Do cosmological data rule out $f(R)$ with $w \neq -1$?

Francesco Pace

JBCA, University of Manchester

30$^{th}$ May 2018, CosmoBack, Marseille

arXiv:1712.05976
Outline

1. Why modify gravity
2. How can we modify General Relativity
3. f(R) models
4. The EoS approach
5. Conclusions
Why modify gravity

- $\Lambda$CDM model works well, but
- Naturalness and coincidence problem
- Based on General Relativity
- Will the $\Lambda$CDM model survive with more accurate data?
How can we modify General Relativity

Lovelock’s Theorem

- Uniqueness of Einstein Tensor in 4D
  - Symmetric
  - Divergenceless
  - Linear function of the second derivatives of the metric tensor
### Evade Lovelock’s theorem

- Extra fields
  - Scalar-Tensor gravity
  - Vector-Tensor gravity
  - Tensor-Tensor gravity
  - Scalar-Vector-Tensor gravity
- Higher dimensions
- Higher derivatives
- Massive gravity
- Non-locality (inverse d’Alembertian)
- Lovelock’s gravity
- ...
Horndeski scalar-tensor action

- Most general model with second order equations of motion
- It depends on four free functions
- It encompasses many models studied in literature
f(R) model

\[ \mathcal{L} = R + f(R) \]

- Simple modifications of gravity (tensor sector)
- Widely studied in literature
- Not affected by gravitational waves constraints
- Need of Chameleon screening mechanism to be viable in the solar system
- Strong constraints from the solar system
Many functional forms proposed

- Background equation of state $w_{de}$ usually assumed to be -1, but not necessarily the case
- Use designer model: specify $w_{de}$ and reconstruct $f(R)$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + U_{\mu\nu} \rightarrow "dark energy contribution"$$

$$f''_R - 2\mathcal{H} f'_R + 2\left(\mathcal{H}' - \mathcal{H}^2\right) f_R = 0 \iff w_{de} = -1$$
Background cosmology

\[ B = \frac{H f_{RR} R'}{(1+f_R)H'} \]

- \( B_0 \approx 10^{-2} \) (CMB)
- \( B_0 \approx 10^{-6} - 10^{-5} \) (LSS, Solar System)
Background cosmology

\[ B = \frac{H f_{RR} R'}{(1+f_R)H'} \]

\[ B_0 \approx 10^{-2} \text{ (CMB)} \]

\[ B_0 \approx 10^{-6} - 10^{-5} \text{ (LSS, Solar System)} \]
The Equation of State (EoS) approach

- At the level of background cosmology all dark energy models are specified by the equation-of-state parameter $w_{de}(a)$
- Is there an equivalent for linear perturbations?
- Yes, and we can identify gauge invariant quantities for scalar, vector and tensor perturbations
- For scalars one finds the entropy perturbation ($w_{de}\Gamma_{de}$) and anisotropic stress $w_{de}\Pi_{de}^S$
- In the fluid language, we are specifying its different properties
- $w_{de}\Gamma_{de} = \left( \frac{\delta P}{\delta \rho} - \frac{dP}{d\rho} \right)\delta$, $w_{de}\Pi_{de}^S \neq 0 \iff \Phi \neq \Psi$
EoS approach

- The EoS approach eliminates the new degrees of freedom induced by the modified gravity theory.
- This is done via the expressions for the perturbed fluid variables.
- Perturbations are a function of scale and time.

\[
\begin{align*}
\omega_{\text{de}} \Gamma_{\text{de}} &= C_{\Gamma_{\text{de}} \Delta_{\text{de}}} \Delta_{\text{de}} + C_{\Gamma_{\text{de}} \Theta_{\text{de}}} \Theta_{\text{de}} + C_{\Gamma_{\text{de}} \Delta_{\text{m}}} \Delta_{\text{m}} + C_{\Gamma_{\text{de}} \Theta_{\text{m}}} \Theta_{\text{m}} + C_{\Gamma_{\text{de}} \Gamma_{\text{m}}} \Gamma_{\text{m}} \\
\omega_{\text{de}} \Pi_{\text{de}}^S &= C_{\Pi_{\text{de}}^S \Delta_{\text{de}}} \Delta_{\text{de}} + C_{\Pi_{\text{de}}^S \Theta_{\text{de}}} \Theta_{\text{de}} + C_{\Pi_{\text{de}}^S \Delta_{\text{m}}} \Delta_{\text{m}} + C_{\Pi_{\text{de}}^S \Theta_{\text{m}}} \Theta_{\text{m}} + C_{\Pi_{\text{de}}^S \Pi_{\text{m}}^S} \Pi_{\text{m}}^S
\end{align*}
\]
### Equations of state

