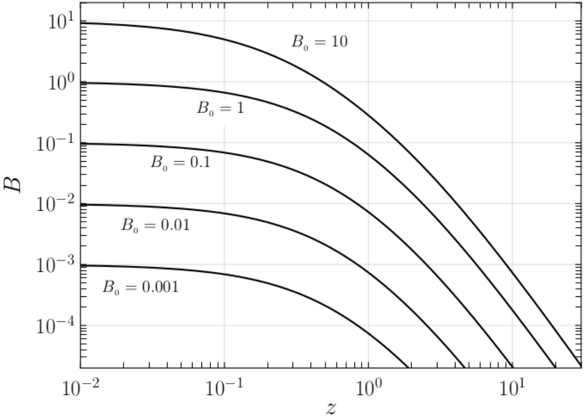
- Simple modifications of gravity (tensor sector)
- Widely studied in literature
- Not affected by gravitational waves constraints
- Need of Chameleon screening mechanism to be viable in the solar system
- Strong constraints from the solar system

Designer f(R) models

- Many functional forms proposed
- Background equation of state $w_{\rm de}$ usually assumed to be -1, but not necessarily the case
- Use designer model: specify $w_{\rm de}$ and reconstruct $f(R)$
- $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \boxed{U_{\mu\nu}} \rightarrow$ "dark energy contribution"

$$f_R'' - 2\mathcal{H} f_R' + 2\left(\mathcal{H}' - \mathcal{H}^2\right) f_R = 0 \Leftrightarrow w_{\rm de} = -1$$

Background cosmology



$$B = \frac{H f_{RR} R'}{(1+f_R) H'}$$

$B_0 \approx 10^{-2}$ (CMB)

$B_0 \approx 10^{-6} - 10^{-5}$ (LSS, Solar System)

Battye, Bolliet, **FP**, 2017

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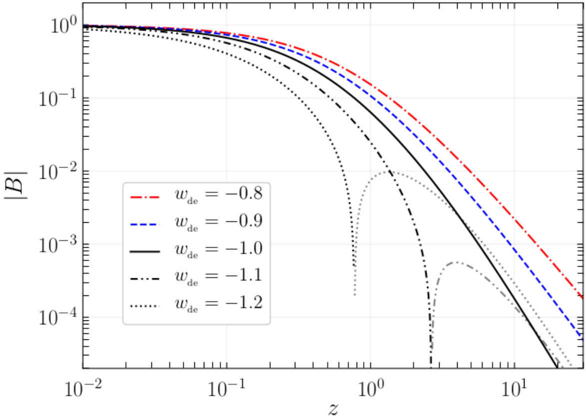

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The Equation of State (EoS) approach

- At the level of background cosmology all dark energy models are specified by the equation-of-state parameter $w_{\rm de}(a)$
- Is there an equivalent for linear perturbations?
- Yes, and we can identify gauge invariant quantities for scalar, vector and tensor perturbations
- For scalars one finds the entropy perturbation ($w_{\rm de}\Gamma_{\rm de}$) and anisotropic stress $w_{\rm de}\Pi^{\rm S}_{\rm de}$
- In the fluid language, we are specifying its different properties
- $w_{\rm de}\Gamma_{\rm de} = \left(\frac{\delta P}{\delta\rho} - \frac{dP}{d\rho}\right)\delta$, $w_{\rm de}\Pi^{\rm S}_{\rm de} \neq 0 \longleftrightarrow \Phi \neq \Psi$

EoS approach

- The EoS approach eliminates the new degrees of freedom induced by the modified gravity theory
- This is done via the expressions for the perturbed fluid variables
- Perturbations are a function of scale and time

$$w_{\rm de}\Gamma_{\rm de} = C_{\Gamma_{\rm de}\Delta_{\rm de}}\Delta_{\rm de} + C_{\Gamma_{\rm de}\Theta_{\rm de}}\Theta_{\rm de} + C_{\Gamma_{\rm de}\Delta_{\rm m}}\Delta_{\rm m} + C_{\Gamma_{\rm de}\Theta_{\rm m}}\Theta_{\rm m} + C_{\Gamma_{\rm de}\Gamma_{\rm m}}\Gamma_{\rm m}$$
$$w_{\rm de}\Pi^{\rm S}_{\rm de} = C_{\Pi^{\rm S}_{\rm de}\Delta_{\rm de}}\Delta_{\rm de} + C_{\Pi^{\rm S}_{\rm de}\Theta_{\rm de}}\Theta_{\rm de} + C_{\Pi^{\rm S}_{\rm de}\Delta_{\rm m}}\Delta_{\rm m} + C_{\Pi^{\rm S}_{\rm de}\Theta_{\rm m}}\Theta_{\rm m} + C_{\Pi^{\rm S}_{\rm de}\Pi^{\rm S}_{\rm m}}\Pi^{\rm S}_{\rm m}$$

Linear Perturbations

Equations of state

$$w_{\rm de}\Pi^S_{\rm de} = \Delta_{\rm de}$$
$$w_{\rm de}\Gamma_{\rm de} = \left\{\frac{1}{3} - w_{\rm de} + \frac{\mathrm{M}^2}{\mathrm{K}^2}\right\}\Delta_{\rm de} + \frac{1}{3}\frac{\Omega_{\rm m}}{\Omega_{\rm de}}\Delta_{\rm m}$$