\[ w_{de} \Pi_{de}^S = \Delta_{de} \]
\[ w_{de} \Gamma_{de} = \left\{ \frac{1}{3} - w_{de} + \frac{M^2}{K^2} \right\} \Delta_{de} + \frac{1}{3} \frac{\Omega_m}{\Omega_{de}} \Delta_m \]

### Growth

\[ \ddot{\Delta}_m + 2H \dot{\Delta}_m - \frac{3}{2} H^2 \Omega_m \Delta_m = -\frac{3}{2} \frac{\Omega_{de}}{\Omega_{de}} \Delta_{de} \]
\[ \ddot{\Delta}_{de} + 2H \dot{\Delta}_{de} + (K^2 + M^2)H^2 \Delta_{de} = -\frac{1}{3} \frac{\Omega_m}{\Omega_{de}} H^2 K^2 \Delta_m \]

### $G_{\text{eff}}$

\[ \Omega_{de} \Delta_{de} = -\frac{1}{3} \frac{K^2}{K^2 + M^2} \Omega_m \Delta_m \Rightarrow \ddot{\Delta}_m + 2H \dot{\Delta}_m - \frac{3}{2} \frac{4K^2 + 3M^2}{3K^2 + 3M^2} H^2 \Omega_m \Delta_m = 0 \]
Growth index

\[ \gamma \equiv \frac{\ln(\Delta'_m/\Delta_m)}{\ln \Omega_m} \]

\[ \gamma_{\Lambda CDM} = \frac{6}{11} \]

- \(\gamma_{ST}\)
- \(k = 0.01\text{ Mpc}^{-1}\)
- \(k = 0.10\text{ Mpc}^{-1}\)
- \(k = 1.00\text{ Mpc}^{-1}\)
\[
\gamma' + \frac{3w_{de} \Omega_{de}}{\ln \Omega_m} \gamma + \frac{\Omega_m^\gamma}{\ln \Omega_m} - \frac{3 \Omega_m^{1-\gamma}}{2 \ln \Omega_m} \varepsilon = \frac{3w_{de} \Omega_{de} - 1}{2 \ln \Omega_m}
\]

\[ \downarrow \quad \Omega_{de} \ll 1 \]

\[
\gamma' + \left(1 - 3w_{de} + \frac{3}{2} \varepsilon\right) \gamma = \frac{3}{2} \left(\frac{1 - \varepsilon}{\Omega_{de}} + \varepsilon - w_{de}\right)
\]

\[ \downarrow \]

\[
\gamma = \frac{3(1 - \varepsilon)}{2 + 3\varepsilon} \frac{\Omega_{m,0}}{\Omega_{de,0}} (1 + z)^{-3w_{de}} + \frac{3(\varepsilon - w_{de})}{2 + 3\varepsilon - 6w_{de}}
\]

\[ \downarrow \]

\[ \gamma_{ST} < \gamma_{wCDM} \rightarrow \text{correlation between } w_{de} \text{ and } \sigma_8 \]
Why modify gravity

How can we modify General Relativity

f(R) models

The EoS approach

Conclusions

Battye, Bolliet, FP, 2017
Conclusions: EoS

- The EoS formalism is a powerful tool to study linear perturbation theory
- It is easy to implement in an Einstein-Boltzmann code
- Proper generation of ICs for generic models
Conclusions: $f(R)$

- It evades GW constraints
- Instabilities for $w_{de} < -1$
- Correlation between $w_{de}$ and $\sigma_8 \rightarrow$ more clustering for less negative $w_{de}$
- $1 + w_{de} > 0.002$ disfavoured at 95% CL
- Ruled out for $w_{de} \neq -1$