Growth

$$\ddot{\Delta}_{\rm m} + 2H\dot{\Delta}_{\rm m} - \frac{3}{2}H^2\Omega_{\rm m}\Delta_{\rm m} = -\frac{3}{2}\Omega_{\rm de}\Delta_{\rm de}$$
$$\ddot{\Delta}_{\rm de} + 2H\dot{\Delta}_{\rm de} + (\mathrm{K}^2 + \mathrm{M}^2)H^2\Delta_{\rm de} = -\frac{1}{3}\frac{\Omega_{\rm m}}{\Omega_{\rm de}}H^2\mathrm{K}^2\Delta_{\rm m}$$

$G_{\rm eff}$

$$\Omega_{\rm de}\Delta_{\rm de} = -\frac{1}{3}\frac{\mathrm{K}^2}{\mathrm{K}^2 + \mathrm{M}^2}\Omega_{\rm m}\Delta_{\rm m} \Longrightarrow \ddot{\Delta}_{\rm m} + 2H\dot{\Delta}_{\rm m} - \frac{3}{2}\frac{4\mathrm{K}^2 + 3\mathrm{M}^2}{3\mathrm{K}^2 + 3\mathrm{M}^2}H^2\Omega_{\rm m}\Delta_{\rm m} = 0$$

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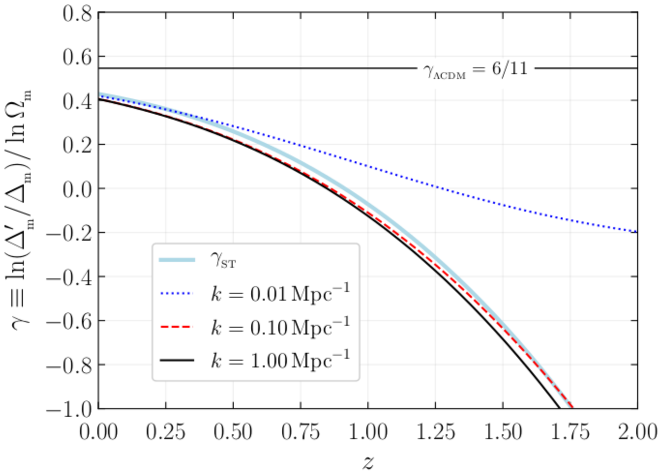
Growth index



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$$\gamma' + \frac{3w_{\rm de}\Omega_{\rm de}}{\ln\Omega_{\rm m}}\gamma + \frac{\Omega_{\rm m}^{\gamma}}{\ln\Omega_{\rm m}} - \frac{3\Omega_{\rm m}^{1-\gamma}}{2\ln\Omega_{\rm m}}\varepsilon = \frac{3w_{\rm de}\Omega_{\rm de} - 1}{2\ln\Omega_{\rm m}}$$

$$\Downarrow \quad \Omega_{\rm de} \ll 1$$

$$\gamma' + \left(1 - 3w_{\rm de} + \frac{3}{2}\varepsilon\right)\gamma = \frac{3}{2}\left(\frac{1-\varepsilon}{\Omega_{\rm de}} + \varepsilon - w_{\rm de}\right)$$

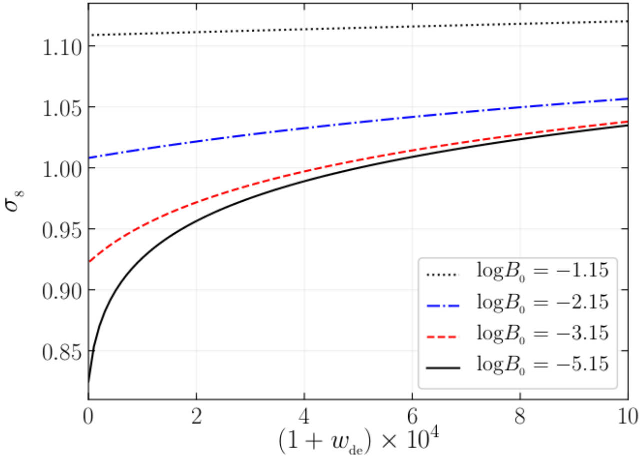
$$\Downarrow$$

$$\gamma = \frac{3(1-\varepsilon)}{2+3\varepsilon}\frac{\Omega_{\rm m,0}}{\Omega_{\rm de,0}}(1+z)^{-3w_{\rm de}} + \frac{3(\varepsilon - w_{\rm de})}{2+3\varepsilon-6w_{\rm de}}$$

$$\Downarrow$$

$\gamma_{\rm ST} < \gamma_{\rm wCDM} \longrightarrow$ correlation between $w_{\rm de}$ and σ_8

σ_8



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Conclusions: EoS

- The EoS formalism is a powerful tool to study linear perturbation theory
- It is easy to implement in an Einstein-Boltzmann code
- Proper generation of ICs for generic models

Conclusions: f(R)

- It evades GW constraints
- Instabilities for $w_{\rm de} < -1$
- Correlation between $w_{\rm de}$ and $\sigma_8 \longrightarrow$ more clustering for less negative $w_{\rm de}$
- $1 + w_{\rm de} > 0.002$ disfavoured at 95% CL
- Ruled out for $w_{\rm de} \neq -1$